A Game-Theoretic Framework for Control of Distributed Renewable-Based Energy Resources in Smart Grids

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Abstract— **Renewable energy plays an important role in distributed energy resources in smart grid systems. Deployment and integration of renewable energy resources require an intelligent management to optimize their usage in the current power grid. In this paper, we establish a game-theoretic framework for modeling the strategic behavior of buses that are connected to renewable energy resources and study the equilibrium distributed power generation at each bus. Our framework takes a cross-layer approach, taking into account the economic factors as well as system stability issues at each bus. We propose an iterative algorithm to compute Nash equilibrium solutions based on a sequence of linearized games. Simulations and numerical examples are used to illustrate the algorithm and corroborate the results.**

I. Introduction

The Smart Grid (SG) initiative aims to modernize the electricity transmission and distribution system to maintain a reliable and secure electricity infrastructure that can meet future demand growth [1]. One of the goals of the SG initiative is to deploy and integrate distributed resources and generation into the current power grid, which can increase system reliability and energy efficiency. Renewable energy plays an essential role in distributed energy resources. Renewable energy sources such as wind, solar energy and geothermal heat can be exploited to supply power through PV arrays, wind turbines, fuel cells, and natural gas reciprocating engines, etc. However, their different physical characteristics as well as relatively high cost will require intelligent management systems to optimize their usage in the current power grid [2].

In this paper, we consider a decentralized energy management scheme to manage the distributed generation of renewable energies in the SG. The grid is conventionally operated in a centralized manner, where the planning of power generation is made to optimize the social welfare in the grid, and the real-time grid operation is monitored and controlled with SCADA supervisory scheme. With deregulation, independent power providers can enter the electricity market and sell cheap power to the grid, and the deployment of distributed renewable energy resources are believed to be able to increase system reliability and efficiency. In addition, climate change concerns, together with high oil prices and government support, are driving increasing renewable energy legislation, incentives and commercialization.

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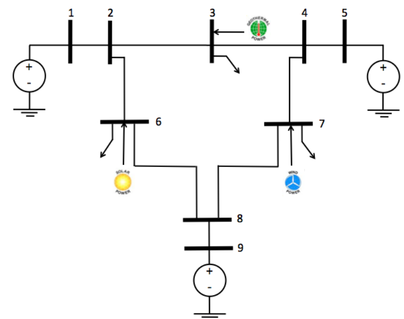

Fig. 1. 9-Bus power system example: buses 3, 6, and 7 are connected to renewable resources that can generate geothermal, solar and wind power, respectively.

Motivated by this, we propose a game-theoretic framework which integrates distributed renewable-based energy resources as distributed resources into power grids. Fig. 1 depicts a 9-bus power system as an example for the integration of renewable resources. Buses 1 and 5 are conventional power plant generators that are used to serve loads in the system. Buses 3, 6, and 7 are connected to renewable energy resources that can generate geothermal, solar, and wind power, respectively. Different from conventional load buses, buses 3, 6, and 7 are capable of their own independent energy generation, which can be used to provision power to their loads or sell it to the power market. Since the decisions of each bus are made independently based on many physical and economic factors such as the load requirement, cost of production and electricity market price, we use a game-theoretic framework to understand the strategic behavior in the multi-agent power grid from a cross-layer perspective. The goal of such a framework is to design a control and automated energy management system for renewable energies in the SG.

Game-theoretic tools have been used to study multi-agent systems such as congestion control in the Internet [7], [8], security and privacy issues in wireless communications [6], mechanism design in pricing issues [9], etc. As the numbers of intelligent devices, agents and players in power grids grow, game-theoretic models become important for understanding strategic interactions among agents that are coupled across different layers in the power grid. Our work is related to [3], [4] and [5]. In [3], a scheme that uses intelligent agents is implemented to relieve line overloads by controlling certain loads in the grid. In addition, a decentralized optimization algorithm is presented to minimize power losses in the distribution network. In [4], a game-theoretic approach is used to control the decision process of individual sources and loads in small-scale and DC power systems. In [5], a differential game-theoretic approach is proposed to provide decentralized control of energy resources in a shipboard power system.

In this paper, we establish a game-theoretic framework to model the operations planning of renewable resources in an AC power system. The optimal decisions made by each bus takes into account economic factors of renewable energy generation and the system stability conditions, as well as the physical AC power flow constraints. Our approach differs from the classical optimal power flow formulation in many aspects [14]–[16]. Firstly, we consider the decentralized voltage angle and magnitude regulation together with economic factors at each bus. Secondly, we use Nash equilibrium as our solution concept instead of optimal power flow, and propose an iterative algorithm to compute the equilibrium power generation strategies.

The rest of the paper is organized as follows. Section II presents a game-theoretic framework among buses that are connected to renewable resources. We also propose metrics such as price of anarchy (PoA) and price of stability (PoS) to measure the loss of efficiency between the equilibrium solution of the game model and its counterpart of the system-wide optimization problem. In Section III, we propose an iterative algorithm based on a sequence of linearized games to compute the Nash equilibrium solution of the game. Finally, conclusions and future work are presented in Section V.

II. System Model

In this section, we propose a game-theoretic framework to model the independent decision making of generators and loads on renewable energy generation. Let $\mathscr{N} := \{s,1,2,\cdots,N\}$ be the set of $N+1$ buses in a power system network, where bus s is the *slack* bus. The slack bus is an arbitrarily selected generator bus whose voltage magnitude V_s and voltage phase angle θ_s are known. Without loss of generality, we can set $V_s = 1$p.u. and $\theta_s = 0$.

The power system is composed of load buses and generator buses. Denote by $\mathscr{N}_d := \{1,2,\cdots,N_d\} \subseteq \mathscr{N}$ the set of $N_d \leqslant N$ buses that are capable of generating and provisioning electricity for themselves independently using renewable resources such as solar, wind and geothermal energies. Buses in the set $\mathscr{N}_d$ can generate power to meet their own need of energy. We let $\mathscr{N}_g := \{N_d+1, N_d+2,\cdots,N\} = \mathscr{N}\backslash\mathscr{N}_d$ denote a set of N_g buses consisting of buses that are either PQ-loads or generators. Note that $N_g + N_d = N$. Each bus $i\in\mathscr{N}$ has its specified load (P_i^l, Q_i^l) if i is a PQ-load bus, and it has its pre-determined generation power (P_i, Q_i) if it is a generation bus. Each bus $i\in\mathscr{N}_d$ has its specified load (P_i^l, Q_i^l) to be serviced and its power (P_i, Q_i) to be determined. For convenience, we can by default set PQ-load bus i's generation power as $P_i = 0, Q_i = 0$, and likewise, we set generation bus i's load power as $P_i^l = 0, Q_i^l = 0$.

Each bus $i\in\mathscr{N}_d$ can also buy an amount of power P_i from utility companies, sell and inject power into the power network, and at the same time, aim to regulate its bus voltage magnitude $V_i \in \mathscr{V} \subset \mathbb{R}_+$ to a nominal level $\hat{V}_i$, and to keep the voltage angle $\theta_i \in [0,\pi]$ close to some reference angle $\bar{\theta}$. The regulation of angles can provide some certainty that the system will be small-signal stable.

Each bus $i\in\mathscr{N}_d$ decides on the amount of power $P_i \in \mathscr{P}_i \subset \mathbb{R}_+$ to generate from its renewable resources at a cost $c_i \in \mathbb{R}_+$. We can represent the goal of bus $i\in\mathscr{N}_d$ by its cost function $U_i : \mathscr{P}_i \times \mathbb{R}_+ \times [0,\pi] \to \mathbb{R}$ given by

$$U_i(P_i, V_i, \theta_i) =$$
$$c_iP_i + \alpha(P_i^l - P_i) + \frac{1}{2}\gamma_i^1(V_i - \hat{V}_i)^2 + \frac{1}{2}\gamma_i^2(\theta_i - \bar{\theta})^2,\ i\in\mathscr{N}_d, \tag{1}$$

where $\gamma_i^1, \gamma_i^2 \in \mathbb{R}_+$ are weighting parameters that indicate the importance or priority of the regulations of voltage magnitude and voltage angle, respectively. The term $P_i^l - P_i$ denotes the power bought or sold by bus $i\in\mathscr{N}_d$ from the power grid at the unit cost $\alpha\in\mathbb{R}$. It can be either positive or negative. When it is positive, it means that bus i buys power from the network, and when it is negative, it means that bus i sells power to the network. The unit cost α is determined by the power market, which depends on the total supply and demand in a wide-area power network. Here, we assume that all the b uses are price-taking entities and α is a given cost parameter. The power generation $P_i, i\in\mathscr{N}$, needs to satisfy the constraints

$$0 \leqslant P_i \leqslant P_{i,\max}, \tag{2}$$

where $P_{i,\max}$ is the maximum power that can be generated by bus i. $\hat{V}_i$ is the nominal voltage level that bus i attempts to regulate at and V_i is the voltage at bus i. We can take $\hat{V}_i = 1$p.u. for all $i\in\mathscr{N}_d$ where we adopt normalized units. The power flow in the power network obeys Kirchoff laws. The power P_i, Q_i, voltage V_i and voltage angle θ_i of every bus $i\in\mathscr{N}$ need to satisfy the power flow equations with reference to the slack bus 0 as follows:

$$P_i = \sum_{j\in\mathscr{H}_i} V_iV_j[G_{ij}\cos(\theta_i-\theta_j) + B_{ij}\sin(\theta_i-\theta_j)], \tag{3}$$
$$Q_i = \sum_{j\in\mathscr{H}_i} V_iV_j[G_{ij}\sin(\theta_i-\theta_j) + B_{ij}\cos(\theta_i-\theta_j)], \tag{4}$$
$$i,j\in\mathscr{N},$$

where $\mathscr{H}_i \subset \mathscr{N}$ is a subset of $N+1$ buses that are directly connected to bus i, $G_{ij}, B_{ij}, i,j\in\mathscr{N}$, are the real part and the imaginary part of the entry (i,j) in the bus admittance matrix $\mathbf{Y}\in\mathbb{C}^{N\times N}$, respectively. The matrix $\mathbf{Y}$ is Hermitian and it is a function of line admittances and shunt load resistance [17]. We can write $\mathbf{Y} = \mathbf{G} + \hat{j}\mathbf{B}$, where $\hat{j} = \sqrt{-1}$, $\mathbf{G}\in\mathbb{R}^{N\times N}$ is the conductance matrix and $\mathbf{B}\in\mathbb{R}^{N\times N}$ is the susceptance matrix. In lossless transmission lines, $\mathbf{G} = 0$. Note that (3) and (4) constitute a system of $2(N+1)$ nonlinear equality constraints. Since the voltage magnitude and angle are known, the nonlinear system can be reduced to a set of $2N$ equations.

We make the following assumptions before we formulate the game-theoretic framework.

(A1) Each bus $i \in \mathcal{N}_d$ can only directly control its real power P_i.

(A2) The generations of reactive power $Q_i, i \in \mathcal{N}_d$, are close to 0 and hence negligible.

(A3) There exists a solution to the power flow equations (3) and (4) when N_d buses all act as loads.

(A4) Each bus is aware of the physical constraints when making decisions.

(A5) Each bus has a high-quality forecast mechanism for the renewable resources connected to the bus.

The assumptions made above are justifiable. According to recent IEEE Standard 1547 [2], distributed resources are not required to absorb and supply reactive power and hence they will not be capable of active voltage regulation. Implementation of renewable sources under assumptions (A1) and (A2) abides by the IEEE Standard 1547 voltage requirement. Note that the loads at the bus can still consume reactive power, i.e., $P_i^l \neq 0,\ i \in \mathcal{N}_d$.

Assumption (A3) suggests that a solution to the power flow equations exists when no bus in the set $\mathcal{N}_d$ produces renewable energies, and all loads of N_d buses need to be provisioned by the power network. This assumption ensures that the power supply in the network can meet the maximum load demand, and hence no blackout can occur in this case. Under this assumption, the power grid system will not incur a blackout when buses start to provision power for themselves, which, from the perspective of engineering practice, leads to the existence of solution to the power flow equations (3) and (4) with renewable energy sources.

Assumption (A4) states that each bus makes decisions by taking the coupled constraints into account, or in other words, the power flow constraints (3) and (4) are common knowledge to all buses in $\mathcal{N}_d$. This is reasonable when all agents are in their planning stage and no sudden topological changes occur in the system. Assumption (A4) can be lifted if we consider an asymmetry in the knowledge of planners. Assumption (A5) allows each bus to acquire high confidence estimation of the maximum amount of renewable energies that can be generated. Hence, we can view $P_{i,\max}, i \in \mathcal{N}_d$, as deterministic quantities instead of a random variable.

We adopt the following vector notations for convenience: $\mathbf{V} := [V_1, V_2, \cdots, V_N]' \in \mathbb{R}^N$, $\mathbf{P} := [P_1, P_2, \cdots, P_N]' \in \mathbb{R}^N$, $\mathbf{Q} := [Q_1, Q_2, \cdots, Q_N]' \in \mathbb{R}^N$, $\theta = [\theta_1, \theta_2, \cdots, \theta_N]' \in \mathbb{R}^N$; $\mathbf{V}_d := [V_1, V_2, \cdots, V_{N_d}]' \in \mathbb{R}^{N_d}$, $\mathbf{P}_d := [P_1, P_2, \cdots, P_{N_d}]' \in \mathbb{R}^{N_d}$, $\mathbf{Q}_d := [Q_1, Q_2, \cdots, Q_{N_d}]' \in \mathbb{R}^{N_d}$, $\theta_d = [\theta_1, \theta_2, \cdots, \theta_{N_d}]' \in \mathbb{R}^{N_d}$.

A. Game-Theoretic Framework

In a decentralized smart grid system, each bus $i \in \mathcal{N}_d$ minimizes its cost function (1) independently subject to the constraints (2), (3) and (4), which leads to a noncooperative game with coupled constraints among N_d buses [10].

Let $\Xi := \langle \mathcal{N}_d, \{\mathscr{P}_i, \Theta_i, \mathscr{V}_i\}_{i \in \mathcal{N}_d}, \{U_i\}_{i \in \mathcal{N}_d}, \mathscr{P} \rangle$ be the continuous-kernel strategic game with a set $\mathcal{N}_d$ of N_d players (or buses). $\{\mathscr{P}_i, \Theta_i, \mathscr{V}_i\}$ is the action set of each player i, where

$$\mathscr{P}_i := \{P_i \in \mathbb{R}_+ : 0 \leqslant P_i \leqslant P_{i,\max}\};$$

$\mathscr{P}$ denotes the feasible set defined by a set of coupled constraints (3) and (4). The feasible set of game outcomes is given by $\mathscr{P}_F := \left(\otimes_{i \in \mathcal{N}_d} \mathscr{P}_i\right) \cap \mathscr{P}$. Note that the utility functions of buses U_i in (1) are independent of each each. However, the game is coupled through the constraints given by the power flow equations (3) and (4). In addition, from (A3), for every fixed $\mathbf{P}_d$, there exists a solution to the power flow equations. Hence, $\theta_d, \mathbf{V}_d$ can be seen as dependent on $\mathbf{P}_d$. Therefore, to indicate the direct dependence on $\mathbf{P}_d$ and to suppress the dependence on θ_d and $\mathbf{V}_d$, we use the notation $U_i(P_i, P_{-i}), i \in \mathcal{N}_d$, where $P_{-i} := [P_1, \cdots, P_{i-1}, P_{i+1}, \cdots, P_{N_d}]'$, to emphasize the coupling of bus i's utility with the power generation of other buses.

The game Ξ falls into a class of constrained games, which has been previously studied in [10], [11], [12]. A Nash equilibrium (NE) solution $\mathbf{P}_d^*$ of the game Ξ is a point where no players can benefit from deviating from it. A precise definition of NE is provided below.

Definition 1 (Nash Equilibrium, [10], [11]): The set of power generation profile $\mathbf{P}_d^*$ for the set of renewable resources $\mathcal{N}_d$ constitutes a *Nash equilibrium* (NE) point if

$$U_i(P_i^*, P_{-i}^*) \leqslant U_i(P_i, P_{-i}^*), \forall P_i \in \Omega_i(P_{-i}^*), i \in \mathcal{N}_d, \quad (5)$$

where $\Omega_i(P_{-i}^*)$ is the *projected* constraint set $\Omega_i(P_{-i}^*) = \{P_i \in \mathscr{P}_i : (P_i, P_{-i}^*) \in \mathscr{P}_F\}$.

B. System-Wide Optimization

The non-cooperative game Ξ formulated in Section II-A describes a scenario where every bus makes independent decisions without centralized coordination. An opposite scenario is where the power system is planned and coordinated in a centralized manner by solving a system-wide optimization problem. Let $U_S : \prod_{i=1}^{N_d} \mathscr{P}_i \times \prod_{i=1}^{N_d} \Theta_i \times \prod_{i=1}^{N_d} \mathscr{V}_i \to \mathbb{R}$ be the aggregate utility of the power system, given by $U_s(\mathbf{z}_d) = \sum_{i=1}^{d} \beta_i U_i(z_i)$, where $z_i = (P_i, \theta_i, V_i)$, $\mathbf{z}_d := (\mathbf{P}_d, \theta_d, \mathbf{V}_d)$, $\beta_i \in [0,1], i \in \mathcal{N}_d$, are the weights on buses such that $\sum_{i=1}^{N_d} \beta_i = 1$. The system-wide optimization problem (SO) is

$$\text{(SO)} \quad \min_{\mathbf{z}_d} U_S(\mathbf{z}_d)$$
$$\mathbf{z}_d \in \mathscr{P}_F$$

From (A4), we have ensured that (SO) is feasible and admits a solution. The optimization problem (SO) fits into the classical optimal power flow framework in [13], [14]. In [15], optimization methods and algorithms for optimal power flow and dispatching problems are summarized. The following proposition connects the game Ξ and (SO).

Proposition 1: Suppose that $\mathbf{z}_d^\circ := (\mathbf{P}_d^\circ, \theta_d^\circ, \mathbf{V}_d^\circ)$ is an optimal solution to (SO). Then, it is also a local NE of the game Ξ in the neighborhood $\mathfrak{N}(\mathbf{z}_d^\circ)$ of $\mathbf{z}_d^\circ$.

Proposition 1 mainly comes from the property of separable utilities of the objective function. To compare NE with the system-wide optimum solution to (SO), we define two metrics to measure the efficiency loss due to the decentralized decision-making [18], [19].

Definition 2 (PoA, PoS, [18], [19]): Consider an N_d-player game Ξ and its associated system-wide optimization problem (SO). Given a weighting profile β, we let $U^{\circ}_{S,\beta} \neq 0$ be the aggregate cost achieved at the optimal solution $\mathbf{z}^{\circ}_{d}$. Denote by $U^{*}_{G,\beta}$ be the aggregate cost achieved under a NE solution $\mathbf{z}^{*} \in \mathscr{Z}^{*}$, where $\mathscr{Z}^{*}$ is the set of NEs of Ξ. The *price of anarchy* (PoA) for the game is defined by

$$\rho_A = \max_{\mathbf{z}\in\mathscr{Z}^*} \frac{U^*_{G,\beta}}{U^{\circ}_{S,\beta}} \tag{6}$$

as the worst-case ratio of the aggregate game cost to the system-wide cost. The *price of stability* (PoS) for the game is defined by

$$\rho_S = \min_{\mathbf{z}\in\mathscr{Z}^*} \frac{U^*_{G,\beta}}{U^{\circ}_{S,\beta}} \tag{7}$$

as the best-case ratio of the aggregate game cost to the system-wide cost.

Remark 1: From Proposition 1, we can conclude that the PoS ρ_S of the game is 1 in the neighborhood of $\mathbf{z}^{\circ}_{d}$ since a global optimal solution $\mathbf{z}^{\circ} \in \mathscr{Z}^{*}$. However, the PoA ρ_A of the game may not be 1 since the reverse statement of the proposition is not true due to lack of uniqueness. The smart grid system can suffer loss of efficiency if it operates at an equilibrium that does not coincide with the system-wide optimal solution.

III. Linearized Game and Iterative Algorithms

The utility functions of the game Ξ are convex in P_i. However, the major difficulty in finding the NE of the game is the set of nonlinear power flow constraints (3) and (4). To simplify the analysis, we can linearize the power flow equations at a given operating point $\Psi_0 := (\mathbf{P}_0, \mathbf{Q}_0, \mathbf{V}_0, \theta_0)$ and hence yield the following linear equations:

$$\begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix} = \mathbf{J}(\Psi_0) \begin{bmatrix} \Delta\mathbf{V} \\ \Delta\theta \end{bmatrix}, \tag{8}$$

where $\Delta\mathbf{P} := \mathbf{P} - \mathbf{P}_0$, $\Delta\mathbf{Q} := \mathbf{Q} - \mathbf{Q}_0$, $\Delta\mathbf{V} := \mathbf{V} - \mathbf{V}_0$, $\Delta\theta := \theta - \theta_0$ and $\mathbf{J}(\Psi_0) \in \mathbb{R}^{(N+1)\times(N+1)}$ is the Jacobian matrix

$$\mathbf{J}(\Psi_0) := \begin{bmatrix} \frac{\partial \mathbf{P}}{\partial \mathbf{V}}\big|_{\Psi_0} & \frac{\partial \mathbf{P}}{\partial \theta}\big|_{\Psi_0} \\ \frac{\partial \mathbf{Q}}{\partial \mathbf{V}}\big|_{\Psi_0} & \frac{\partial \mathbf{Q}}{\partial \theta}\big|_{\Psi_0} \end{bmatrix},$$

where $\frac{\partial \mathbf{P}}{\partial \mathbf{V}} = \left[\frac{\partial P_i}{\partial V_j}\right]_{i,j=1,2,\cdots,N}$, $\frac{\partial \mathbf{P}}{\partial \theta} = \left[\frac{\partial P_i}{\partial \theta_j}\right]_{i,j=1,2,\cdots,N}$, $\frac{\partial \mathbf{Q}}{\partial \mathbf{V}} = \left[\frac{\partial Q_i}{\partial V_j}\right]_{i,j=1,2,\cdots,N}$, $\frac{\partial \mathbf{Q}}{\partial \theta} = \left[\frac{\partial Q_i}{\partial \theta_j}\right]_{i,j=1,2,\cdots,N}$ can be analytically found as follows. Since the generation bus and load bus have their predetermined power, we can let $P_{i,0} = P_i, Q_{i,0} = Q_i$ for generation buses and let $P_{i,0} = P^l_i, Q_{i,0} = Q^l_i$ for load buses.

Assume that $\mathbf{J}(\Psi_0)$ is nonsingular and has its inverse $\mathbf{W}(\Psi_0)$. Hence

$$\begin{bmatrix} \Delta\mathbf{V} \\ \Delta\theta \end{bmatrix} = \mathbf{W} \begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix}, \tag{9}$$

where $\mathbf{W}(\Psi_0)$ can be partitioned into

$$\mathbf{W}(\Psi_0) = \begin{bmatrix} \mathbf{W}^A & \mathbf{W}^B \\ \mathbf{W}^C & \mathbf{W}^D \end{bmatrix},$$

where $\mathbf{W}^l = [w^h_{ij}]_{i,j=1,2,\cdots,N}, h = A, B, C, D$, are N-by-N real matrices, which yield

$$\Delta\mathbf{V} = \mathbf{W}^A \Delta P \tag{10}$$

and

$$\Delta\theta = \mathbf{W}^C \Delta P \tag{11}$$

when ΔQ is negligible.

Therefore, with linearized relations (10) and (11), we can find the NE of Ξ with linearized constraints [11]. Let $\Xi^l_0 := \langle \mathscr{N}_d, \{\bar{\mathscr{P}}_i\}_{i\in\mathscr{N}_d}, \{\bar{U}_i\}_{i\in\mathscr{N}_d}, \mathscr{P}^l_0\rangle$ be the linearized version of the game Ξ around operating point Ψ, where $\mathscr{P}^l_0$ denotes the feasible set defined by a set of linearized coupled constraints (10) and (11). The set $\bar{\mathscr{P}}_i$ is defined by $\bar{\mathscr{P}}_i := \{\Delta P_i \in \mathbb{R} : 0 \leqslant \Delta P_i + P_{i,0} \leqslant P_{i,\max}\}$, and the cost function $\bar{U}_i$ is defined in terms of $\Delta\mathbf{P}$.

The linearized game Ξ^l_0 can be solved by substituting (9) into (1). The objective function U_i can be rewritten in terms of $\Delta\mathbf{P}$ as follows.

$$\begin{aligned} \bar{U}_i(\Delta P_i, \Delta P_{-i}) = (c_i - \alpha)\Delta P_i + \frac{\gamma^1_i}{2}\left(\sum_{j\in\mathscr{N}} w^A_{ij}\Delta P_j + \widetilde{V}_i\right)^2 \\ + \frac{\gamma^2_i}{2}\left(\theta_{i,0} + \sum_{j\in\mathscr{N}} w^C_{ij}\Delta P_j\right)^2, \end{aligned} \tag{12}$$

where $\widetilde{V}_i = V_{i,0} - \hat{V}_i$. Note that, in (12), for buses $j \in \mathscr{N}_g$, $\Delta P_j = 0$ as their generation powers are fixed, and hence we can replace $\mathscr{N}$ with $\mathscr{N}_d$ in (12). In addition, the utility functions are convex in ΔP_i, and hence, by taking the derivative with respect to ΔP_i, we arrive at the first-order necessary and sufficient condition for optimal response of bus i to ΔP_{-i}:

$$\begin{aligned} (c_i - \alpha) + \gamma^1_i w^A_{ii}\widetilde{V}_i + \gamma^2_i w^C_{ii}\theta_{i,0} + \gamma^1_i \sum_{j\in\mathscr{N}_d} w^A_{ij} w^A_{ii}\Delta P_j \\ + \gamma^2_i \sum_{j\in\mathscr{N}_d} w^C_{ij} w^C_{ii}\Delta P_j = 0, \quad i \in \mathscr{N}_d, \end{aligned}$$

which can be written into the matrix form $\mathbf{A}_d\Delta\mathbf{P}_d = \mathbf{b}_d$, where $\xi_d := [\xi_i]_{i\in\mathscr{N}_d} \in \mathbb{R}^{N_d\times N_d}$, $\xi_i := \gamma^1_i w^A_{ii}\widetilde{V}_i + \gamma^2_i w^C_{ii}\theta_{i,0}$,

$$\mathbf{b}_d := \begin{bmatrix} \alpha - c_1 \\ \alpha - c_2 \\ \vdots \\ \alpha - c_{N_d} \end{bmatrix} + \xi_d \in \mathbb{R}^{N_d},$$

and $\mathbf{A}_d \in \mathbb{R}^{N_d\times N_d}$ is described in (13). Assume that $\mathbf{A}_d$ is nonsingular, we obtain the NE solution $\mathbf{P}^{\star} = \Delta\mathbf{P}^{\star} + \mathbf{P}_0$ for the linearized game Ξ^l_0 with reference to Ψ_0, where

$$\Delta\mathbf{P}^{\star}_d = \mathbf{A}^{-1}_d \mathbf{b}_d, \tag{14}$$

and $\Delta P^{\star}_i = 0$, for $i \in \mathscr{N}_g$. To take into account the local constraints (2), we need to project the solution $\Delta\mathbf{P}^{*}_d + \mathbf{P}_{d,0}$ onto the convex and compact set $\mathscr{P}^L_d := \{\mathbf{P}_d \in \mathbb{R}^{N_d} : 0 \leqslant P_i \leqslant P_{i,\max}, i = 1, 2, \cdots, N_d\}$, i.e, $\mathbf{P}^*_d = \mathrm{Proj}_{\mathscr{P}^L_d}(\Delta\mathbf{P}^{\star}_d + \mathbf{P}_0)$. The above results are summarized in the following proposition.

Proposition 2 (Existence and uniqueness of NE): There exists a NE of the linearized game Ξ^l_0. Furthermore, the NE is unique if $\mathbf{A}_d$ is nonsingular.

$$\mathbf{A}_d := \begin{bmatrix} \gamma_1^1 (w_{11}^A)^2 + \gamma_1^2 (w_{11}^C)^2 & \gamma_1^1 w_{11}^A w_{12}^A + \gamma_1^2 w_{11}^C w_{12}^C & \cdots & \gamma_1^1 w_{11}^A w_{1N_d}^A + \gamma_1^2 w_{11}^C w_{1N_d}^C \\ \gamma_2^1 w_{22}^A w_{21}^A + \gamma_2^2 w_{22}^C w_{21}^C & \gamma_2^1 (w_{22}^A)^2 + \gamma_2^2 (w_{22}^C)^2 & \cdots & \gamma_2^1 w_{22}^A w_{2N_d}^A + \gamma_2^2 w_{22}^C w_{2N_d}^C \\ \vdots & \ddots & \ddots & \vdots \\ \gamma_{N_d}^1 w_{N_d N_d}^A w_{N_d 1}^A + \gamma_{N_d}^2 w_{N_d N_d}^C w_{N_d 1}^C & \gamma_{N_d}^1 w_{N_d N_d}^A w_{N_d 2}^A + \gamma_{N_d}^2 w_{N_d N_d}^C w_{N_d 2}^C & \cdots & \gamma_{N_d}^1 (w_{N_d N_d}^A)^2 + \gamma_{N_d}^2 (w_{N_d N_d}^C)^2 \end{bmatrix} \quad (13)$$

A. Iterative Algorithm

In this subsection, we propose an iterative algorithm to find the NE of the game Ξ in Subsection II-A. Since the operating point Ψ_0 is chosen arbitrarily in Subsection III, we can continue to construct a linearly constrained game Ξ_{k+1}^l around the NE solution $\mathbf{P}_k^*$ of the linearized game Ξ_k^l, where $\mathbf{P}_k^* = \Delta\mathbf{P}_k^* + \mathbf{P}_{k-1}^*$ and $\Delta\mathbf{P}_k^*$ solves (14). We iterate the process until we achieve $\|\Delta\mathbf{P}_k\| \leqslant \delta$ for some precision $\delta > 0$.

Algorithm 1 Iterative Algorithm to Find NE of the Game Ξ

Initialization :
Set the prices $\alpha, c_i, i = 1, 2, \cdots, N_d$.
Specify (P_i^l, Q_i^l) for load bus and (P_i, Q_i) for generation bus.
$\Psi_0 := (\mathbf{P}_0, \mathbf{Q}_0, \mathbf{V}_0, \theta)$ // Initialize with nominal values of each bus.
Calculate matrices **G** and **B**.
Iterative update:
while $\|\Delta\mathbf{P}_{d,k}\| > \delta$ **do**
  Construct linearized game Ξ_k^l around operating point Ψ_{k-1}.
  Calculate Jacobian **J** and its inverse **W**.
  Find $\mathbf{P}_{d,k}^*$ that satisfies the constraint.
  Obtain NE $\mathbf{P}_{d,k}^* = \Delta\mathbf{P}_{d,k}^* + \mathbf{P}_{d,k-1}^*$ of the linearized game Ξ_k^l.
  Define new operating point Ψ_k using $\mathbf{P}_d^*, \mathbf{V}_d^*, \theta_d^*$.
end while

Proposition 3: If Algorithm 1 converges, then it converges to a NE of Ξ.

IV. Simulations and Illustrations

In this section, we first illustrate the game-theoretic framework with a numerical example based on a 3-bus power system. Fig. 6 depicts a power system configuration in which a generator supplies power to two loads through three transmission lines. Two load buses are capable of generating solar energies independently, and they can either consume the generated power or sell it to the power market. In smart grid systems, load buses can make individual decisions for renewable power generation in order to maximize their profits without coordination. Hence, the two load buses can be viewed as players in the non-cooperative game described in Section II-A. We interface *MATLAB* with *PowerWorld* [20] as a tool to compute the Nash equilibrium using Algorithm 1.

We let the cost of production of solar energy to be proportional to the price of solar panel and take α as the average electricity price during an operation period. Hence, we set the parameters of the objective functions U_1, U_2 as

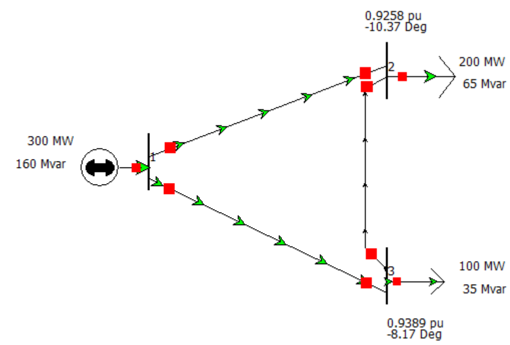

Fig. 6. *PowerWorld* system configuration of a 3-bus power network

follows: $c_1 = c_2 = 60\$/\text{MWh}; \alpha_1 = \alpha_2 = 142\$/\text{MWh}; P_2^l = 200\text{MW}, P_3^l = 100\text{MW}; \hat{V}_2 = \hat{V}_3 = 1\text{p.u.}$ with $1\text{p.u.} = 230\text{kV}$. Let $\gamma_1 = 0.4, \gamma_2 = 0.04$, which implies that voltage magnitude regulation is emphasized more than angle difference minimization. This is because voltage magnitude directly affects the quality of power supply at the load.

We show in Fig.2 the renewable power generation by bus 2 and bus 3, respectively, at each iteration. We can see that the algorithm converges to the Nash equilibrium $P_1^* = 128.8753\text{MW}$ and $P_2^* = 14.3430\text{MW}$. In Fig. 3, we show the values of cost functions at each iteration and they converge to equilibrium costs as P_1 and P_2 converge to the Nash equilibrium.

We investigate further the influences of the renewable energy unit cost and electricity market price on individual decision-makers. In Figs. 4 and 5, we fix the production cost of unit renewable energy and vary the market price α. It can be seen from Fig. 4 that when the electricity price increases, both players increase their amount of renewable power generations at the Nash equilibrium. In addition, it can be seen from Fig. 5 that as the market price increases, voltage profiles on buses are closer to 1p.u., which is a consequence of higher level of renewable power generation. This observation agrees with the intuition that the voltage profile improves, i.e., closer to 1p.u., when load buses generate more energy on their own.

V. Conclusion

In this paper, we have presented a game-theoretic framework to model independent decision-making of buses connected to renewable resources. The game formulation has taken into account economic factors as well as physical constraints. We have proposed an iterative algorithm to compute the Nash equilibrium of the nonlinear constrained game. Through numerical examples and simulations, we have demonstrated the equilibrium solution and the convergence of the algorithm. Our framework differs from the classical optimal power flow problem, and game theory is shown to be

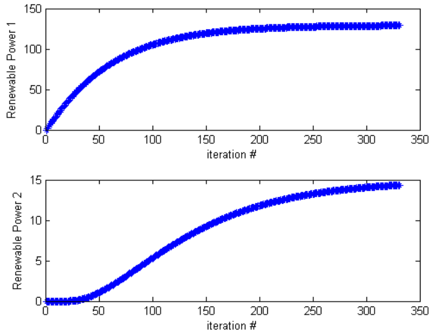

Fig. 2. Renewable power vs. iterations

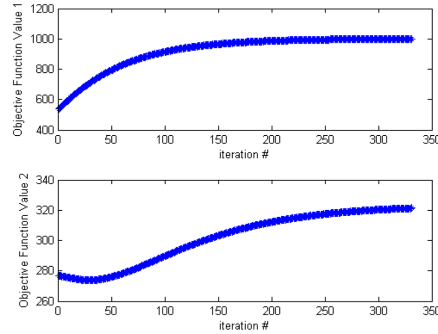

Fig. 3. Cost function vs. iterations

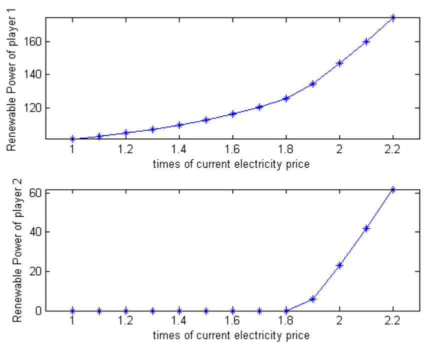

Fig. 4. Renewable power generations at buses 1 and 2 vs. electricity market prices

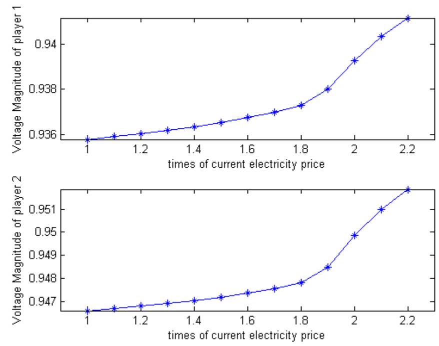

Fig. 5. Voltage magnitudes at buses 1 and 2 vs. electricity market prices

a useful tool for understanding strategic behaviors in smart grid systems. As future work, we intend to explore security aspects of distributed renewable energy generation within this framework. We can modify the model to incorporate malicious agents whose goals are purely for their economic benefits rather than system stability. It will also be interesting to understand the robustness and resilience of the distributed energy system framework.

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