COMPARISON OF STATISTICAL ALGORITHMS FOR POWER SYSTEM LINE OUTAGE DETECTION

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ABSTRACT

We propose a statistical algorithm for detecting line outages in a power system and show that it has better performance than other schemes proposed in the literature. Our algorithm is based on the Cumulative Sum (CuSum) test from the Quickest Change Detection (QCD) literature. It exploits the statistical properties of the measured voltage phase angles before, during, and after a line outage, whereas other methods in the literature only utilize the change in statistics that occurs at the instant of outage.

Index Terms— Power systems, line outage detection, quickest change detection, CuSum test, Shewhart test.

1. INTRODUCTION

Many tools for power system monitoring and control rely on a model of the power system that is obtained offline, which can be inaccurate due to topology errors. Thus, rapid detection of line outages in a power system is crucial for maintaining reliable and stable operation. As an example, in the 2011 San Diego blackout, operators were unable to determine overloaded lines because the network model was out of date [1]. This lack of situational awareness limited the operators' ability to identify and prevent the next critical contingency, leading to instability and cascading failures. Similarly, during the 2003 Northeast blackout, operators failed to initiate the correct control schemes because they had an inaccurate model of the power system and could not identify the loss of key transmission elements [2]. These blackouts highlight the importance of developing online measurement-based techniques to detect and identify system topological changes in a timely manner. This paper addresses the issues discussed above by proposing a real-time algorithm for detecting line outages; this algorithm is based on the theory of quickest change detection (QCD).

Earlier detection algorithms found in the literature, which we collectively refer to as meanshift tests, exploit the fact that

when a line outage occurs, the expected value of some measured variables (e.g., voltage phase angles), at the exact time of change is different from the average pre-change values [3]–[8]. The methods used in these papers can be equivalently formulated as a log-likelihood ratio test that only uses the most recent measurement to make a decision on whether or not an outage has occurred; additionally, none of these detection schemes exploit the persistent change in the covariance of the observations after the occurrence of a line outage.

As in [9], the algorithm proposed in this paper is based on adapting the Cumulative Sum (CuSum) test from the QCD literature (see, e.g., [10], [11]) to the line outage detection problem. Our algorithm not only takes the persistent covariance change into consideration, but it also exploits past observations to detect the occurrence of an outage. In [9], the statistics for each individual line are compared to a common predetermined threshold, and an outage is declared if one of these statistics crosses the threshold. In this paper, we present a method for setting a different threshold for each line outage statistic by taking the dissimilarity between the pre- and post-change distribution into consideration. This difference between pre- and post-change distributions is described by the Kullback-Leibler (KL) divergence, a metric that quantifies the distance between two distributions. In addition, we compare the performance of our test to that of the Shewhart test, the meanshift test, and the algorithm of [9], and observe notable improvements in terms of performance.

2. POWER SYSTEM MODEL

In this section, we present the pre- and post-outage statistical model of the power system voltage angles used for formulating the proposed line outage detection algorithm.

2.1. Pre-outage Incremental Power Flow Model

Consider a power system network represented by a graph with N nodes and L edges denoted by $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E}$, respectively. Let $(m, n) \in \mathcal{E}$ denote the transmission line between buses m and n. Let $P_i[k] := P_i(k\Delta t)$, $\Delta t > 0$, $k = 0, 1, 2, \ldots$, denote the kth measurement sample of active power injections into bus i. Similarly, let

$\theta_i[k]$, $k = 0, 1, 2, \ldots$, denote bus i's k^{th} voltage angle measurement sample. Furthermore, define variations in voltage phase angles between consecutive sampling times $k\Delta t$ and $(k+1)\Delta t$ as $\Delta\theta_i[k] := \theta_i[k+1] - \theta_i[k]$. Similarly, variations in the active power injections at bus i between two consecutive sampling times are defined as $\Delta P_i[k] = P_i[k+1] - P_i[k]$.

Now, using the standard DC power flow assumptions (see, e.g., [12]), namely, i) flat voltage profile, ii) negligible line resistance, and iii) small phase angle differences, we can decouple the real and reactive power flow equations and relate the variations in the voltage phase angles to the variations in the real power flow as follows:

$$\Delta P[k] \approx H_0 \Delta\theta[k], \tag{1}$$

where $\Delta P[k]$, $\Delta\theta[k] \in \mathbb{R}^{(N-1)}$ and $H_0 \in \mathbb{R}^{(N-1)\times(N-1)}$. Note that the $N-1$ dimension of the vectors is the result of omitting the reference bus equation. Then, by denoting $M_0 := H_0^{-1}$, we can rewrite (1) as follows:

$$\Delta\theta[k] \approx M_0 \Delta P[k]. \tag{2}$$

2.2. Post-Outage Incremental Power Flow Model

Now, suppose an outage for line (m, n) occurs at time $t = t_f$, where $(\lambda - 1)\Delta t \le t_f < \lambda\Delta t$. In order to relate the post-outage $\Delta\theta[k]$ to $\Delta P[k]$ as in (2), we first express the change in matrix H_0 resulting from the outage as the sum of the pre-change matrix and a perturbation matrix, $\Delta H_{(m,n)}$, i.e., $H_{(m,n)} = H_0 + \Delta H_{(m,n)}$, where the only non-zero terms in the matrix $\Delta H_{(m,n)}$ are $\Delta H_{(m,n)}[n, n] = -\Delta H_{(m,n)}[m, n] = -\Delta H_{(m,n)}[n, m] = \Delta H_{(m,n)}[m, m] = -1/X_{(m,n)}$ with $X_{(m,n)}$ denoting the imaginary part of the impedance of the outaged line [12]. By denoting $M_{(m,n)} := H_{(m,n)}^{-1}$, the post-outage relation between the changes in the voltage angles and the real power injection becomes

$$\Delta\theta[k] \approx M_{(m,n)} \Delta P[k]. \tag{3}$$

2.3. Statistics of $\{\Delta\theta[k]\}_{k\ge 1}$

By attributing the small variations in the real power injections, $\Delta P[k]$, to random fluctuations in electricity consumption, we can model the entries of $\Delta P[k]$ as identically distributed (i.i.d.) random variables with a zero-mean joint Gaussian probability density function (p.d.f.), i.e., $\Delta P[k] \sim \mathcal{N}(0, \Sigma)$. We also assume that the entries of $\Delta P[k]$ are uncorrelated, which results in Σ being a diagonal matrix. Since $\Delta\theta[k]$ are measured, and given (2) and (3), we have that $\Delta\theta[k] \sim f_\infty = \mathcal{N}(0, M_0\Sigma M_0^T)$ for the pre-outage voltage phase angles and $\Delta\theta[k] \sim f_{(m,n)}^\sigma = \mathcal{N}(0, M_{(m,n)}\Sigma M_{(m,n)}^T)$ for the post line (m, n) outage voltage phase angles. During the instant of change, it was previously shown in [9] that $\Delta\theta[k] \sim f_{(m,n)}^\mu = \mathcal{N}\big(-M_0 P_{(m,n)}[\lambda]r_{(m,n)}, M_0\Sigma M_0^T\big)$,

where $P_{(m,n)}[\lambda]$ is the pre-outage line flow across (m, n) [9] and $r_{(m,n)} \in \mathbb{R}^{N-1}$ is a vector with the mth entry equal to 1, the nth entry equal to -1, and all other entries equal to 0.

In summary, the p.d.f. of $\Delta\theta[k]$ before, during, and after an outage in line (m, n) is given by

$$\Delta\theta[k] \sim \begin{cases} f_\infty, & \text{if } k \le \lambda - 1, \\ f_{(m,n)}^\mu, & \text{if } k = \lambda, \\ f_{(m,n)}^\sigma, & \text{if } k \ge \lambda + 1. \end{cases} \tag{4}$$

3. QUICKEST CHANGE DETECTION

With the statistical model for $\{\Delta\theta[k]\}_{k\ge 1}$ in place, the problem of detecting a line outage can be formulated equivalently as a problem of detecting a change in the probability distribution of the sequence of observations $\{\Delta\theta[k]\}_{k\ge 1}$ as quickly as possible given false alarm constraints. This problem of detecting a change in the statistical behavior of a process is known in the literature as the quickest change detection (QCD) problem. We refer the reader to [10], [11] for a more in-depth survey of QCD algorithms.

3.1. Preliminaries

Suppose that at some unknown time instant $\lambda \ge 1$, a change in the statistics of the observation sequence, $\{\Delta\theta[k]\}_{k\ge 1}$, occurs due an outage in line (m, n). Therefore, we have that before, during, and after the statistical change, the distribution of $\Delta\theta[k]$ is described by (4).

The goal in QCD is to find a stopping time, τ, on the observation sequence, at which time the line outage is declared, such that the delay in decision making, measured by $\mathbb{E}_\lambda[\tau - \lambda | \tau \ge \lambda]$ is small, while guaranteeing that the occurrence of false alarm events is rare, i.e., $\mathbb{E}_\infty[\tau] \ge \gamma$, where γ a desired lower bound [11]. Here $\mathbb{E}_\lambda$ denotes the expectation under the probability measure which is induced on the observations when a change occurs at time λ, and $\mathbb{E}_\infty$ denotes the expectation under the pre-change distribution. This is a multi-objective optimization problem and several ways to formulate the tradeoff between these two quantities are discussed in [11].

For the intuition behind the proposed detection algorithm, it is useful to introduce the Kullback-Leibler (KL) divergence, which is a measure of distance between two distributions. In particular, the KL divergence between two probability density functions, f and g, is defined as:

$$D(f \parallel g) := \int f(x) \log \frac{f(x)}{g(x)} dx = \mathbb{E}_f\left[\log \frac{f(X)}{g(X)}\right]. \tag{5}$$

It is easy to show that $D(f \parallel g) \ge 0$, with equality if and only if $f = g$.

3.2. The CuSum Algorithm

In our setting, the line in which the outage occurs is unknown, i.e., the post-change distribution induced on the observation sequence $\{\Delta\theta[k]\}_{k\geq 1}$ is unknown. Since there are $|\mathcal{E}|$ lines, we have $|\mathcal{E}|$ different post-change scenarios.

We now present the CuSum algorithm, which detects statistical changes based on the idea of using past observation to accumulate log-likelihood ratios. We present two versions of this algorithm, one proposed in [9], in which a common threshold is used for all CuSum statistics and another in which the thresholds are selected based on each line's KL divergence.

We define the CuSum statistic corresponding to line (m, n) outage recursively as:

$$W^{\text{CU}}_{(m,n)}[k+1] = \max\left\{ W^{\text{CU}}_{(m,n)}[k] + \log \frac{f^{\sigma}_{(m,n)}(\Delta\theta[k+1])}{f_{\infty}(\Delta\theta[k+1])}, \right.$$
$$\left. \log \frac{f^{\mu}_{(m,n)}(\Delta\theta[k+1])}{f_{\infty}(\Delta\theta[k+1])}, 0 \right\},$$
$$(6)$$

with $W^{\text{CU}}_{(m,n)}[0] = 0$ for all $(m, n) \in \mathcal{E}$. The CuSum stopping time is defined as:

$$\tau^{\text{CU}} = \inf_{(m,n)\in\mathcal{E}} \left\{ \inf\{k \geq 1 : W^{\text{CU}}_{(m,n)}[k] > A^{\text{CU}}_{(m,n)}\} \right\}. \quad (7)$$

We now present ways of choosing the thresholds for the CuSum test. It can be shown (see, e.g., [13]) that by choosing

$$A^{\text{CU}}_{(m,n)} = \log\gamma - \log\beta_{(m,n)}, \quad (8)$$

with $\beta_{(m,n)}$ being a positive constant independent of γ, the expected delay for each possible outage differs from from the corresponding minimum delay among the class of stopping times $\mathcal{C}_{\gamma} = \{\tau : \mathbb{E}_{\infty}(\tau) \geq \gamma\}$, as $\gamma \to \infty$, by a bounded constant.

A choice of thresholds for the CuSum algorithm is obtained by setting $\beta_{(m,n)} = \frac{1}{L}$ for all $(m, n) \in \mathcal{E}$. This way we get a common threshold, i.e., $A^{\text{CU}}_{(m,n)} = A^{\text{CU}} = \log(\gamma L)$ for all $(m, n) \in \mathcal{E}$. It can be shown (see, e.g., [14]) that by choosing the thresholds this way, we can guarantee that $\mathbb{E}_{\infty}[\tau^{\text{CU}}] \geq \gamma$.

Using the results in [13], another choice of the thresholds could be based on a relative performance loss criterion, i.e.,

$$\beta_{(m,n)} = \frac{1}{D(f^{\sigma}_{(m,n)} \parallel f_{\infty})L(\zeta_{(m,n)})^2)}, \quad (9)$$

where

$$\zeta_{(m,n)} = \lim_{b\to\infty} \mathbb{E}^{\sigma}_{(m,n)}\left[e^{\{-(S_{(m,n)}[\tau^{\text{b}}_{(m,n)}]-b)\}} \right], \quad (10)$$

with

$$\tau^{\text{b}}_{(m,n)} = \inf\{k \geq 1 : S_{(m,n)}[k] \geq b\}, \quad (11)$$

and

$$S_{(m,n)}[k] = \sum_{l=1}^{k} \log \frac{f^{\sigma}_{(m,n)}(\Delta\theta[l])}{f_{\infty}(\Delta\theta[l])}. \quad (12)$$

This choice of threshold depends on the asymptotic overshoot of an SPRT-based test, which is often used in hypothesis testing [10]. As we show later through case studies, these thresholds result in performance gains.

4. OTHER ALGORITHMS FOR CHANGE DETECTION

In this section, we present some other change detection algorithms that can be shown to be equivalent to other techniques proposed in the literature. For example, the line outage detection algorithm proposed in [5] can be shown to be equivalent to a log-likelihood ratio test that only uses the most recent measurements.

4.1. Meanshift Test

The meanshift test is a "one-shot" detection scheme in that the algorithm uses only the most recent observation to decide whether a change in the mean has occurred and ignores all past observations. The meanshift statistic corresponding to line (m, n) is defined as follows:

$$W^{\text{MS}}_{(m,n)}[k] = \log \frac{f^{\mu}_{(m,n)}(\Delta\theta[k])}{f_{\infty}(\Delta\theta[k])}. \quad (13)$$

The decision maker declares a change when one of the $|\mathcal{E}|$ statistics crosses a corresponding threshold, $A^{\text{MS}}_{(m,n)}$. The stopping time for this algorithm is defined as:

$$\tau^{\text{MS}} = \inf_{(m,n)\in\mathcal{E}} \left\{ \inf\{k \geq 1 : W^{\text{MS}}_{(m,n)}[k] > A^{\text{MS}}_{(m,n)}\} \right\}. \quad (14)$$

The meanshift test ignores the persistent covariance change that occurs after the outage. In particular, note that the meanshift test is using the likelihood ratio between the distribution of the observations before and at the changepoint. More specifically, assuming that an outage occurs in line (m, n), the expected value of the statistic at the changepoint is given by

$$\mathbb{E}^{\mu}_{(m,n)}\left[\log \frac{f^{\mu}_{(m,n)}(\Delta\theta[k])}{f_{\infty}(\Delta\theta[k])} \right] = D(f^{\mu}_{(m,n)} \parallel f_{\infty}) > 0. \quad (15)$$

On the other hand, after the changepoint ($k > \lambda$), the expected value of the statistic is given by

$$\mathbb{E}^{\sigma}_{(m,n)}\left[\log \frac{f^{\mu}_{(m,n)}(\Delta\theta[k])}{f_{\infty}(\Delta\theta[k])} \right] =$$
$$= D(f^{\sigma}_{(m,n)} \parallel f_{\infty}) - D(f^{\sigma}_{(m,n)} \parallel f^{\mu}_{(m,n)}), \quad (16)$$

which could be either positive or negative.

4.2. Shewhart Test

Similar to the meanshift test, the Shewhart test is also a "one-shot" detection scheme. This test attempts to detect a change on the observation sequence through the meanshift and the change in the covariance of the data. The Shewhart test statistic for line (m, n) outage is defined as:

$$W^{SH}_{(m,n)}[k] = \max \left\{ \log \frac{f^{\mu}_{(m,n)}(\Delta\theta[k])}{f_{\infty}(\Delta\theta[k])}, \log \frac{f^{\sigma}_{(m,n)}(\Delta\theta[k])}{f_{\infty}(\Delta\theta[k])} \right\}, \tag{17}$$

where the first log-likelihood ratio is used to detect the meanshift, while the second log-likelihood ratio is used to detect the persistent change in the covariance. The stopping time is:

$$\tau^{SH} = \inf_{(m,n)\in\mathcal{E}} \left\{ \inf\{k \geq 1 : W^{SH}_{(m,n)}[k] > A^{SH}_{(m,n)}\} \right\}. \tag{18}$$

Since the Shewhart test exploits the covariance change in addition to the meanshift statistic, it should perform better than the meanshift test, at least as the meantime to false alarm goes to infinity, which is verified in the case studies.

All the detection algorithms presented in this paper can also be used to identify the outaged line, which is estimated to be the line with the largest statistic at the stopping time, τ. Denote $\hat{L}$ to be the estimated outaged line. Then,

$$\hat{L} = \arg \max_{(m,n)\in\mathcal{E}} W_{(m,n)}[\tau]. \tag{19}$$

5. SIMULATION RESULTS AND DISCUSSION

In this section, we demonstrate the effectiveness of our proposed line outage detection algorithm on a IEEE 14-bus system. We applied the CuSum algorithm to the IEEE 14-bus test system for an outage in line $(2, 5)$, with the thresholds chosen according to (8). The entries of $\Delta P[k]$ are sampled from a zero-mean Gaussian p.d.f. with covariance matrix, $\Sigma = \text{diag}(0.5)$. We simulate a the line outage at $k = 10$ and the results are shown in Fig. 1. The $W^{CU}_{(2,5)}[k]$ statistic
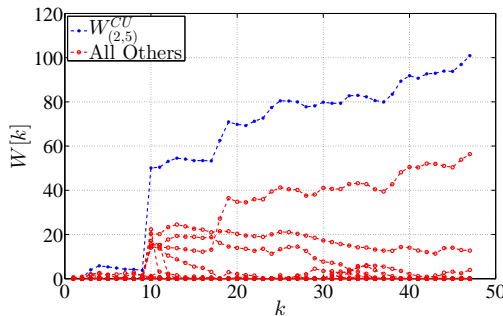
(blue) crosses the threshold of $A^{CU}_{(2,5)} = 100$ at $\tau^{CU} = 57$, resulting in a detection delay of 47 samples. As expected, all of the other CuSum statistics (red) either remain close to zero, or increase at a slower rate.

Next, we perform Monte Carlo simulations for the Shewhart, meanshift, and CuSum algorithms to obtain plots of average detection delay versus mean time to false alarm. The values for the average detection delay are obtained by simulating an outage in line $(4, 5)$ and running the corresponding detection algorithms for different thresholds until a detection of the outaged line is declared. For computing the mean time to false alarm, the detection algorithms are executed for the power system under normal operation until a false alarm occurs. Since false alarm events are in general rare, averaging many sample runs would incur significant computation time. In order to reduce the simulation time, importance sampling is used for the meanshift and Shewhart tests. For our simulations, we found that the error bounds for all the simulated values are within 5% of the means.

Figure 2 shows the average detection delay versus mean time to false alarm for all of the detection methods mentioned in this paper. As evidenced by the crossing of the Shewhart and meanshift plots, for small values of mean time to false alarm, the meanshift test performs better than the Shewhart test. It can be verified from QCD theory that the slope of Delay versus log(mean time to false alarm) for the Shewhart and CuSum tests is given by $\frac{1}{D(f^{\sigma}_{(m,n)} \| f_{\infty})}$ for large mean time to false alarm [11].

From the plots, we conclude that for the same value of mean time to false alarm, both CuSum-based algorithms have a much lower average detection delay compared to the Shewhart and meanshift algorithms. In addition, the figure shows that when we use varied thresholds for the CuSum test as opposed to a fixed threshold, even lower detection delay can be achieved for the same mean time to false alarm. This illustrates that our algorithm is an improvement over that of [9]. Lastly, simulation results demonstrate that the detection delay scales exponentially with the selected thresholds for both the meanshift and Shewhart tests, and linearly for the CuSum-based tests.
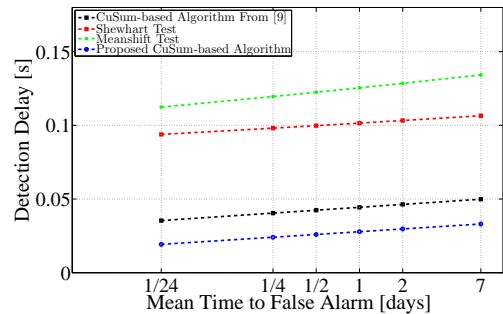

Fig. 1. Example of a run of the CuSum for the 14-bus system.

Fig. 2. Detection delay vs. mean time to false alarm.

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