Stochastic Hybrid Systems Models for Performance and Reliability Analysis of Power Electronic Systems

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Abstract

In this chapter, we propose stochastic hybrid systems (SHS) models for performance and reliability analysis of power electronic systems. The state space of an SHS is composed of: i) a discrete state that describes the possible configurations of the system, and ii) a continuous state that captures a physics-based or behavioral model of the system associated with performance metrics of interest. Transitions of the discrete state are random, and occur at rates that can be functions of the continuous state. Similarly, the evolution of the continuous state is discrete-state dependent. Transitions of the discrete state are accompanied by reset maps that determine how the transitions affect the continuous state. The proposed framework expands on conventional reliability models such as Markov models by providing the capability to monitor a dynamical system that captures some notion of performance of the power electronic system. Applications of the framework are demonstrated to model the expected accumulated revenue of a two-inverter residential-scale photovoltaic system.

1. Introduction

Stochastic hybrid systems (SHS) are a class of stochastic processes with a state space composed of a discrete state and a continuous state. The transitions of the discrete state are random, and the rates at which these transitions occur are, in general, a function of the value of the continuous state. For each value that the discrete state takes—referred subsequently as *modes* of the system—the evolution of the continuous state is described by a stochastic differential equation. The vector fields that govern the evolution of the continuous state in each mode depend on the operational characteristics of the system in that mode. Reset maps associated with mode transitions define how the discrete and continuous states map into post-transition

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discrete and continuous states. The SHS-based framework we outline is built on theoretical foundations developed in [1, 2].

The SHS formalism described above is well suited to model system dynamics in a variety of uncertain environments. The set in which the discrete state takes values describes the possible configurations that the system can adopt. For instance, in the context of power electronic systems reliability modeling, in addition to one (or more) nominal (non-faulted) operational modes, other operational modes include those that arise due to faults (and repairs) in components that comprise the system. Modes may also model discretizations of uncertain generation and load values in several applications including renewable-based resources, energy storage, and electric vehicle systems [3, 4, 5]. The continuous state captures the evolution of variables associated with the system's performance. For instance, physics-based models could include inductor currents and capacitor voltages, network voltages and currents, and electrical frequency as the continuous states in the SHS. Similarly, behavioral models could be formulated to describe system economics, expended repair costs, availability, energy yield, and incentives to participate in demand-response programs. Reset maps in this context are enabling and improve modeling accuracy since they can describe instantaneous impacts of mode transitions (arising from failures and repairs in the constituent elements of the power electronic system) on the continuous states.

An SHS-based model is completely characterized by the combined distribution of the continuous and discrete states. However, the coupling between the discrete and continuous states—given the generality afforded to the transition rates and reset maps in the model formulation—renders the problem of obtaining the combined distribution analytically intractable in most practical applications. In fact, the combined distribution can only be recovered in a few special cases. For instance, if the evolution of the discrete state does not depend on the continuous state, the model boils down to a continuous-time Markov chain; the probability distribution in this case is fully characterized and specified by the Chapman-Kolmogorov equations [6]. However, given the difficulty in obtaining the combined distribution of the discrete and continuous states, we focus on a method to compute any arbitrary number of their raw moments. To this end, we leverage the approach outlined in [1, 2], and demonstrate how to formulate a family of nonlinear ODEs, the solutions of which yield the moments of the continuous state. In lieu of the availability of the complete distribution, the moments are useful in many contexts pertaining to dynamic risk assessment in the system. For instance, the moments can be used to compute bounds on the probabilities of events when the continuous states do not satisfy performance specifications.

The SHS framework encompasses a variety of commonly used stochastic modeling and analysis tools including: i) jump linear systems (linear flows with no jumps in the continuous state); ii) discrete-space continuous-time Markov chains (no continuous state and constant/time-varying transition rates for the discrete state); iii) Markov reward models (constant rate of growth in the continuous state); and iv) piecewise deterministic Markov processes (no diffusion terms in the stochastic differential equations that govern the evolution of the continuous state). Given the generality offered by the SHS modeling formalism, they have been applied to study a host of systems such as communication networks, financial systems, air-traffic management systems, bulk power systems, and biological systems (e.g., see [7] and references therein).

The goals of this chapter are to provide a concise introduction to SHS (Section 2.1), demonstrate how the moments of the continuous state can be recovered (Section 2.2) and leveraged for dynamic risk assessment (Section 2.4), and establish the links between SHS and Markov reliability models (Section 2.5). As an application example, we demonstrate how the SHS-based approach can be leveraged to model the accumulated revenue in operating a photovoltaic (PV) system under an uncertain environment characterized by failures and repairs in the constituent inverters (Section 3).

2. Fundamentals of Stochastic Hybrid Systems

This section begins with a brief overview of stochastic hybrid systems (SHS). For a particular class of SHS, leveraging the results in [1, 2], we demonstrate how to formulate a family of ODEs, the solutions of which can yield the moments of the discrete and continuous states of the SHS.

2.1. Evolution of Continuous and Discrete States

In the most general sense, an SHS is a combination of a continuous-time, discrete-state stochastic process $Q(t) \in \mathcal{Q}$, coupled with a continuous-time, continuous-state stochastic process $X(t) \in \mathbb{R}^n$. Let $\mathcal{Q}_i^+ \subseteq \mathcal{Q}$ denote the set of all modes that $Q(t)$ can transition to, given that $\Pr\{Q(t) = i\} = 1$; similarly, let $\mathcal{Q}_i^- \subseteq \mathcal{Q}$ denote the set of all modes from which $Q(t)$ can transition to mode i. The evolution of $Q(t)$ and $X(t)$ can be described with the aid of the following functions:

$$\lambda_{ij}(x,t), \quad \lambda_{ij} \colon \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^+, \tag{1}$$

$$\phi_{ij}(q,x), \quad \phi_{ij} \colon \mathcal{Q} \times \mathbb{R}^n \to \mathcal{Q} \times \mathbb{R}^n. \tag{2}$$

The λ_{ij}'s are the *transition rates* that govern the times when the system switches from mode i to mode j, and the ϕ_{ij}'s are the *transition reset maps* that tell us how the discrete and continuous states change when there is a reset.[3]

We now provide an intuitive description of how the discrete and continuous state evolve in an SHS. Without loss of generality, as a particular example consider the SHS system in mode i at time t, i.e., $\Pr\{Q(t) = i\} = 1$. In a small time interval $[t, t+\tau)$ the probability of a transition out of mode i is given by

$$\sum_{j \in \mathcal{Q}_i^+} \lambda_{ij}(X(t),t)\tau + o(\tau), \tag{3}$$

[3] We point out that the notation adopted for reset maps is slightly cumbersome; in particular, for the $i \to j$ transition, it follows that $\phi_{ij}(q,\cdot) = \phi_{ij}(i,\cdot) = (j,\cdot)$. Nevertheless, we persist with this notation for clarity.

and the probability of a particular $i \to j$ transition is given by

$$\lambda_{ij}(X(t), t)\tau + o(\tau). \tag{4}$$

If the $i \to j$ transition occurs, the new values of Q and X (i.e., the initial conditions for the post-transition evolution) are defined to be

$$\phi_{ij}(Q((t+\tau)^-), X((t+\tau)^-)) = \phi_{ij}(i, X((t+\tau)^-)) = (j, X(t+\tau)), \tag{5}$$

where $f(t^-) := \lim_{s \to t} f(s)$. The probability that no transition out of state i occurs in the time interval $[t, t+\tau)$ is given by

$$1 - \tau \sum_{k \in \mathcal{Q}_i^+} \lambda_{ik}(X(t), t). \tag{6}$$

Between transitions, $X(t)$ evolves according to

$$\frac{d}{dt} X(t) = f(Q(t),\, X(t),\, t), \tag{7}$$

where $f : \mathcal{Q} \times \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$. In general, the evolution of the continuous state can be governed by a stochastic differential equation. For details of this more general setting, readers are referred to [1, 2].

The SHS model described above affords flexibility and generality to model a variety of stochastic phenomena of interest in power electronic systems. With a particular emphasis on reliability modeling, the elements of the set $\mathcal{Q}$ index different *operational modes*, including the nominal (non-faulted) mode and any modes that arise due to faults (and repairs) in the components comprising the power electronic system. Similarly, $X(t)$ denotes the underlying dynamic states of the power electronic system of interest. The states of $X(t)$ could be derived from physics-based models. In this case, they could represent, for instance, inductor currents and capacitor voltages in the power converter model. Based on the desired modeling resolution, the dynamics in (7) could represent an averaged or switching time-scale model of the power converter. Alternately, one could investigate other behavioral models that describe a particular attribute of interest for the power electronic system under study. For example, in Section 3, we provide a numerical case study focused on PV system economics. In this setting, $X(t)$ represents the accumulated revenue of the PV system.

2.2. Test Functions, Extended Generator, and Moment Evolution

The evolution of the discrete and continuous states in the general SHS model described in Section 2.1 is tightly coupled. In particular, the vector field f that governs $X(t)$ (see (7)) is discrete-state dependent. Concurrently, transitions of the discrete state depend on the value of the continuous state, since the transition rates, λ_{ij} in general are functions of the continuous state, $X(t)$ (see (1)). This tight interplay challenges the analysis of SHS, and indeed, it is intractable to obtain the distribution of the discrete and continuous states in closed form except in some elementary cases. Therefore, we focus instead on computing the moments of the discrete and continuous states; the general procedure for this is described next.

Consider the SHS model described in Section 2.1. We define a *test function*, $\psi(q,\, x)$, $\psi\colon\, \mathcal{Q}\times\mathbb{R}^n\to\mathbb{R}$, a linear operator given by

$$\begin{aligned}(L\psi)\,(q,x) =& \frac{\partial}{\partial x}\psi(q,x)\cdot f(q,x,t)\\ &+\sum_{i,j\in\mathcal{Q}}\lambda_{ij}(x,t)\Big(\psi(\phi_{ij}(q,x))-\psi(q,x)\Big),\end{aligned}\tag{8}$$

where $\partial\psi/\partial x\in\mathbb{R}^{1\times n}$ denotes the gradient of $\psi(q,\, x)$ with respect to x, $\lambda_{ij}(x,t)$ is the transition rate for the $i\to j$ transition, and $\phi_{ij}(q,x)$ denotes the corresponding reset map for the discrete and continuous states. The definition of the test function and the generator above follows from [1, 2, 8]. The evolution of the expected value of the test function $\mathbb{E}\left[\psi(Q(t),\, X(t))\right]$, is governed by Dynkin's formula, which can be stated in differential form as follows [1, 8]:

$$\frac{d}{dt}\mathbb{E}[\psi(Q(t),\, X(t))] = \mathbb{E}[(L\psi)(Q(t),\, X(t))].\tag{9}$$

Dynkin's formula indicates that the time rate of change of the expected value of a test function evaluated on the stochastic process is given by the expected value of the generator. Given the definition of the generator in (8), this makes intuitive sense. The first term in (8) captures the total derivative of the test function with respect to time, and the second term captures the impact of incoming and outgoing transitions on the test function [9].

By judicious choice of test functions, (9) can be used to obtain ODEs that describe the evolution of relevant conditional moments of interest. From this, the law of total expectation will yield the desired moments of the continuous states. For the SHS model where the discrete state $Q(t)$ takes values in the set $\mathcal{Q}$, we define the following family of test functions:

$$\psi_i^{(m)}(q,\, x) := \begin{cases} x^m & \text{if} \quad q=i \\ 0 & \text{if} \quad q\neq i\end{cases},\forall i\in\mathcal{Q},\tag{10}$$

where

$$\begin{aligned} m &:= (m_1,\, m_2,\ldots,\, m_n)\in\mathbb{N}^{1\times n},\\ x^m &:= x_1^{m_1}x_2^{m_2}\ldots x_n^{m_n}.\end{aligned}\tag{11}$$

With this definition in place, note that the mth order conditional moment of the continuous state—conditioned on the discrete state being in mode i—is given by the expected value of the test function. In particular, we have $\forall i\in\mathcal{Q}$:

$$\mu_i^{(m)}(t) := \mathbb{E}[\psi_i^{(m)}(q,x)] = \mathbb{E}[X^m(t)|Q(t)=i]\pi_i(t),\tag{12}$$

where $\pi_i(t)$ denotes the occupational probability of mode i, i.e.,

$$\pi_i(t) := \Pr\{Q(t) = i\}. \tag{13}$$

2.3. Evolution of the Dynamic-state Moments

Now that various conditional moments are defined, we explain how the law of total expectation is applied to obtain the moments of $X(t)$ from them. We then derive ODEs that govern the evolution of the conditional moments of $X(t)$ [2, 8].

Suppose we want to compute $\mathbb{E}[X^m(t)]$, for some $m \in \mathbb{N}^{1\times n}$. Applying the law of total expectation, it follows that this is given by

$$\mathbb{E}[X^m(t)] = \sum_{i\in\mathcal{Q}} \mathbb{E}[X^m(t)|Q(t) = i]\,\pi_i(t) = \sum_{i\in\mathcal{Q}} \mu_i^{(m)}(t). \tag{14}$$

Therefore, at each time t, to obtain $\mathbb{E}[X^m(t)]$, we need to know the conditional moments of $X(t)$, $\mu_i^{(m)}(t)$, $\forall i \in \mathcal{Q}$. Dynkin's formula (9) then yields ODEs that govern the evolution of $\mu_i^{(m)}(t)$. In particular, the evolution of $\mu_i^{(m)}(t)$, $\forall i \in \mathcal{Q}$, is given by

$$\dot{\mu}_i^{(m)}(t) = \frac{d}{dt}\mathbb{E}[\psi_i^{(m)}(q,x)] = \mathbb{E}[(L\psi_i^{(m)})(q,x)]. \tag{15}$$

Simulating a family of relevant ODEs of the form in (15), and applying (14) yields the desired moment of interest.

2.4. Leveraging Continuous-state Moments for Dynamic Risk assessment

While the combined distribution of the discrete and continuous states would completely characterize the SHS, we have already noted how this is intractable to recover. Nonetheless, the moments of the continuous states convey important information about the distribution, and in fact, upper bounds on the probability that the power electronic system dynamic states satisfy certain performance requirements can be obtained with a few lower-order moments.

Suppose performance requirements establish the maximum and minimum values that the continuous state $x(t)$ can take at any time by $\mathcal{R}_x := [x^{\min}, x^{\max}]$. In spite of mode transitions (including those triggered by failures and repairs), we are interested in studying whether the continuous state satisfies the performance requirements, i.e., we are interested in answering whether $x(t) \in \mathcal{R}_x$, $\forall t$. This can be accomplished by establishing the following probabilistic notion of risk, $\rho_x(t)$, which quantifies the probability that the continuous state does not confirm to the performance requirements at time $t > 0$,

$$\rho_x(t) := \Pr\{X(t) \notin \mathcal{R}_x\} = 1 - \Pr\left\{x^{\min} \le X(t) \le x^{\max}\right\}. \tag{16}$$

The moments of $X(t)$, i.e., $\mathbb{E}[X^m(t)]$, $m \in \mathbb{N}^+$ that can be obtained from (14)-(15) can be utilized to establish an upper bound on $\rho_x(t)$ using moment inequalities. For example, consider the following Chebyshev-based moment inequality [10], which yields an upper bound on $\rho_x(t)$, that we denote by $\overline{\rho}_x(t)$:

$$\rho_x(t) \leq 1 - \frac{4\left(\left(\mathbb{E}[X(t)] - x^{\min}\right)\left(x^{\max} - \mathbb{E}[X(t)]\right)\right)}{(x^{\max} - x^{\min})^2} - \frac{4\sigma^2_{X(t)}}{(x^{\max} - x^{\min})^2} =: \overline{\rho}_x(t), \tag{17}$$

where $\sigma_X(t)$ is the standard deviation of $X(t)$,

$$\sigma_{X(t)} := \left(\mathbb{E}[X^2(t)] - (\mathbb{E}[X(t)])^2\right)^{1/2}. \tag{18}$$

Essentially, (17) indicates how upper bounds on the probability that the dynamic states do not meet a priori specified performance specifications can be obtained simply from a few lower-order moments, the evolution of which is recovered from the solution of a nonlinear ODE.

One approach to compute $\rho_x(t)$ would be based on repeated Monte Carlo simulations. In each simulation, the transition rates would determine when mode transitions are triggered. Repeated simulations would yield the distribution of the continuous state $X(t)$, from which (16) could be numerically computed. This approach is indeed easy to conceptualize and implement; however, it is computationally burdensome and accuracy is directly related to the number of simulations. On the other hand, the SHS-based alternative is analytical and repeated simulations are not required. Also, the bound in (17) is conservative in the sense that the actual probability of violating a performance objective is always lower. More precise estimates of $\rho_x(t)$ can be computed if higher-order moments are known.

2.5. Recovering Markov Reliability and Reward Models from SHS

A major appeal of SHS is that a wide variety of stochastic modeling frameworks can be recovered as special cases of the most general SHS formalism. In this section, we demonstrate how Markov reliability models and Markov reward models [11, 12] can be recovered as special cases of the most general SHS formalism described in Section 2.

2.5.1. Continuous-time Markov Chains and Markov Reliability Models

A continuous-time discrete-state stochastic process $Q(t)$ is called a continuous-time Markov chain (CTMC) if it satisfies the Markov property:

$$\Pr\left\{Q(t_r) = i | Q(t_{r-1}) = j_{r-1}, \ldots, Q(t_1) = j_1\right\} = \Pr\left\{Q(t_r) = i | Q(t_{r-1}) = j_{r-1}\right\}, \tag{19}$$

for $t_1 < \cdots < t_r$, $\forall\, i,\, j_1, \ldots, j_{r-1} \in \mathcal{Q}$, and for $r > 1$ [6]. The chain Q is said to be *homogeneous* if it satisfies

$$\Pr\left\{Q(t) = i | Q(s) = j\right\} = \Pr\left\{Q(t-s) = i | Q(0) = j\right\}, \ \forall i,\, j \in \mathcal{Q},\ 0 < s < t. \tag{20}$$

With the states of the CTMC, i.e., the entries of the set $\mathcal{Q}$, denoting different operational modes of the system of interest, we recover a Markov reliability model. Transitions of the discrete state are triggered by failures and subsequent repair actions that aim to restore functionality [11]. Continuous-time Markov chains are commonly used for system reliability and availability modeling in many application domains. In addition to power and energy systems [13, 14, 15, 16, 17, 18], these include: computer systems [19], communication networks [20], electronic circuits [21, 22], and phased-mission systems [23, 24].

The problem of interest in Markov reliability models is to determine the distribution of the discrete state, $Q(t)$, at any instant of time $t > 0$. Recall, from (13) that the occupational probability of mode i is denoted by $\pi_i(t)$. Let us denote the entries of the column vector of occupational probabilities by $\{\pi_q(t)\}_{q\in\mathcal{Q}}$. The evolution of $\pi(t)$ is governed by the Chapman-Kolmogorov equations [12]:

$$\frac{d}{dt}\pi(t) = \Lambda\pi(t), \tag{21}$$

where $\Lambda \in \mathbb{R}^{|\mathcal{Q}|\times|\mathcal{Q}|}$ is the Markov chain generator matrix that is composed of component failure and repair rates. In particular, let

$$\lambda_{ij}(t), \quad \lambda_{ij}\colon \mathbb{R}^+ \to \mathbb{R}^+, \tag{22}$$

denote the transition rate for the $i \to j$ transition. Then the generator matrix, Λ, is constructed as follows:

$$[\Lambda]_{ij} = \begin{cases} \lambda_{ji}(t) & \text{if} \quad i \neq j, j \in \mathcal{Q}_i^- \\ -\sum_{\ell\in\mathcal{Q}_i^+}\lambda_{i\ell}(t) & \text{if} \qquad i = j. \end{cases} \tag{23}$$

The evolution of the ith occupational probability from (21), (23) is therefore given by

$$\dot{\pi}_i(t) = \sum_{j\in\mathcal{Q}_i^-}\lambda_{ji}(t)\pi_j(t) - \sum_{k\in\mathcal{Q}_i^+}\lambda_{ik}(t)\pi_i(t). \tag{24}$$

Markov reliability and availability models can be readily recovered from the general SHS formulation in Section 2 by ignoring the continuous states, $X(t)$, and the reset maps $\phi(\cdot)$. When the continuous states are ignored, the transition rates in (1) are now either constants or functions of time, thus recovering the formulation in (22). Therefore, using (15) we can recover the Chapman-Kolmogorov differential equations that govern the occupational probabilities of the CTMC that underlies the Markov reliability model. To this end, choosing $m = (0, 0, \ldots, 0)$ in (12) recovers the discrete-state occupational probabilities

$$\mu_i^{(0,0,\ldots,0)}(t) = \Pr\{Q(t) = i\} = \pi_i(t). \tag{25}$$

Subsequently, the moment ODEs in (15) boil down to

$$\dot{\mu}_i^{(0,\ldots,0)}(t) = \sum_{j\in\mathcal{Q}_i^-}\lambda_{ji}(t)\mu_j^{(0,\ldots,0)}(t) - \sum_{k\in\mathcal{Q}_i^+}\lambda_{ik}(t)\mu_i^{(0,\ldots,0)}(t), \tag{26}$$

which are precisely the Chapman-Kolmogorov differential equations for the occupational probabilities of the CTMC (24).

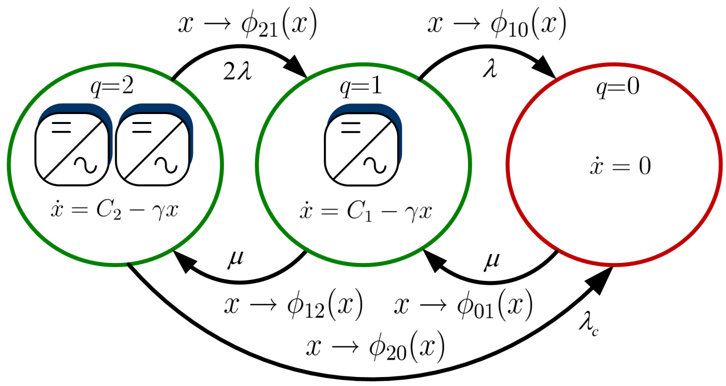

Figure 1: State transition diagram for the PV-system reliability model.

2.5.2. *Markov Reward Models*

A Markov reward model comprises a Markov chain $Q(t)$ taking values in the set $\mathcal{Q}$ (which describes the possible system operational modes) and an *accumulated reward* $X(t)$, which captures some performance measure of interest. The most commonly studied Markov reward models are rate-reward models (see, e.g., [25, 26], and the references therein). The accumulated reward in rate-reward models evolves according to

$$\frac{dX(t)}{dt} = f(Q(t)),$$

where $f\colon \mathcal{Q} \rightarrow \mathbb{R}$ is the (discrete-state-dependent) reward growth rate. Impulses in the accumulated reward capture one-time effects due to failures or repairs of components in the system. The Markov reward formalism can also be recovered as a special case of the most general SHS formulation; in particular, with the choice $f(q, x, t) = f(q)$ in (7), we recover the Markov reward modeling framework.

3. Application of SHS to PV System Economics

This case study demonstrates how the SHS framework can be applied to model the accumulated revenue in a residential-scale PV system with multiple inverters. The sample PV system is the Gable Home: a net-zero, solar-powered house built for the U. S. Department of Energy's 2009 Solar Decathlon [27]. The PV electrical system consists of forty 225 W mono-crystalline modules. The dc power sourced by the PV modules is converted to utility-compatible ac power by two 5 kW grid-tied inverters.

The state-transition diagram that illustrates the reliability model for the inverters in the system is depicted in Fig 1. The CTMC that describes the reliability model takes values in the set $Q = \{0, 1, 2\}$. In operational mode 2, both inverters are functioning, in operational mode 1, a single inverter is functioning, and in operational mode 0, both inverters have failed. The failure rate, repair rate, and common-cause failure rate are denoted by λ, μ, and λ_c, respectively. From the state-transition diagram in Fig. 1, it follows that the transition rates are

$$\lambda_{21} = 2\lambda, \quad \lambda_{20} = \lambda_{\mathrm{c}},$$
$$\lambda_{10} = \lambda, \quad \lambda_{01} = \lambda_{12} = \mu. \tag{27}$$

The reward of interest is the accumulated revenue of operating the PV system, denoted by $X(t)$. The constant rate at which the accumulated revenue grows in the i operational mode is denoted by C_i \$/yr. Additionally, we factor in a degradation rate (that captures natural wear and tear and depreciation) denoted by γ in each operational mode. The dynamical systems that govern the evolution of the accumulated revenue in the three modes are specified by

$$f(q,x) = \begin{cases} C_2 - \gamma x & \text{if} \quad q = 2, \\ C_1 - \gamma x & \text{if} \quad q = 1, \\ 0 & \text{if} \quad q = 0. \end{cases} \tag{28}$$

Transitions due to failures are associated with impulses that model one-time expenses in replacing or repairing the inverters. In particular, the impulse change in accumulated revenue as a result of a failure transition from operational mode i to mode j is denoted by C_{ij} \$.[4] The reset maps that describe how the discrete and continuous states are affected by the transitions are given by

$$\phi_{21}(q,x) = (1, x - C_{21}), \quad \phi_{20}(q,x) = (0, x - C_{20}),$$
$$\phi_{10}(q,x) = (0, x - C_{10}), \quad \phi_{01}(q,x) = (1, x), \quad \phi_{12}(x) = (2, x). \tag{29}$$

The problem of interest is to determine the moments of the accumulated revenue of the PV system, i.e., $\mathbb{E}[X(t)]$, $\mathbb{E}[X^2(t)]$. We address this problem with the SHS-based framework. To this end, begin by defining test functions for each state of the CTMC:

$$\psi_i^{(m)}(q,\, x) = \begin{cases} x^m & \text{if} \quad q = i \\ 0 & \text{if} \quad q \neq i \end{cases}, \; i \in \mathcal{Q} = \{0, 1, 2\}. \tag{30}$$

[4]The cost parameters could be modeled to be time-dependent to factor inflation or cash flow streams. For instance, following along the model in [13], we could model $C_i(t) = C_i e^{-\delta t}$ and $C_{ij}(t) = C_{ij} e^{-\delta t}$; where δ is the discount rate that represents future costs by a discounted value [13].

From (8), the extended generators are given by

$$\begin{aligned}
\left(L\psi_0^{(m)}\right)(q,\,x) &= -\mu\psi_0^{(m)}(q,x) + \lambda\left(\psi_1^{(1)}(q,x) - C_{10}\psi_1^{(0)}(q,x)\right)^m \\
&+ \lambda_{\mathrm{c}}\left(\psi_2^{(1)}(q,x) - C_{20}\psi_2^{(0)}(q,x)\right)^m, \qquad (31)\\
\left(L\psi_1^{(m)}\right)(q,x) &= mc_1(t)\psi_1^{(m-1)}(q,x) - m\gamma\psi_1^{(m)}(q,x) - (\lambda+\mu)\,\psi_1^{(m)}(q,x) \\
&+ 2\lambda\left(\psi_2^{(1)}(q,x) - C_{21}\psi_2^{(0)}(q,x)\right)^m + \mu\psi_0^{(m)}(q,x), \qquad (32)\\
\left(L\psi_2^{(m)}\right)(q,x) &= mc_2(t)\psi_2^{(m-1)}(q,\,x) - m\gamma\psi_2^{(m)}(q,x) \\
&- (2\lambda+\lambda_{\mathrm{c}})\,\psi_2^{(m)}(q,x) + \mu\psi_1^{(m)}(q,x). \qquad (33)
\end{aligned}$$

Applying Dynkin's formula in (15) to (31)-(33), we obtain the following set of differential equations for the conditional moments of mth order,

$$\begin{aligned}
\frac{d}{dt}\mu_0^{(m)}(t) &= -\mu\mu_0^{(m)}(t) + \lambda\left((-1)^m C_{10}^m\pi_1(t) + \sum_{k=0}^{m-1}\binom{m}{k}\mu_1^{(m-k)}(t)(-1)^k C_{10}^k\right) \\
&+ \lambda_{\mathrm{c}}\left((-1)^m C_{20}^m\pi_2(t) + \sum_{k=0}^{m-1}\binom{m}{k}\mu_2^{(m-k)}(t)(-1)^k C_{20}^k\right), \qquad (34)
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt}\mu_1^{(m)}(t) &= mc_1(t)\mu_1^{(m-1)}(t) - m\gamma\mu_1^{(m)}(t) - (\lambda+\mu)\,\mu_1^{(m)}(t) \\
&+ 2\lambda\left((-1)^m C_{21}^m\pi_2(t) + \sum_{k=0}^{m-1}\binom{m}{k}\mu_2^{(m-k)}(t)(-1)^k C_{21}^k\right) + \mu\mu_0^m(t), \qquad (35)
\end{aligned}$$

$$\frac{d}{dt}\mu_2^{(m)}(t) = mc_2(t)\mu_2^{(m-1)}(t) - m\gamma\mu_2^{(m)}(t) - (2\lambda+\lambda_{\mathrm{c}})\,\mu_2^{(m)}(t) + \mu\mu_1^{(m)}(t), \qquad (36)$$

where $\pi_0(t)$, $\pi_1(t)$, and $\pi_2(t)$ are the occupational probabilities of the different modes. The m-order moment of the accumulated revenue is given by

$$\mathbb{E}[X^m(t)] = \mu_0^{(m)}(t) + \mu_1^{(m)}(t) + \mu_2^{(m)}(t). \qquad (37)$$

Notice that substituting $m = 0$ in (34)-(36) recovers the Chapman-Kolmogorov equations: $\dot{\pi}(t) = \Lambda\pi(t)$, where $\pi(t) = [\pi_0(t),\, \pi_1(t),\, \pi_2(t)]^{\mathrm{T}}$, and Λ is given by:

$$\Lambda = \begin{bmatrix} -\mu & \lambda & \lambda_{\mathrm{c}} \\ \mu & -(\lambda+\mu) & 2\lambda \\ 0 & \mu & -(2\lambda+\lambda_{\mathrm{c}}) \end{bmatrix}.$$

For illustration, we consider the following simulation parameters. The transition rates are assumed to be $\lambda = 0.1\,\mathrm{yr}^{-1}$, $\lambda_{\mathrm{c}} = 0.001\,\mathrm{yr}^{-1}$, and $\mu = 30\,\mathrm{yr}^{-1}$ [16]. The impulse costs are assumed to be a fraction, ρ, of the upfront inverter installed cost, $C_{\mathrm{inverter}} = \$2850$; with this model we have $C_{21} = C_{10} = \rho C_{\mathrm{inverter}}$, and

$C_{20} = 2\rho C_{\text{inverter}}$. We set the nominal value of ρ to be 6 %. The nominal degradation rate γ, is set to be 0.7% following [28]. The rate at which revenue is accumulated for the two-inverter system, C_2, is assumed to be 1125 \$/yr, and we assume that $C_1 = C_2/2$. This is computed using the National Renewable Energy Laboratory PVWatts® Calculator[5] for the location of Springfield, IL, assuming total system losses of 14%, inverter efficiency of 96%, and a dc-ac size ratio of 1.1 (given the inverter rating of 5 kW).

We demonstrate the impact of the model parameters on the expected accumulated revenue of the PV system. For comparison, we also model the evolution of the accumulated revenue with an alternate investment of $2C_{\text{inverter}}$ that accrues revenue at a discount rate d. Figure 2 plots the expected accumulated revenue with the PV system and also depicts the accumulated revenue with the alternate revenue for discount rates of $d = 1,\ 3,\ 5\%$ The intersection of the curves provides an idea of when the PV-inverter investment is expected to be competitive with the alternate investment, i.e., the expected payback time for the inverter cost. Figure 3 plots the expected accumulated revenue with the PV system for degradation rates of $\gamma = 0.1, 5, 10\%$, and also depicts the accumulated revenue with the alternate investment for a nominal discount rate of $d = 1\%$. For instance, the expected payback time only increases by around 3 yrs for an increase in degradation rate by two orders of magnitude. With the proposed repair model, the expected payback time more than doubles as the fraction of upfront inverter costs expended in repairs varies from 10% to 100%.

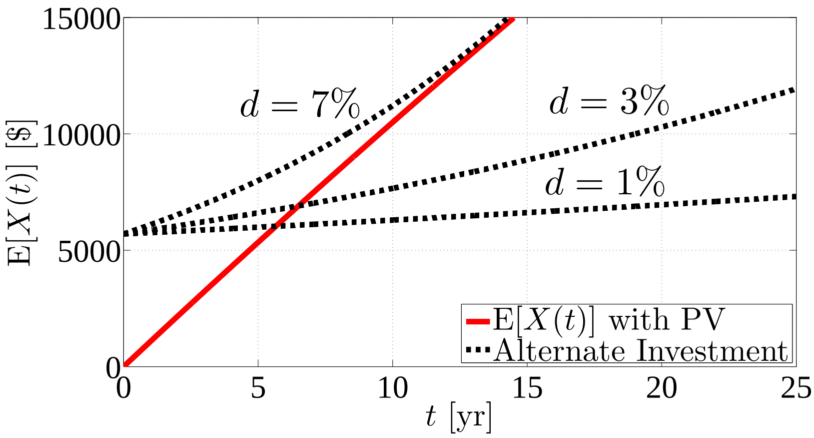

Figure 2: Comparing the PV investment with an alternative investment that accrues value at discount rates $d = 1, 3, 7\%$.

[5][Online] Available: http://pvwatts.nrel.gov/

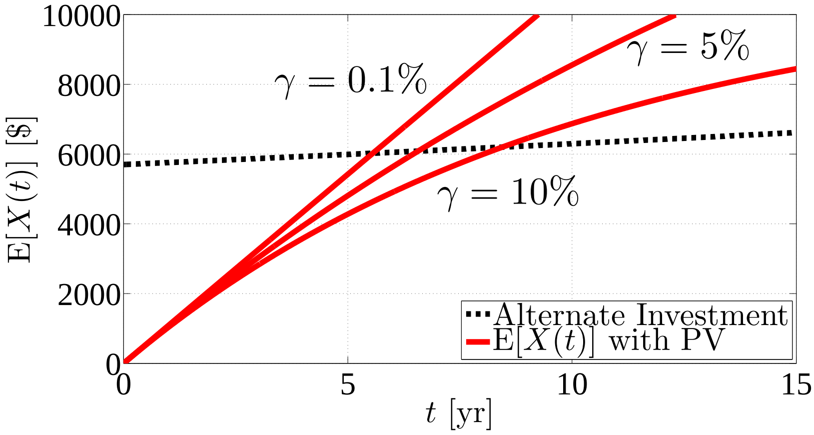

Figure 3: Comparing the PV investment for different degradation rates, $\gamma = 0.1, 5, 10\%$ with the alternative investment.

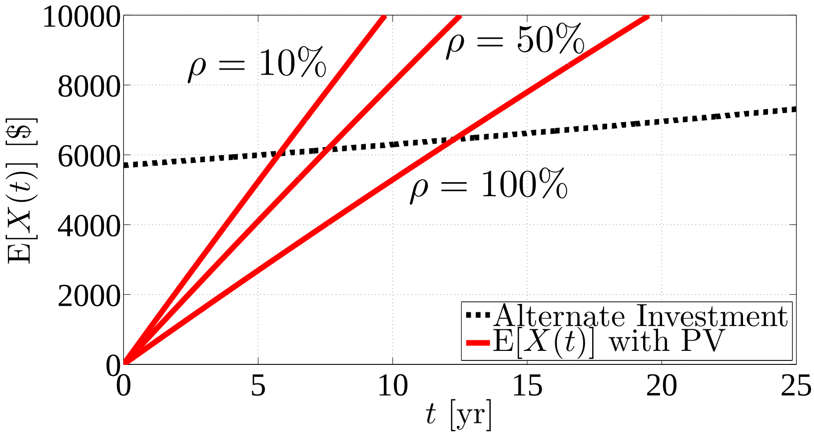

Figure 4: Comparing the PV investment with an alternative investment as the fraction of upfront inverter costs expended in repair is varied from $\rho = 10\%$ to $\rho = 100\%$.

4. Concluding Remarks

This chapter introduced an SHS-based framework to analyze the performance and reliability of power electronic systems. Expanding on conventional reliability models that only include a discrete state space, the SHS formulation includes continuous states from physics-based or behavioral models of a power converter. Analytical methods based on Dynkin's formula were leveraged to obtain the moments of the continuous state. We demonstrated how the moments can be utilized in dynamic risk assessment. Applications of the framework to model the expected accumulated revenue in a PV system were presented.

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