A Framework for Multi-Level Reliability Evaluation of Electrical Energy Systems

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***Abstract*—This paper proposes a framework for multi-level reliability evaluation of electrical energy systems. Reliability, independent of the criteria (deterministic or probabilistic) used to measure it, is often treated as a binary variable, i.e, the system is considered to be reliable if it meets some intended function and unreliable otherwise. This binary approach is appropriate in systems with a single intended function or in which degraded performance is not allowed. Electrical energy systems are highly distributed and have multiple functions, i.e., deliver power to different loads. Furthermore, electrical energy systems are dynamic, both in operational conditions and physical structure, which has an additional impact on system performance, and therefore on reliability. In this context, the paper presents *multiple levels of reliability* and defines the concept of *degraded performance* for power systems work. The proposed multi-level reliability evaluation framework explicitly models both structural and operational uncertainty. It will become increasingly important as user-based reliability considerations become more prominent in electric power systems.**

I. INTRODUCTION

The 2030 smart grid vision calls for the construction of the future US electric system to provide clean, affordable, efficient and reliable electric power [1]. To accomplish these objectives, several features have been identified as key enablers of this smart grid: self-healing from power disturbances, active participation by customers, resiliency against physical and cyber attack, asset optimization, and new product definition. The introduction of these features requires a redesign of the electricity grid to integrate new technologies, such as electrical generation based on renewable energy, advanced power electronics, and smart metering devices. The integration of these technologies poses great challenges as they will modify the structure and operation of the grid in fundamental ways. In this regard, integration will introduce new elements of uncertainty at all levels, thus increasing the difficulty to provide efficient and reliable system design and operation. For example, the penetration of renewable-based generation will increase operating uncertainty on the supply side.

The integration of advanced power electronics systems for power flow control, including both FACTS devices and distribution-level devices, is another source of uncertainty in system design and operation. Despite extensive research in the area, the impact on system operation, particularly in terms of reliability and performance, is not completely understood. Other sources of uncertainty in the design are caused by the integration of intelligent loads such as plug-in vehicles (both as loads and as potential distributed energy storage), and microgeneration.

The effect of uncertainty has substantial impact on system performance, and therefore on reliability. In general terms, reliability can be defined as the ability of a device or system to perform a required function under stated conditions for a stated period of time [2]. In this regard, the definition of reliability is binary, i.e., either the system meets its intended function or it does not. However, it is not clear that this binary definition of reliability is suitable for analyzing electric power systems. The grid is a highly distributed system. Reliability can be measured, among other ways, as the ability of the system to deliver the total demand. However, if a contingency occurs and not all demand can be supplied, the grid usually continues to deliver functionality to a subset of the load, and can be said to operate in a *degraded mode*.

The question that arises, and that other researchers have addressed in the context of electrical systems [3], is the necessity to define different levels of reliability or what is established in this paper as *performance levels*. The purpose of this paper is twofold. First, the need for revisiting the binary definition of reliability is discussed. Second, a formal framework for multi-level definition and evaluation of reliability in electrical energy systems is proposed. This framework explicitly includes models for both structural and operational uncertainty.

The idea of treating performance and reliability issues jointly was first proposed in the 1970s in the field of fault-tolerant computing, where it was acknowledged that, for distributed systems in which degraded performance was acceptable, reliability and performance issues must not be treated separately [4], [5], [6]. In this context, Meyer coined the term *performability* to refer to this idea of jointly quantifying, in the presence of faults, performance degradation and overall computer reliability, and laid down a rigorous probabilistic framework for performability analysis [7]. The framework presented in this paper has been inspired by the work on performability of computer systems, although there are a few fundamental differences between performability and the proposed framework. First, the proposed framework is explicitly formulated in the context of dynamic systems that can be described by state-space models. Second, instead of using probabilistic descriptions of operational uncertainty, a set-membership approach to uncertainty modeling developed in control theory is employed. Third, the definition of performance measure is not treated as probabilistic as a probabilistic description of the structural uncertainty is not required.

The structure of this paper is as follows. Section II discusses the sources of uncertainty in an electrical energy system. Section III discusses the meaning of reliability and gives an overview of approaches to quantify reliability using both deterministic and probabilistic criteria. In Section IV, a framework is presented for assessing reliability using a multi-level definition of system reliability. Section V discusses the potential applications of the proposed framework for achieving the 2030 Vision. Concluding remarks are presented in Section VI.

II. SOURCES OF UNCERTAINTY IN ELECTRICAL ENERGY SYSTEMS

The operation of electrical energy systems is subject to uncertainty. This uncertainty can be classified into *operational uncertainty* and *structural uncertainty*. Operational uncertainty is usually associated with changes on the demand or on the supply side. For example, in the simplified dc power distribution system of Fig. 1, the power demanded by the three loads is not fixed but can change over time. Without control over these fluctuations in the load, this variability is considered a source of operational uncertainty. The same is true on the supply side of this example, where there could be uncontrolled fluctuations in the value of the dc voltage. Structural uncertainty is associated with changes in the physical structure of the system. For example, in Fig. 1, if a fault causes the line linking buses 1 and 4 to open, the system structure changes. Other faults may also result in a structural modification. It is important to note that structural uncertainty is not always caused by faults. In the same dc power distribution example, the system might be designed such that whenever the combined demand falls below some threshold, one of the sources switches off.

In a more general context, generation based on stochastic renewable resources, such as solar or wind, can be regarded as a source of operational uncertainty. Solar resources might be designed to switch in and out to follow the day-night cycle, which could produce structural uncertainty. Although these types of uncertainty are not at all new to electric power systems, the extension of operational uncertainty to a significant portion of the generation capacity has been a cause of concern, while the need to support structural reconfiguration to lower power levels presents a major design challenge.

III. RELIABILITY AS THE MEASURE OF THE IMPACT OF UNCERTAINTY ON SYSTEM PERFORMANCE

The sources of uncertainty discussed in Section II will affect system performance. Reliability is often used as a global measure of system performance under uncertain conditions, both operational and structural. In this section, the meaning of reliability and how reliability is quantified using both deterministic and probabilistic approaches is discussed.

A. Definitions of Reliability

The interpretation and assessment methods for reliability depend on the application domain. In the *Authoritative Dictionary of IEEE Standards Terms*, for example, there are three general definitions of reliability and several other domain-oriented ones. [2]. Among these general definitions of reliability are the following quoted from [2].

> **Reliability (1)** is the *ability* of an item to perform a *required function* under *stated conditions* for a stated period of time.
> **Reliability (2)** is the *probability* that a system will perform its *intended functions* without failure, within design parameters, under *specific operating conditions*, and for a specified period of time.

There is a fundamental difference between these definitions in that definition 2 defines reliability as a probability, implying a method to measure reliability, whereas definition 1 does not specify any method to quantify reliability. This has an impact on the design criteria. Whereas the first definition does not imply whether deterministic or probabilistic criteria are to be used to measure reliability, the second one does by defining reliability in probabilistic terms.

Different engineering fields have more specialized definitions of reliability. For example, In the context of the bulk power system, the North American Electric Reliability Corporation (NERC) defines reliability in [8] as follows.

> **Reliability** is the degree of performance of the elements of the bulk electric system that results in electricity being delivered to customers within accepted standards and in the amount desired.

The definition is complemented by two additional concepts: *adequacy* and *operational reliability*, which are defined also in [8] as follows.

> **Adequacy** is the ability of the electric system to supply the aggregate electric power and energy requirements of the electricity consumers at all times, taking into account scheduled and reasonably expected unscheduled outages of system components.
> **Operational reliability** is the ability of the electric system to withstand sudden disturbances such as electric short circuits or unanticipated loss of system components.

None of these definitions imply a method to measure reliability; however, it is common practice to use frequency and duration methods to measure reliability in bulk power systems, which implies that a probabilistic method for measuring reliability is in place.

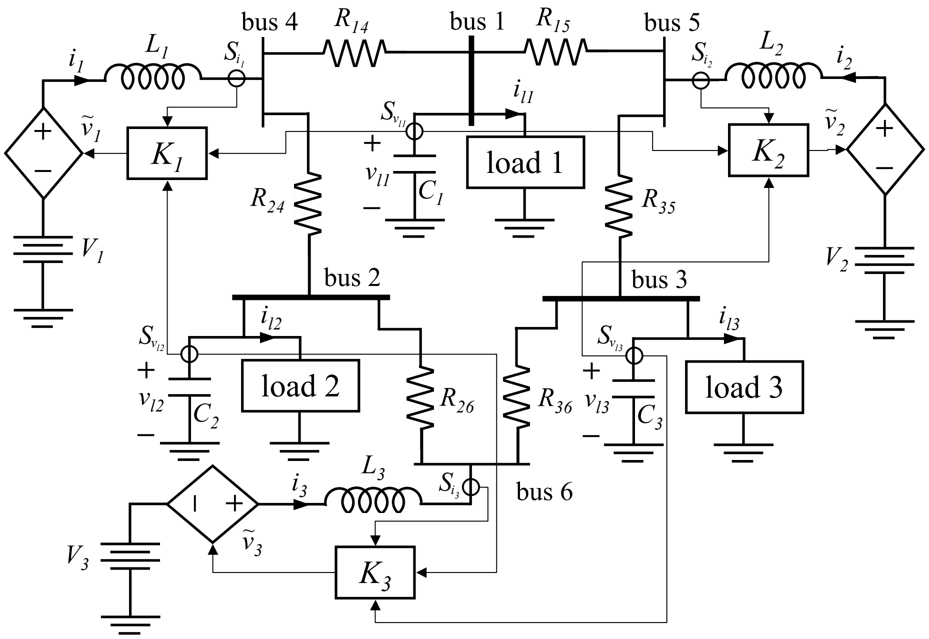

Fig. 1. Abstraction of dc power distribution system.

B. Deterministic vs Probabilistic Measure of Reliability

Two approaches can be taken when measuring reliability. On the one hand, a deterministic approach, given worst-case operational conditions and deterministic contingency criteria, can be used to measure system reliability. In this regard, the system is consider to be reliable if it delivers its intended function for the highest possible load demand and for any combination of k component outages, where $k = 1, 2, \ldots N$ with N the number of components in the system, and unreliable otherwise.

In the context of the dc power system of Fig. 1, consider that the intent of the system is to deliver the power demanded by the three loads, maintaining the voltages at each load bus within some tolerance. The loads are uncontrolled but will not exceed a known limit. Then, based on a deterministic $N-k$ contingency criterion, the system must be designed in such a way that, for all combinations of k component outages and worst-case operational conditions, the sources are able to deliver the required power and the voltage at all load buses are kept within the predefined tolerances. From this example, it is clear how considering worst-case operational conditions combined with deterministic reliability criteria is likely to result in an extremely over-designed system.

On the other hand, instead of using deterministic criteria, a probabilistic measure of reliability could be based, for example, on assessing the likelihood that for any combinations of k or fewer component outages and all possible operational conditions, the sources are able to deliver the required power and the voltages at all load buses are kept within the predefined tolerances. In the previous paragraph, for the system to be reliable, it must deliver its intended function after k component outages under worst-case operational conditions, which is equivalent to say that the system is reliable only if it delivers its intended function with *probability 1*. If this requirement is relaxed to allow some likelihood that the system will not perform after k component faults, a probabilistic measure to assess system reliability emerges.

When using probabilistic criteria to measure system reliability, less conservative designs result than when using deterministic criteria. However, it is important to note that there are trade-offs between deterministic and probabilistic reliability criteria in terms of, for example, maintenance cost. A less conservative design will most likely need service more often that the more conservative design. Of course this also depends on how the more conservative system is designed. For example, if component redundancy was used to achieve the $N-k$ contingency criteria (with probability 1), then even if the more conservative design is less likely to fail, it also has more components, thus increasing the likelihood of individual component outages and raising maintenance needs.

It is also possible to think about using both deterministic and probabilistic criteria to assess reliability. For example, it is common to impose an $N-1$ deterministic criterion, i.e., the system must continue full function in the event of any single fault, and then impose some probabilistic criteria under more severe conditions, e.g., the overall probability of system failure must be smaller that some predetermined requirement. This is the usual procedure in aircraft and aerospace system design, where system design emphasizes tolerance of any first (and sometimes second) component fault with probability 1; and then some probabilistic target is imposed on the overall system reliability, e.g., 10^{-9} catastrophic failures per operating hour as the upper limit for commercial aircraft [9]. In a power grid, two other probabilistic measures of failure relate to the point of end use: the probablity that any given user is without power at any given time, and the fraction of total load not served at a given moment in time. Without some consideration of all these measures, an individual customer may experience poor service with many extended outages that occur in a system otherwise assessed as highly reliable.

C. Need for a New Framework to Evaluate Reliability

Current reliability evaluation techniques in power systems do not include explicitly operational uncertainty. Reliability is evaluated for worst-case results based on a set of pre-defined operational conditions. However, if useful global measures of reliability are to be obtained, it is necessary to include a model of system operational uncertainty. A binary approach is useful for first contingencies, but more generally is not suited for evaluation of grid reliability given the distributed functionality. In this regard, the system is still useful operating in a degraded mode serving part of the load. The use of probabilistic reliability criteria is challenging. The probabilities of component outages are usually difficult to estimate. Even if estimates exist, there is uncertainty in the validity of these probabilities and especially in contigent probabilities that reflect cascading failures.

A system that allows for degraded performance operation can achieve higher levels of flexibility in system design and operation. For example, consider again the dc power system of Fig. 1. Think of each i-labeled load and source, controller and their corresponding interconnecting elements as a single control area. For example, control area 1 comprises power source 1, controller K_1, and the line linking buses 1 and 4. Under nominal conditions, the voltage at each load is controlled by its own control area voltage source. However, the voltage at load 1 is also monitored by the controller of area 2. If a failure were to occur in any of the elements of control area 1, it would immediately alter the voltage at load 1. Since controller K_2 is continuously monitoring the voltage at load 1, it would detect this fault. Depending on the loading level of source 2, K_2 could adjust its settings to “help” load 1. If the power delivery capability of power source 1 is close to being maximum when its assistance to load 1 s needed, two options are possible based on a “priority-deferral” scheme. First if load 1 is more critical than load 2, K_2 could switch off load 2 (the corresponding switches and information flows from K_2 are not displayed) and help load 1 immediately. Second, if load 1 is not as critical as load 2, it could defer until the loading of source 2 decreases and assistance to load 1 becomes possible. To implement and evaluate schemes like the one described, it is necessary to change current thinking about reliability of electrical systems.

IV. A FRAMEWORK FOR MULTI-LEVEL RELIABILITY EVALUATION

In this section, a framework is proposed for assessing reliability based on a multi-level definition of system performance. The framework is formulated using state-space representations of the system dynamics used in switching system theory, which allows explicit description of structural uncertainty. Rather than using a probabilistic approach, operational uncertainty is modeled using set-membership uncertainty modeling techniques commonly used in control for verification of reachable sets in switching systems. The traditional definition of reliability is substituted by a multi-level definition by dividing the state space in disjoint regions, each of which has a defined level of reliability. In order to measure reliability within this multi-level reliability framework, a global measure is defined that will quantify the extent to which the system will be operating on each performance level.

A. Operational and Structural Uncertainty: Reachability Sets

Dynamic systems that can be described by a family of continuous-time dynamic models with discrete transitions among members of the family are called switched systems, where the switching among members can be controlled or uncontrolled [10]. Many electrical systems can be described as controlled or uncontrolled switched systems. For example, a buck dc-dc switching converter has two configurations, and the switching between them is controlled in any of a number of ways to regulate the output voltage. A power distribution system can be thought of as having several configurations, where the switching between them can be controlled, e.g., a source is connected or disconnected as needed, or uncontrolled, e.g., a random fault causes some element to disconnect from the rest of the system. Thus, uncontrolled switching can be regarded as a source of uncertainty in the system configuration. The purpose of many electrical systems is to deliver power to loads at a specified voltage. The uncontrolled loads are a bounded source of uncertainty in the system inputs.

Here the focus is on electrical systems that can be described by a family of continuous-time dynamic models with controlled or uncontrolled transitions among them and subject to unknown but bounded inputs. These systems can be represented using a family of state-space representations of the form

$$\begin{gathered}\dot{x} = f_k(x, w),\\ k \in \mathcal{K},\ w \in \Omega_w\end{gathered} \tag{1}$$

where $x \in \mathbb{R}^n$, $\mathcal{K} = \{1, 2, \ldots, l\}$, and $\Omega_w \subset \mathbb{R}^m$ is bounded. Furthermore, the model will be specialized to the class of electrical systems where each $f_k(x, w)$ is linear and the set Ω_w is defined by an ellipsoid. This specialization is not crucial, but supports analytical solutions to the model. If nonlinear models are to be used, then it is necessary to apply numerical techniques. Given LTI system models, dynamic behavior can be described by a family of state-space represntations of the form

$$\begin{gathered}\frac{dx(t)}{dt} = A_k x(t) + B_k w,\\ k \in \mathcal{K} = \{1, 2, \ldots, l\},\\ w \in \mathcal{W} = \{w : w' Q^{-1} w \leq 1\},\end{gathered} \tag{2}$$

where $x \in \mathbb{R}^n$ represents the system state variables (e.g., voltage and currents), and $w \in \mathbb{R}^m$ represents uncontrolled inputs, such as variations in load demands. Matrices A_k and B_k are constant, and $\mathcal{K}$ represents the number of possible configurations the system can adopt due to component faults or other controlled switching actions.

For each $k \in \mathcal{K}$, the steady-state set of reachable states $\mathcal{R}_k$ is to be computed. This reachability problem was solved in [11] by computing a family of ellipsoids, the intersection of which yields $\mathcal{R}_k$:

$$\mathcal{R}_k = \bigcap_{\eta} \mathcal{X}_k(\eta), \ \forall \eta : \ \eta'\eta = 1. \tag{3}$$

where, for each particular η, the ellipsoid $\mathcal{X}_k(\eta)$ is defined by

$$\mathcal{X}_k(\eta) = \{x : x'\Psi_k^{-1}(\eta)x \leq 1\}, \tag{4}$$

with $\Psi_k(\eta)$ is obtained by solving the nonlinear Riccati equation given by

$$A\Psi_k(\eta) + \Psi_k(\eta)A' + \frac{\sqrt{\eta' B_k Q B_k \eta}}{\sqrt{\eta' \Psi_k(\eta)\eta}}\Psi_k(\eta) + \frac{\sqrt{\eta' \Psi_k(\eta)\eta}}{\sqrt{\eta' B_k Q B_k \eta}} BQB'. \tag{5}$$

The reader is referred to [11] for a detailed explanation of the derivation of (5) and (3).

B. Multi-Level Reliability Definition: Performance Levels

As discussed before, there are systems intended to fulfill a single intended function, which can be defined by a set of dynamic performance requirements. In these systems, the performance requirements constrain the state vector x to a region of the state space. In these systems, it may be appropriate to define reliability as a binary variable, where the system is considered to be operational and delivering its function whenever the state vector x is within the region of the state-space and failed otherwise. In most electrical systems, the system can deliver its intended function only partially under all possible conditions. In this case, it is possible to think of having not a single set of performance requirements, but several sets of performance requirements, constraining the state vector x to various regions of the state space.

Following this rationale, and depending on the system configuration k and the input w, a multi-level reliability definition emerges, where the system intended function is divided into different *performance levels*. To formally define these different performance levels, divide the state-space into s disjoint sets given by

$$\Pi_j = \begin{cases} x : x'P_1^{-1}x \leq 1, & \text{for } j = 1 \\ x : x'P_{j-1}^{-1}x > 1, \ x'P_j^{-1}x \leq 1, & \text{for } 1 < j < s \\ x : x'P_{s-1}^{-1}x > 1, & \text{for } j = s \end{cases} \tag{6}$$

Then, performance levels $\mathcal{P} = \{1, 2, \ldots, s\}$ are established through the map $\mathcal{F} : \mathbb{R}^n \to \mathcal{P}$ defined by

$$\mathcal{P} = j, \forall x \in \Pi_j, \tag{7}$$

where it is assumed that the lower the value of $\mathcal{P}$ is, the better the system is performing. In other words, performance levels with lower indexes are preferred to performance levels with higher indexes. The idea can be generalized to the case in which there is a deterministic input and the system is to operate as close as possible to the trajectory that results from this deterministic input.

It is important to note that although ellipsoids centered at the origin were used here to define the regions for (6), it is possible to partition the state-space using other criteria, which will depend on the nature of the system.

C. A Global Measure of System Performance

For every configuration k, the reachability set $\mathcal{R}_k$ could span several regions Π_j, therefore there is uncertainty associated with the performance level $\mathcal{P}$ in which the system is operating. Therefore, in order to quantify the extent to which the system will be operating in each performance level, a measure of system performance needs to be established. It might be possible to establish a probabilistic measure, but to do this, probabilistic descriptions of the switching process and the system state x are needed. Instead, assume that no information is available on such probabilistic descriptions. This leads to a more general measure of performance $\mu : \mathcal{P} \to \mathbb{R}$, where for each $j \in \mathcal{P}$

$$\mu(j) = \sum_{k=1}^{l} w_{k,j} \cdot \frac{\text{vol}(\mathcal{R}_k \cap \Pi_j)}{\text{vol}(\mathcal{R}_k)}, \tag{8}$$

where vol$(\cdot)$ is the generalization to an n-dimensional space of the concept of area in a 2-dimensional space or volume in a 3-dimensional space. Then (8) is just a weighted sum of the portion of volume of each $\mathcal{R}_k, \ \forall k = 1, \ldots l$ that intersects with the region of the state-space defining the jth performance level.

The choice of the weight $w_{j,k}$ can be based on engineering judgment, experimental data, and economic factors. Depending on the choice of $w_{j,k}$, the resulting performance measure will have very different significance. For example, if there are only two performance levels and only the first one is acceptable, then the traditional frequency interpretation of reliability will result if weights are chosen $w_{k,1}, \forall k \in \mathcal{K}$ proportional to the fraction of time the system spends in configuration k operating in region Π_j and $w_{k,2} = 0, \forall k \in \mathcal{K}$. In order to enable the definition of other performance measures, write $w_{k,j}$ as

$$w_{k,j} = \pi_{k,j}^1 \pi_{k,j}^2 \ldots \pi_{k,j}^q, \tag{9}$$

where $\pi_{k,j}^1, \pi_{k,j}^2, \ldots \pi_{k,j}^2$ are associated with metrics of interest for assessing system performance. Then, consider the following cases:

1) Case 1: Choose $\pi_{k,j}^1$ proportional to the fraction of time the system spends in configuration k operating in region Π_j, and $\pi_{k,j}^2, \ldots, \pi_{k,j}^q = 1, \forall k \in \mathcal{K}, \forall j \in \mathcal{P}$. Then the performance measure μ is an extension of the traditional frequency interpretation of reliability to the case where several levels of performance are allowed, and can be interpreted as the fraction of time the system spends operating at each performance level.

2) Case 2: Choose $\pi^1_{k,j}$ as in case 1, $\pi^2_{k,j}$ as the economic benefit (or loss) to operate in region Π_j when the system is in configuration k and $\pi^3_{k,j}, \ldots, \pi^q_{k,j} = 1, \forall k \in \mathcal{K}, \forall j \in \mathcal{P}$, then the performance measure μ can be interpreted as the expected economic benefit (or loss) resulting from system operation at a particular performance level.

3) Case 3: Another, more generalized measure of system performance can be obtained by choosing $\pi^1_{k,j}$ and $\pi^2_{k,j}$ as in case 2, and choosing $\pi^3_{k,j}$ to be a measure of, social welfare when the system is in configuration k.

V. THE ROLE OF THE PROPOSED FRAMEWORK IN ACHIEVING THE 2030 VISION

The proposed framework is intended to help realize some of the key enablers of the 2030 vision such as new product definitions and asset optimization. In terms of asset optimization, the proposed framework, by enabling the definition and quantification of multiple levels of reliability, will allow system planners to address degraded performance criteria at the design stage, as opposed to worst-case and binary reliability criteria. Designers can optimize system design and operation to accommodate loads that may require different levels of reliability. A critical load such as the computer server room in a financial institution is likely to have more stringent time-based reliability requirements than dryers in a laundromat business, for example. A power glitch lasting just one second might have very different impacts on these two loads. However, it would not be unusual today to have both such loads on a single distribution feeder, leading to similar levels of reliability from a source perspective (and therefore requiring the financial business to purchase separate power support equipment at considerable expense). This is a good example where the multiple-level reliability criteria could be used to optimize the power supply system design and operation to provide different levels or reliability.

The proposed framework for reliability design and operation is an important tool as stochastic resources grow as a fraction of supply. New large loads such as plug-in vehicles can be time-shifted based on reliability targets, for example. An important issue is how to choose levels of reliability, another aspect which is linked directly with the definition of new products for the 2030 vision. Customers may be able to decide their own level of reliability, or even choose different reliability levels for different components. In other words, rather than system planners using predefined levels of reliability criteria, each customer could decide a level of reliability demand and pay accordingly. A business may decide that point-of-sale terminals and communications require extreme reliability, while some individual processes can be interrupted for a minute or so with little impact. The expectations may require extensive distribution redesign, or may instead imply that the utility should be a provider of uninterruptible power equipment to deliver extreme reliability on a local basis. The multi-level concept allows highly variable resources to drive the system among different reliability levels, while meeting customer expectations and optimizing overall operating costs.

One important factor that must be captured as this work progresses is the time basis of utility grid reliability. On the one hand, loads differ greatly in their sensitivity to interruptions, from industrial processes with only a cycle or so of capability to manage power loss to bulk heating applications or time-shfitable battery charging that can move around by several hours. Today's user base perceives a vast difference between ten one-second outages and one ten-second outage. On the other hand, reliability requirements tend to differ at various times each day. A one-second outage at 3:00 am has much different system-wide impact than the same outage at 3:00 pm. An important challenge is to expand the proposed framework for time-aware reliability levels.

VI. CONCLUDING REMARKS

The need for revising binary-based techniques definitions for evaluating reliability of electric power systems has been discussed. It has been argued that the power grid is highly distributed, and therefore it is not possible to define a single level of reliability. An approach that establishes multiple levels of performance instead appears to be more suitable. To tackle this problem, a multiple-level reliability framework was proposed based on concepts used in system theory, such as switching system representations and reachability analysis.

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