Detection of Impulsive Effects in Switched DAEs with Applications to Power Electronics Reliability Analysis

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***Abstract*— This paper presents an analytical framework for detecting the presence of jumps and impulses in the solutions of switched differential algebraic equations (switched DAEs). The framework can be applied in the early design stage of fault-tolerant power electronics systems to identify design flaws that could jeopardize its reliability. The system is described by a switched differential algebraic equation, accounting for both fault-free system configurations and the configurations that arise after component faults, where each configuration p is defined by a pair of matrices (E_p, A_p). For each configuration p, the so called *consistency projector* is obtained from the pair (E_p, A_p). Based on the consistency projectors of all possible configurations, conditions for impulse-free and jump-free solutions of the switched DAE are established. A case-study of a dual redundant buck converter is presented to illustrate the framework.**

***Index Terms*— Reliability, Fault Tolerance, Differential-Algebraic Equation**

I. Introduction

There are certain safety- and mission-critical systems, e.g., shipboard, aircraft, and automotive, where electrical power provided by power electronics-based systems is key to maintain system operation. In this regard, these power electronics-based systems must be designed to operate even in the presence of component faults, i.e., they must be fault tolerant. In a power electronics system, key elements to achieving fault tolerance are: component redundancy, a fault diagnosis system, and a reconfiguration system that, upon information provided by the diagnosis system, removes faulty components and usually substitutes them with redundant ones.

To design effective fault-tolerant power electronics systems, it is necessary to conduct extensive analysis during the design stage to ensure that there are no single uncovered faults, which might cause the system to fail despite the presence of redundancy. These uncovered faults can be caused by i) a poor design of the fault diagnosis and reconfiguration systems; or ii) the propagation of the fault to other parts of the system, e.g., a component short-circuit might cause sudden jumps or impulses on certain voltages and currents, destroying other components and defeating the purpose of component redundancy.

The second problem above motivates this paper. While there is considerable work devoted to the first problem in the power electronics literature (see, e.g., [1], [2], [3], [4], [5]), and more broadly in the control literature (see, e.g., [6], [7], [8]); to the authors knowledge, there are no analytical techniques to address the second problem. Thus, the focus of this paper is to provide an analytical framework to identify component faults in power electronics systems that can propagate to other parts of the system.

We formulate the problem by using a switched differential algebraic equation (switched DAE) description, which naturally captures algebraic constraints imposed by Kirchhoff's laws and the switching behavior of power electronics circuits. Additionally, it provides a natural way to describe switching events that originate from component faults. We present an analytical framework based on the system's DAE description that allows to identify component faults that might introduce hazardous jumps and impulses in voltages and currents, destroying elements that are originally non-faulty and causing the system to fail after a single fault.

It is important to note that proposed framework is meant to be utilized during the system design process as an analysis tool to help uncovering weak design point. The framework can also be utilized as an off-line tool to detect weak points in power electronics systems that are already deployed with the objective of providing guidance on how to fix these design flaws. We do not envision the utilization of the framework in on-line applications. Additionally, although not discussed in detail, the framework has the potential to be used to gather information about the description of the power electronics system switching logic, which can have application in computer simulation environments. Finally, while the application proposed in the paper is power electronics, the method can be applied to other domains.

In the remainder of this introductory section, we provide some modeling background needed throughout the paper, and a precise statement of the problem to be addressed. Section II provides the necessary mathematical background to address the problem, while Section III provides the solution in the form of an algorithm amenable for computer implementation. Section IV illustrates the application of the results to a dual-redundant buck converter. Concluding remarks are discussed in Section V.

A. Modeling Background

In circuit analysis, it is common to model the relations between voltages and currents by an ordinary differential equation of the form [9]

$$\dot{z} = \widetilde{A}z + \widetilde{B}u,$$

where z is a vector containing the circuit state variables, e.g., inductor current and capacitor voltages; u is a vector containing circuit inputs, e.g., voltage or current sources; and where the matrices $\widetilde{A}$, $\widetilde{B}$ are function of the circuit physical parameters and a result of Kirchhoff's laws. However, this description is always obtained by simplifying the more natural description given by a differential algebraic equation (DAE,

This work was supported by NFS grants CNS-0614993, ECCS-0821153 and DFG grant Wi1458/10-1.

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also called descriptor systems or singular systems) [10]:

$$E\dot{x} = Ax + Bu, \tag{1}$$

where E is in general a singular matrix and all matrices are functions of the circuit physical parameters. The matrix E is singular because this description still contains the algebraic relations given by Kirchhoff's laws and algebraic relations given by elements like resistors.

B. Switched DAE models for Power Electronics Systems

The DAE description in (1) can be extended to include the effect of switching actions (common in power electronics circuits), which results in a family of DAE descriptions

$$E_p\dot{x} = A_px + B_pu, \quad p \in \mathcal{P}, \tag{2}$$

where the system switches between the differenent configurations $p \in \mathcal{P}$. Note that the meaning of the state variable x is unchanged by the switches, only the matrices E_p, A_p, B_p change. Simplification of these DAEs yields a family of conventional ODE descriptions

$$\dot{z}_p = \tilde{A}_pz_p + \tilde{B}_pu, \quad p \in \mathcal{P}, \tag{3}$$

where the state variables in z_p depend on the switches, because the simplifications from (2) to (3) is based on algebraic relations which can differ in different configurations. In particular, the knowledge of the ODE description alone is not sufficient to analyse the circuits behaviour at a switch because the relationship between the different state variables z_p, $p \in \mathcal{P}$, is not given and it is necessary to define certain jump maps based on physical insight and/or by going back to the DAE description (2).

Because the DAE description (2) still contains algebraic constraints, not all initial values for x are possible. It is therefore a long standing question (already studied in the 1950s [11]) what initial value $x(0+)$ will result after a switch at $t = 0$ and for given $x(0-)$. The value $x(0-)$ does not necessarily fulfill the algebraic constraints which are active after the switch, therefore a jump in the state is expected. For electrical circuits additional physical properties like conservation of charge and flux linkages [11] and, more recently, passivity and energy minimization [12] were invoked to solve this problem. It turns out that, by utilizing the quasi-Weierstrass form for regular matrix pencils [13], the DAE description (2) already uniquely defines the jump from $x(0-)$ to $x(0+)$ via the so called *consistency projector*, and no additional physical properties have to be invoked.

Another property of DAE descriptions, as already addressed in [10] and [11] is the possible presence of impulses in the solutions (derivatives of jumps). For studying impulses in the solutions, it is necessary to embed the problem into a distributional framework which is, from a mathematical point of view, not so straightforward as pointed out in [14, Sec. 1.1]. In the latter, the piecewise-smooth distributional solution framework is introduced to analyze the impulsive solutions of DAEs.

C. Problem Statement

We assume that nominal behavior (no faults) of a power electronics circuit is described by

$$E_p\dot{x} = A_px, \quad p \in \mathcal{P}, \tag{4}$$

where $\mathcal{P}$ is a finite set of all nominal configurations (determined by the state of switches -open/closed- in the circuit). Formally, we do not consider inputs in the description (4). However constant inputs can be easily included into the description by adding the equation $\dot{u} = 0$. A similar trick can be used as well to allow for sinusoidal inputs, for details see Section III. Faults in any component will cause the pairs (E_p, A_p) to change abruptly. Thus, (4) can be extended to also cover faulty converter configurations as follows

$$E_p\dot{x} = A_px, \quad p \in \mathcal{P} \cup \mathcal{F}, \tag{5}$$

where the additional elements in $\mathcal{F}$ index the power electronics circuit configurations that arise due to faults. A component fault will cause a sudden switching, resulting in a transition from a non-faulty configuration $p \in \mathcal{P}$ to a faulty configuration $q \in \mathcal{F}$. Depending on the nature of the fault, this could induce some of the state variables to suddenly jump or even experience an impulse. This phenomena could affect some parts of the circuit that were not originally affected by the fault, and perhaps cause the system to fail to provide power to the load after a single initiating event despite the presence of redundancy in the case of fault-tolerant designs.

This paper addresses the problem described above, and provides an analytical framework to establish conditions based on the pairs $(E_p,\ A_p)$, with $p \in \mathcal{P} \cup \mathcal{F}$, that point out to transitions from non-faulty configurations $p \in \mathcal{P}$ to $q \in \mathcal{F}$ that causes impulses and undesired jumps in the system state variables.

II. Mathematical Background

The DAE representation of a power electronics system given in (5), including both non-faulty and faulty configurations, can be further formalized by introducing a switching signal $\sigma : \mathbb{R} \to \mathcal{P} \cup \mathcal{F}$, and thus the *switched DAE* description resulting is

$$E_\sigma\dot{x} = A_\sigma x \tag{6}$$

where $(E_p, A_p) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times n}$. In general the switching signal is unknown since the switching might be caused by faults. It is only assumed that the switching signal *behaves mathematically well* in the sense that only finitely many switches in every finite time interval occurs, i.e. no chattering (see e.g. [15]) occurs. No further assumptions are made on the switching signal.

A. Existence and uniqueness of (distributional) solutions

The switched DAE (6) can be interpreted as a time-varying DAE with piecewise-smooth coefficients, and therefore the *piecewise-smooth distributional solution framework* introduced in [16], [14] can be used in the problem of jump and impulse detection. In particular, solutions of (6) are piecewise-smooth distributions, i.e., $x \in (\mathbb{D}_{\mathrm{pw}\mathcal{C}^\infty})^n$. Roughly speaking, piecewise-smooth distributions are the sum of a piecewise-smooth functions and Dirac impulses and its derivatives. At some time $t \in \mathbb{R}$, the impulsive part of a piecewise-smooth distribution $x \in (\mathbb{D}_{\mathrm{pw}\mathcal{C}^\infty})^n$ is denoted by $x[t]$; the left and right sided evaluation of x is denoted by $x(t-)$ and $x(t+)$. For existence and uniqueness of solution the following property is essential.

Definition 1: A matrix pair $(E, A) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times n}$ is called *regular* if, and only if, the polynomial $\det(sE - A) \in \mathbb{R}[s]$ is not identically zero.

If all matrix pairs (E_p, A_p) in the switched DAE (6) are regular, then the switched DAE always has a (global) solution for any given initial value, including also inconsistent initial conditions, i.e. the following results hold.

Theorem 2 ([16]): Consider the switched DAE (6) with regular matrix pairs (E_p, A_p), $p \in \mathcal{P} \cup \mathcal{F}$. Then for all initial trajectory $x^0 \in (\mathbb{D}_{\text{pw}\mathcal{C}^\infty})^n$ there exists a unique $x \in (\mathbb{D}_{\text{pw}\mathcal{C}^\infty})^n$ which solves the initial trajectory problem:

$$x_{(\infty,0)} = x^0_{(-\infty,0)},$$
$$(E_\sigma \dot{x})_{[0,\infty)} = (A_\sigma x)_{[0,\infty)},$$

where D_M denotes the distributional restriction of $D \in \mathbb{D}_{\text{pw}\mathcal{C}^\infty}$ to the interval $M \subseteq \mathbb{R}$.

A very important consequence of this result is that the jumps induced by the switches are already uniquely defined by the regular matrix pairs (E_p, A_p) and no additional physical properties like conservation of charge or some energy minimization argument has to be invoked. This is an interesting mathematical property of switched DAEs (6) which seems to be not so well known in the electrical circuits community.

B. Consistency projectors

For the classical DAE $E\dot{x} = Ax$ with a fixed regular matrix pair (E, A) all solutions evolve within a so called *consistency space* $\mathfrak{C}_{(E,A)} \subseteq \mathbb{R}^n$ and are uniquely given by the (consistent) initial value $x(0) = x_0 \in \mathfrak{C}_{(E,A)}$. When inconsistent initial conditions are present, the solutions are not classical anymore, i.e., they may have jumps or impulses (derivatives of jumps). Inconsistent initial condition can be induced by switching because the consistency spaces for different configuration need not to coincide.

The following Theorem provides a result that allows the definition of the so called consistency projectors directly in terms of the system matrices. It is based on the *Wong sequences* introduced in [17].

Theorem 3 ([13]): Consider a regular matrix pair $(E, A) \in \mathbb{R}^{n\times n}$, $n \in \mathbb{N}$, and let the *Wong sequences* be given by

$$\mathcal{V}_0 = \mathbb{R}^n, \qquad \mathcal{V}_{i+1} = A^{-1}(E\mathcal{V}_i),\ i = 0, 1, \ldots,$$
$$\mathcal{W}_0 = \{0\}, \qquad \mathcal{W}_{i+1} = E^{-1}(A\mathcal{W}_i),\ i = 0, 1, \ldots,$$

where $B\mathcal{M} := \{\ Bx \in \mathbb{R}^n \mid x \in \mathcal{M}\ \}$ and $B^{-1}\mathcal{M} := \{\ x \in \mathbb{R}^n \mid Bx \in \mathcal{M}\ \}$ for some matrix $B \in \mathbb{R}^{n\times n}$ and some set $\mathcal{M} \subseteq \mathbb{R}^n$. Then there exists $i^* \in \{0, 1, \ldots, n\}$ such that

$$\mathcal{V}_0 \supset \mathcal{V}_1 \supset \ldots \supset \mathcal{V}_{i^*} = \mathcal{V}_{i^*+1} = \ldots =: \mathcal{V}^*,$$
$$\mathcal{W}_0 \subset \mathcal{W}_1 \subset \ldots \subset \mathcal{W}_{i^*} = \mathcal{W}_{i^*+1} = \ldots =: \mathcal{W}^*,$$

with $\mathcal{V}^* \oplus \mathcal{W}^* = \mathbb{R}^n$ and $\mathfrak{C}_{(E,A)} = \mathcal{V}^*$, i.e. $\mathcal{V}^*$ is the consistency space. Furthermore, for full rank matrices $V \in \mathbb{R}^{n\times n_1}$, $W \in \mathbb{R}^{n\times n_2}$, $n_1 + n_2 = n$, with $\operatorname{im} V = \mathcal{V}^*$ and $\operatorname{im} W = \mathcal{W}^*$ the matrices $T = [V, W]$ and $S^{-1} = [EV, AW]$ are invertible and put the matrix pair (E, A) into *quasi-Weierstrass form*:

$$SET = \begin{bmatrix} I & 0 \\ 0 & N \end{bmatrix}, \quad SAT = \begin{bmatrix} J & 0 \\ 0 & I \end{bmatrix},$$

where $J \in \mathbb{R}^{n_1\times n_1}$ is some matrix and $N \in \mathbb{R}^{n_2\times n_2}$ is nilpotent.

Definition 4 (Consistency projector): For a regular matrix pair (E, A) let $V \in \mathbb{R}^{n\times n_1}$ and $W \in \mathbb{R}^{n\times(n-n_1)}$ be given as in Theorem 3. The *consistency projector* for the pair (E, A) is

$$\Pi_{(E,A)} := [V, W] \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} [V, W]^{-1} = [V, 0][V, W]^{-1}$$

where $I \in \mathbb{R}^{n_1\times n_1}$ is an identity matrix of size $n_1 \times n_1$.

Note that Theorem 3 ensures that the matrix $[V, W] \in \mathbb{R}^{n\times n}$ is indeed invertible and it is easy to see that the projector matrix $\Pi_{(E,A)} \in \mathbb{R}^{n\times n}$ does not depend on the spefic choise of V and W. Furthermore, $\operatorname{im} \Pi_{(E,A)} = \mathcal{V}^* = \mathfrak{C}_{(E,A)}$, i.e. the column space of the matrix $\Pi_{(E,A)}$ is exactly the consistency space $\mathfrak{C}_{(E,A)}$ and $\Pi_{(E,A)}x = x$ for all $x \in \mathfrak{C}_{(E,A)}$.

The consistency projectors define the jumps at switching times, i.e. the following results holds.

Theorem 5 (Consistency projectors and solutions, [14]): Consider the switched DAE (6) with regular matrix pairs (E_p, A_p) and corresponding consistency projectors Π_p and consistency space $\mathfrak{C}_p$. Every solution $x \in (\mathbb{D}_{\text{pw}\mathcal{C}^\infty})^n$ has the following properties

$$x(t+) = \Pi_{\sigma(t+)} x(t-),$$

in particular $x(t+) = x(t-)$ if $\sigma(t+) = \sigma(t-)$.

C. Main theoretical result for impulse and jump detection

We can now formulate the two main theoretical tools for our detection algorithm.

Theorem 6 (Sufficient condition for impulse freeness, [14]): Consider (5) with regular matrix pairs (E_p, A_p) and corresponding consistency projectors $\Pi_p := \Pi_{(E_p,A_p)}$, $p \in \mathcal{P} \cup \mathcal{F}$, as in Definition 4. If

$$\boxed{E_q(I - \Pi_q)\Pi_p = 0,} \tag{7}$$

holds for $p, q \in \mathcal{P} \cup \mathcal{F}$ then all solutions $x \in (\mathbb{D}_{\text{pw}\mathcal{C}^\infty})^n$ of (6) with a switch from p to q at $t \in \mathbb{R}$ (i.e. $\sigma(t-) = p$ and $\sigma(t+) = q$) are impulse free at this switching time, i.e. $x[t] = 0$.

Note that if (7) holds for all $p, q \in \mathcal{P} \cup \mathcal{F}$ then all solutions of the switched DAE (6) are impulse free no matter what the switching signal is.

The following results gives a sufficient conditions for jump-freeness of the switched DAE (6).

Theorem 7 (Sufficient condition for jump freeness, [14]): Use the same notation as in Theorem 6. If

$$\boxed{(I - \Pi_q)\Pi_p = 0,} \tag{8}$$

holds for $p, q \in \mathcal{P} \cup \mathcal{F}$ then all solutions $x \in (\mathbb{D}_{\text{pw}\mathcal{C}^\infty})^n$ of (6) with a switch from p to q at $t \in \mathbb{R}$ are jump and impulse free at this switching time, i.e. $x(t+) = x(t-)$ and $x[t] = 0$.

III. Jump and Fault Detection Framework

Based on the results presented in the previous section, we provide a framework to detect whether or not transitions among certain configurations can induce impulses or jumps in some state variables. The framework is based on Definitions 1 and 4, and the results provided in Theorems 6 and

7. The following steps determine the algorithmic framework, some comments on the algorithmic implementations are given afterwards.

1. Identify which system components are subject to faults and how these faults affect the system dynamic description, producing different configurations.
2. Treat constant voltage and current sources as state variables by adding a differential equation of the form $\dot{x}_i = 0$ for each constant source i.
3. Treat sinusoidal voltage and current sources as state variables by adding two differential equations $\dot{x}_j = \omega_j x_{j+1}$, $\dot{x}_{j+1} = -\omega_j x_j$ for each sinusoidal source j with frequency ω_j.
4. Model each configuration via a linear homogeneous DAE $E_p\dot{x} = A_p x$, with $p \in \mathcal{P} \cup \mathcal{F}$, where x is the same for all configurations.
5. Check whether all matrix pairs (E_p, A_p), with $p \in \mathcal{P} \cup \mathcal{F}$, are regular as defined in Definition 1. If one of the matrix pairs is not regular then the model is not appropriate and therefore the DAE description or circuit model needs to be redefined.
6. Calculate the Wong sequences $\mathcal{V}_0, \mathcal{V}_1, \ldots$ and $\mathcal{W}_0, \mathcal{W}_1, \ldots$ as in Theorem 3 for each matrix pair (E_p, A_p), $p \in \mathcal{P} \cup \mathcal{F}$, until the spaces do not change.
7. Calculate the consistency projector matrices $\Pi_{(E_p,A_p)}$, $p \in \mathcal{P} \cup \mathcal{F}$, as in Definition 4.
8. Check condition (7) for all relevant pairs (p, q). If this condition is not fulfilled then impulses can occur. If condition (7) is fulfilled, but not condition (8), then no impulses can occur but some of the states have jumps. If both conditions are fulfilled no jumps or impulses can occur.

Comments on Step 1: In general, this step cannot be automated because it is often the case that only certain components are subject to failure. For electrical circuits with resistors, inductors, capacitors, and sources it is of course possible to assume that all these elements are subject to failure and include, for example, two failure modes per element into the failure set $\mathcal{F}$: short circuit, i.e. zero voltage drop, and open circuit, i.e. zero current flow. However, this would make the number of failure modes grow exponentially in the number of elements and is therefore not feasible for non-trivial circuits.

Comments on Step 2 and 3: We explicitely mention here constant and sinusoidal input signals because of their important role in power electronics systems. However, more general time-varying input signals can also be included, as long as they can be expressed as solutions of homogeneous differential equations. For example, the family of ramp signal given by $u(t) = at+b$, $a, b \in \mathbb{R}$, can be encoded by the two differential equations $\dot{x}_j = x_{j+1}$ and $\dot{x}_{j+1} = 0$.

Comments on Step 4: Obtaining the matrices (E_p, A_p) can be done fully automatically by the following procedure, provided the faults are restricted to short and open circuits. Let the nominal system (with possible switches) be given as a graph whose edges are the electrical elements in the circuit. Additionally, edges can also represent ideal wires (in faulty configurations where an element is a short circuit) or a non-existent connection (for faulty configuration where the element is destroyed such that the current flow is interrupted). For each edge introduce the current through the element and the voltage drop over the element as state variables and, for each element, add a row in the matrix pair (E_p, A_p) describing the property of the element, e.g. $0 = v - Ri$ for a resistor or $C\frac{\mathrm{d}}{\mathrm{d}t}v = i$ for a capacitor. Finally, add the independet algebraic equations stemming from the Kirchhoff's current and voltage laws. This yields a square description $E_p\dot{x} = A_p x$ of the same size for each configuration.

We would like to highlight here again, that we do not simplify algebraic equations because in general these simplification eliminate certain state variables. In the presence of switches (intended or induced by faults) this elimination will in general lead to different state variables in the different configurations, hence the resulting switched system cannot be described as a switched DAE (6). However, if the same algebraic equation is present in all configurations then this simplification can be safely done.

Finally, it is possible to have symbolic entries in the matrices E_p and A_p and do the calculation in Step 6-8 symbolically. This is often feasible because the resulting matrices E_p and A_p are sparse.

Comments on Step 5:

Although the process described in Step 4 always yields square matrices E_p and A_p, it is not guaranteed that regular matrix pairs are obtained. For example, when an ideal current source is connected in serial with a switch, then if the switch is open then the property of the ideal current source (given by $\frac{\mathrm{d}}{\mathrm{d}t}i = 0$ or, equaivelently, $i(t) = i_0$ for all $t \in \mathbb{R}$) and the property $i = 0$ after opening the switch contradicts each other (at least when the current source is not trivial and provides a zero current). Note that also in the distributional framework no solution exists in this situation.

Comments on Step 6: If the matrices E_p, A_p are given symbolically or with rational entries the short following Matlab program can be used to calculate the preimages $E^{-1}(A\mathcal{V}_i)$ or $A^{-1}(E\mathcal{W}_i)$:

Listing 1. Matlab function for calculating a basis of the preimage $A^{-1}(\operatorname{im} S)$ for some matrices A and S

```
function V=getPreImage(A,S)
[m1,n1]=size(A); [m2,n2]=size(S);
if m1==m2 | m2==0
    H=null([A,S]);
    V=colspace(H(1:n1,:));
else
    error('Both matrices must have same number of rows');
end;
```

The subspaces $\mathcal{V}^*$ and $\mathcal{W}^*$ can then be calculated[1] as follows:

Listing 2. Matlab function for calculating a basis of the space $\mathcal{V}^*$ as given in Theorem 3

```
function V = getVspace(E,A)
[m,n]=size(E);
if size(E)==size(A)
    V=eye(n,n);
    oldsize=n+1; newsize=n;
    while ~(newsize==oldsize)
       EV=colspace(E*V);
       V=getPreImage(A,EV);
       oldsize=newsize;
       newsize=rank(V);
```

[1]It seems that in the recent version of Matlab (R2009a) the symbolic toolbox has a bug when calling `null(M)` for a full column rank matrix $M \in \mathbb{R}^{n\times m}$ because it returns a 1×0 empty matrix instead of a $m \times 0$ empty matrix which prevents the proposed algorithm from working correctly when E is invertible or $E = 0$.

```
    end;
else
    error('Matrices E and A must have the same size');
end;
```

Listing 3. Matlab function for calculating a basis of the space $\mathcal{W}^*$ as given in Theorem 3

```
function W = getWspace(E,A)
[m,n]=size(E);
if size(E)==size(A)
    W=zeros(n,1);
    oldsize=-1; newsize=0;
    while ~(newsize==oldsize);
       AW=colspace(A*W);
       W=getPreImage(E,AW);
       oldsize=newsize;
       newsize=rank(W);
    end;
else
    error('Matrices E and A must have the same size');
end;
```

If the matrices E_p and A_p contain numerical values then the kernel and image calculations need to be more sophisticated because a numerical matrix generically always has full rank.

Comments on Step 7 and 8: Once the matrices V_p and W_p from Step 6 are obtained, the Steps 7 and 8 are simple matrix multiplications. For large systems the necessary matrix inversion in Step 7 might lead to problems. If the calculations are carried out numerically (instead of symbolically or rational) then checking conditions (7) or (8) is also not always trivial because one has to decide whether a small non-zero entry is indeed non-zero or just some numerical error.

Finally, a symbolic implementation of the algorithm may provide information on how to choose parameters appropriately to remove identified weak design points.

IV. Dual-Redundant Buck Converter Case Study

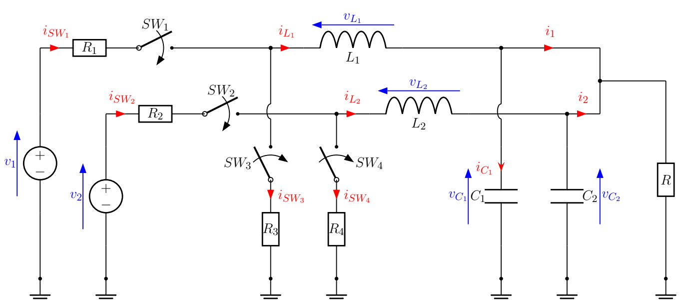

Fig. 1. Dual-redundant buck converter architecture.

Consider the dual-redundant buck converter of Fig. 1. The purpose of this redundant design is to ensure reliable power delivery to the load R even in the presence of component faults. The two fault free configurations are the "ON" configuration, where SW_1 and SW_2 are closed and SW_3 and SW_4 are open, and the "OFF" configuration where SW_1 and SW_2 are open and SW_3 and SW_4 are closed, in particular all four switches are synchronized. It is now of interest how the circuit behaves in the following fault scenarios: i) some of the switches get stuck in a fixed position, ii) a short-circuit occurs in C_1. As common state variables for all configuration, we choose $x = (\mathbf{v}^\top, \mathbf{i}^\top)^\top$ with $\mathbf{v}^\top = (v_1, v_2, v_{L_1}, v_{L_2}, v_{C_1}, v_{C_2})$ and $\mathbf{i}^\top = (i_{L_1}, i_{L_2}, i_{SW_1}, i_{SW_2}, i_{SW_3}, i_{SW_4}, i_1, i_2, i_{C_1})$ where v_1, v_2 are the input voltages, modeled as constant state variables by $\frac{\mathrm{d}}{\mathrm{d}t} v_1 = 0 = \frac{\mathrm{d}}{\mathrm{d}t} v_2$, the variables v_{L_1}, v_{L_2}, v_{C_1}, v_{C_2} stand for the voltages in inductors and capacitors respectively, i_{L_1}, i_{L_2}, i_{SW_1}, i_{SW_2}, i_{SW_3}, i_{SW_4}, i_{C_1} are the currents through the inductors, switches and capacitor C_1, and finally, i_1, i_2 are the currents flowing to the load from each converter.

The following equations hold independently of the position of the switches:

$$L_1 \tfrac{\mathrm{d}}{\mathrm{d}t} i_{L_1} = v_{L_1}, \quad i_{L_1} = i_{SW_1} - i_{SW_3},$$
$$L_2 \tfrac{\mathrm{d}}{\mathrm{d}t} i_{L_2} = v_{L_2}, \quad i_{L_2} = i_{SW_2} - i_{SW_4},$$

and

$$i_{C_1} = i_{L_1} - i_1, \quad v_{C_1} = v_{C_2},$$
$$C_2 \tfrac{\mathrm{d}}{\mathrm{d}t} v_{C_2} = i_{L_2} - i_2, \quad v_{C_2} = R(i_1 + i_2).$$

If one of the switches is open, then the corresponding current is zero, otherwise it holds that

$$\begin{aligned} SW_1 \text{ closed:} \quad & v_1 = R_1 i_{SW_1} + v_{L_1} + v_{C_1}, \\ SW_3 \text{ closed:} \quad & R_3 i_{SW_3} = v_{L_1} + v_{C_1}, \\ SW_2 \text{ closed:} \quad & v_2 = R_2 i_{SW_2} + v_{L_2} + v_{C_2}, \\ SW_4 \text{ closed:} \quad & R_4 i_{SW_4} = v_{L_2} + v_{C_2}, \end{aligned}$$

where it was assumed that the switches are non-ideal, in the sense that they are behaving as resistors when they are closed, otherwise the switches are assumed to be ideal. Finally, if C_1 is not short-circuited, it holds that $i_{C_1} = C_1 \frac{\mathrm{d}}{\mathrm{d}t} v_{C_1}$, otherwise, $v_{C_1} = 0$ holds.

These equations directly yield 32 matrix pairs (E_p, A_p), $p = 0, \ldots, 31$. Let $0, 1, \ldots, 15$ denote the configurations where C_1 is not short-circuited and furthermore identify each switching position with a binary quadruple and the corresponding number, i.e. all switches open correspond to the quadruple $(0, 0, 0, 0)$ and number 0, only switch SW_2 closed correspond to $(0, 1, 0, 0)$ and the number 4 and all switches closed correspond to $(1, 1, 1, 1)$ and the number 15. For the configurations where C_1 is short-circuited just add 16 to these numbers. The non-faulty "ON" configuration is then given by

$$(E_{12}, A_{12}) =$$
$$\left(\begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&L_1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&L_2&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&C_2&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&C_1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}, \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&-1&0&-1\\
0&0&0&0&0&0&0&1&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&1&0&-1&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&1&0&-1&0&-1&0&0&0\\
0&0&0&0&1&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&-R&-R&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
-1&0&1&0&1&0&0&0&R_1&0&0&0&0&0&0\\
0&-1&0&1&0&1&0&0&0&R_2&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0
\end{bmatrix}\right)$$

and the "OFF" configuration is

$$(E_3, A_3) =$$
$$\left(\begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&L_1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&L_2&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&C_2&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&C_1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}, \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&-1&0&-1\\
0&0&0&0&0&0&0&1&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&1&0&-1&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&1&0&-1&0&-1&0&0&0\\
0&0&0&0&1&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&-R&-R&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&-1&0&-1&0&0&0&0&0&R_3&0&0&0&0\\
0&0&0&-1&0&-1&0&0&0&0&0&R_4&0&0&0
\end{bmatrix}\right)$$

It is easy to check using a Matlab program that all matrix pairs (E_0, A_0), (E_1, A_1), …, (E_{31}, A_{31}) are regular by calculating $\det(sE_p - A)$, $p = 0, \ldots, 31$. It is not difficult to calculate the consistency projectors as in Definition 4 and to check the conditions (7) and (8).

For example, it turns out that condition (7) is fulfilled for the two non-faulty configurations $\{12, 3\}$, hence arbitrary switching between the "ON" and "OFF" configurations does not result in impulses in the solution. However, (8) is not fulfilled, i.e., jumps in certain state variables cannot be excluded, and it can be seen that a transition from the "OFF" configuration forces i_{SW_1} and i_{SW_2} to zero immediately.

To check whether and which faulty configurations can induce impulses in the state variables the condition (7) must be check for each pair $p, q \in \{0, \dots, 31\}$, the result of this check is given in the matrix $\mathcal{I} \in \{\square, \blacksquare\}^{32\times 32}$ given in Figure 2, where $\mathcal{I}_{p,q} = \square$ if, and only if, no impulse can occur at a switch from configuration p to configuration q.

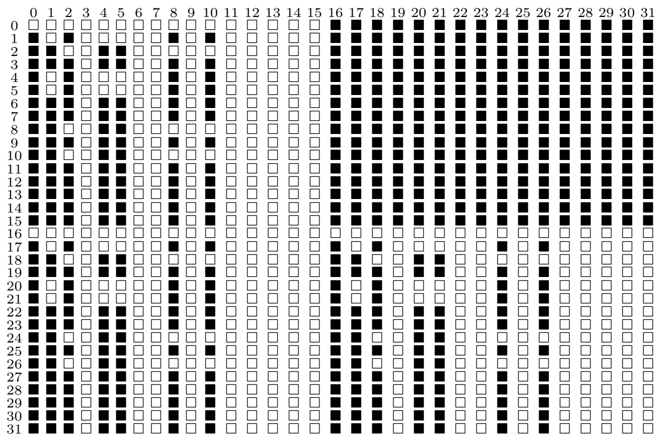

Fig. 2. Impulse matrix $\mathcal{I}$ for $(E_0, A_0) - (E_{31}, A_{31})$, where $\mathcal{I}_{p,q} = \square$ if, and only if, a transition from configuration p to q does not produce impulses.

Of special interest is row 12 in $\mathcal{I}$ corresponding to the non-faulty "ON" configuration, because it can be clearly seen, which faulty configurations might produce impulses. For example, when a faulty switch back to the "OFF" configuration occurs in the sense that the switches SW_1 and SW_2 are opened first and switches SW_3 and SW_4 are closed with a small delay (i.e., going from configuration 12 to 3 via the faulty configuration 0), then impulses can occur. On the other hand, if in the same situation SW_3 and SW_4 are closed first and SW_1 and SW_2 are opened later (i.e., going from 12 to 3 via 15) no impulses can occur. Furthermore, the matrix $\mathcal{I}$ reveals that a short-circuit of C_1 (i.e. a transition from some configuration $0, \dots, 15$ to some configuration $16, \dots, 31$) can always produce impulses.

V. Conclusions

We presented an analytical framework for detecting undesired jumps and impulses in the states of power electronics circuits in the presence of sudden component faults. The framework could be useful during the early design stage of a power electronics system to detect design flaws and help the designer to modify the design accordingly. Although not discussed in detail, the framework is also useful for designing the switching logic between non-faulty configurations as it provides information on which transitions must be avoided.

In the example discussed, only constant inputs were considered. As discussed, sinusoidal inputs can also be naturally included by augmenting the state space. Another way to include more general time-varying inputs is to approximate them by a piecewise constant function which can be considered as several constant inputs and corresponding switches, each of which would yield different configuration of the resulting switched DAE. However, this might add additional phenomena (induced by the jumps in the input) not present in the original system.

We are aware that diodes play an important role in electrical systems, however our framework cannot be applied to circuits with diodes. The reason is that the presence of diodes leads to state-dependent switching or, from another viewpoint, to non-linearity. The distributional framework cannot directly be applied to nonlinear systems and the necessary modifiactions to the mathematical theory is ongoing research.

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