

Online Computation of Power System Linear Sensitivity Distribution Factors

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Abstract—Linear sensitivity distribution factors (DFs) are commonly used in power systems analyses, e.g., to determine whether or not the system is N-1 secure. This paper proposes a method to compute linear sensitivity distribution factors (DFs) in near real-time without relying on the system power flow model. Instead, through linear least-squares estimation (LSE), the proposed method only uses high-frequency synchronized data collected from phasor measurement units (PMUs) to estimate the injection shift factors (ISFs). Subsequently, ISFs can be used to compute other DFs. Such a measurement-based approach is desirable since it is adaptive to changes in system operating point and topology. We illustrate the value of the proposed measurement-based DF estimation approach over the traditional model-based method through several examples.

I. INTRODUCTION

In order to monitor operational reliability, power system operators rely heavily on online studies using a model of the system obtained offline [1]. One such study is N-1 contingency analysis, with which operators determine whether or not the system will meet operational reliability requirements in case of outage in any one particular asset (e.g., a generator or transmission line) [2]. In general, these model-based online studies may include repeated computations of power flow solutions using the full nonlinear system model, a linearized model, or, in the simplest case, linear sensitivity distribution factors (DFs) such as injection shift factors (ISFs), power transfer distribution factors (PTDFs), line outage distribution factors (LODFs), and outage transfer distribution factors (OTDFs). For example, in the context of N-1 contingency analysis, ISFs and LODFs are used, in conjunction with an estimate of the system's current operating point, to predict the change in operating point in the event that an outage in certain generating facilities or transmission lines occurs. These post-contingency operating point predictions are then used to determine whether or not the system is N-1 secure.

Conventional model-based studies are not ideal because (i) an accurate model containing up-to-date network topology is required, and (ii) the results from such model-based studies may not be applicable if the actual system evolution does not match any predicted operating points due to unforeseen circumstances such as equipment failure, outages in external areas, or unpredictable levels of renewable generation. For example, in the 2011 San Diego blackout, operators could not detect that certain lines were overloaded or close to being

overloaded because the network model was not up-to-date, which caused state estimator results to be inaccurate [1]. Thus, traditional model-based techniques may no longer satisfy the needs of monitoring and protection tasks, and therefore it is important to develop measurement-based power system monitoring tools that are adaptive to changes in operating point (such as generation or load variations) and topology (such as outage of a transmission line). In this regard, phasor measurement units (PMUs) are an enabling technology for the development of such measurement-based monitoring tools.

Unlike current system measurements, PMUs measure voltages, currents, and frequency at a very high speed (usually 30 measurements per second) [3], and phasors measured at different locations by different devices are time-synchronized [4]. In this paper, we propose a method to estimate linear sensitivity DFs that exploits only measurements obtained from PMUs in near real-time without relying on the system power flow model. These online DFs can be used in numerous applications in power system static security assessment, including contingency analysis, post-contingency generation re-dispatch, congestion management, and model validation. In particular, we rely on real power bus injection and line flow data obtained from PMUs to compute DFs through linear least-squares estimation (LSE).

Distribution factors are widely known and used in power systems analyses [5], [6]. Existing approaches to computing DFs typically employ so-called DC approximations, which can provide fast DC contingency screening [7]. They do not, however, have the flexibility of adapting to changes in network topology or generation and load variations, which can all affect the actual linear sensitivities significantly. Recent attention has been given to the computation of the line outage distribution factor due to their prominent role in revealing and ameliorating cascading outages [8], [9]. Additionally, work has been done in the area of detecting line outages using PMU measurements [10], [11]. Such proposed approaches still largely rely on a model of the system and utilize the so-called DC approximation. In [12], phasor measurements were used in online contingency analysis by monitoring buses that had been classified as high-risk by an offline study. Other applications for PMU measurements include monitoring, protection, and control of power networks (see e.g., [13] and references therein).

II. DF COMPUTATION APPROACH

Distribution factors are linearized sensitivities used online in contingency analysis and remedial action schemes. A key distribution factor is the injection shift factor (ISF), which quantifies the redistribution of power through each transmission line following a change in generation or load on a particular bus. In essence, the ISF captures the sensitivity of the flow through a line with respect to changes in generation or load. Other DFs are the PTFDF, LODF, and OTDF [7], which can all be derived from the ISFs. In this section, we describe the proposed approach to estimate ISFs using LSE and then the computation of other DFs once the ISF estimates have been obtained.

A. Computation of ISFs

The ISF of line L_{k-l} (assume positive real power flow from bus k to l) with respect to bus i , denoted by Ψ_{k-l}^i , is a linear approximation of the sensitivity of the active power flow in line L_{k-l} with respect to the active power injection at node i with the slack bus defined and all other quantities constant. Let $P_i(t)$ and $P_i(t+\Delta t)$, respectively, denote the active power injection at bus i and times t and $t+\Delta t$, $\Delta t > 0$ and small. Define $\Delta P_i(t) = P_i(t+\Delta t) - P_i(t)$ and denote the change in active power flow in line L_{k-l} resulting from $\Delta P_i(t)$ by $\Delta P_{k-l}^i(t)$. Then, based on the definition of ISF, it follows that

$$\Psi_{k-l}^i := \frac{\partial P_{k-l}}{\partial P_i} \approx \frac{\Delta P_{k-l}^i(t)}{\Delta P_i(t)}. \quad (1)$$

In order to obtain Ψ_{k-l}^i , we also need $\Delta P_{k-l}^i(t)$, which are not readily available from PMU measurements. We assume that the net variation in active power through line L_{k-l} , denoted by $\Delta P_{k-l}(t)$, however, is available from PMU measurements. We express this net variation as the sum of active power variations in line L_{k-l} due to active power injection variations at each bus i :

$$\Delta P_{k-l}(t) = \Delta P_{k-l}^1(t) + \dots + \Delta P_{k-l}^n(t). \quad (2)$$

Equivalently, by substituting (1) into (2), we can rewrite (2) as

$$\Delta P_{k-l}(t) \approx \Delta P_1(t)\Psi_{k-l}^1 + \dots + \Delta P_n(t)\Psi_{k-l}^n,$$

where $\Psi_{k-l}^i \approx \frac{\Delta P_{k-l}^i}{\Delta P_i}$, $i = 1, \dots, n$. Suppose $m+1$ sets of synchronized measurements are available. Let $\Delta P_i[j] = \Delta P_i(j\Delta t)$ and $\Delta P_{k-l}[j] = \Delta P_{k-l}(j\Delta t)$, $j = 1, \dots, m$, and define $\Delta P_{k-l} = [\Delta P_{k-l}[1], \dots, \Delta P_{k-l}[j], \dots, \Delta P_{k-l}[m]]^T$, $\Delta P_i = [\Delta P_i[1], \dots, \Delta P_i[j], \dots, \Delta P_i[m]]^T$, and $\Psi_{k-l} = [\Psi_{k-l}^1, \dots, \Psi_{k-l}^i, \dots, \Psi_{k-l}^n]^T$. Further, suppose $m > n$, then we obtain the following overdetermined system:

$$\Delta P_{k-l} = [\Delta P_1 \quad \dots \quad \Delta P_i \quad \dots \quad \Delta P_n] \Psi_{k-l}. \quad (3)$$

For ease of notation, let ΔP represent the $m \times n$ matrix $[\Delta P_1, \dots, \Delta P_i, \dots, \Delta P_n]$. Then, the system in (3) is of the form $\Delta P_{k-l} = \Delta P \Psi_{k-l}$.

The vector of ISFs for line L_{k-l} , $\Psi_{k-l} = [\Psi_{k-l}^1, \dots, \Psi_{k-l}^n]^T$, can be obtained by solving the following LSE problem:

$$\min_{\Psi_{k-l}} e^T e, \quad (4)$$

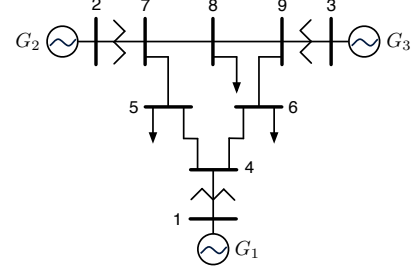


Fig. 1: Network topology for WECC 3-machine 9-bus system.

TABLE I: Comparison of ISFs obtained for Example 1.

Line	Actual (p.u.)	Model-based (p.u.)	LSE (p.u.)
ΔP_{4-5}	-0.2970	-0.3196	-0.3022
ΔP_{4-6}	-0.1734	-0.1804	-0.1749
ΔP_{7-8}	+0.1838	+0.1804	+0.1830

where $e = \Delta P_{k-l} - \Delta P \Psi_{k-l}$, the solution to which is $\hat{\Psi}_{k-l} = (\Delta P^T \Delta P)^{-1} \Delta P^T \Delta P_{k-l}$. In doing so, we make two key assumptions: (i) the ISFs are approximately constant across the $m+1$ measurements, and (ii) the regressor matrix has full column rank.

Example 1 (3-Machine 9-Bus System): We illustrate the concepts described above with the Western Electricity Coordinating Council (WECC) 3-machine 9-bus system, the one-line diagram for which is shown in Fig. 1. In order to simulate PMU measurements of slight fluctuations in active power injection at each bus, we create times-series data for the active power injection at each bus. In particular, the injection at node i , denoted by P_i , is given by

$$P_i[j] = P_i^0[j] + \sigma_1 P_i^0[j] v_1 + \sigma_2 v_2, \quad (5)$$

where $P_i^0[j]$ is the nominal power injection at node i at instant $j\Delta t$, and v_1 and v_2 are pseudorandom values drawn from standard normal distributions with 0-mean and standard deviations $\sigma_1 = 0.1$ and $\sigma_2 = 0.1$, respectively. The first component of variation, $\sigma_1 P_i^0[j] v_1$ represents the inherent fluctuations in generation and load, while the second component, $\sigma_2 v_2$, represents random measurement noise.

In this example, 601 sets of synchronized line flow and bus injection data are acquired, i.e., $m = 600$. For each set of bus injection data obtained at a particular time instant, we compute the power flow, allowing the slack bus to absorb all power imbalances, and compute the active power flow through line L_{k-l} for that time instant. By taking the difference between consecutive line flow measurements, we obtain the vector P_{k-l} in (3). Similarly, we obtain the regressor matrix on the right-hand side of (3) by taking the differences between consecutive values of bus power injections. Suppose a 0.5 p.u. increase is applied to G_2 at bus 2 and the slack bus absorbs the resulting power imbalance. Table I shows a comparison between the corresponding effect on three lines computed from actual power flow solution, linearized model-based approximation, and our proposed measurement-based method. It is evident that our measurement-based approach provides more accurate results than the model-based one. ■

TABLE II: Comparison of change in active power line flows with outage in line L_{8-9} .

Line	Actual (p.u.)	Model-based (p.u.)	LSE (p.u.)
ΔP_{1-4}	0.0071	0.0	0.0027
ΔP_{4-5}	0.2374	0.2410	0.2301
ΔP_{5-7}	0.2349	0.2410	0.2297
ΔP_{4-6}	-0.2303	-0.2410	-0.2274
ΔP_{6-9}	-0.2290	-0.2410	-0.2247
ΔP_{7-8}	0.2458	0.2410	0.2441
ΔP_{3-9}	0.0	0.0	0.0
ΔP_{2-7}	0.0	0.0	0.0

B. Computation of Other Distribution Factors

Once the ISFs are obtained via online estimation, we can compute other relevant linear sensitivity distribution factors. In this section, we describe the algorithm to obtain PTDFs and subsequently LODFs.

1) *Power Transfer Distribution Factor (PTDF)*: The PTDF, denoted by $\Phi_{k-l}^{k'l'}$, approximates the sensitivity of the active power flow on line L_{k-l} with respect to an active power transfer of a given amount of power, $\Delta P_{k'l'}$, from bus k' to l' [7]. The PTDF can be computed as a superposition of an injection at bus k' and a withdrawal at bus l' , where the slack bus accounts for the power imbalance in each case. Thus,

$$\Phi_{k-l}^{k'l'} = \Psi_{k-l}^{k'} - \Psi_{k-l}^{l'},$$

where $\Psi_{k-l}^{k'}$ and $\Psi_{k-l}^{l'}$ are the line flow sensitivities in line L_{k-l} with respect to injections at buses k' and l' , respectively.

2) *Line Outage Distribution Factor (LODF)*: The LODF, denoted by $\Xi_{k-l}^{k'-l'}$, approximates the active power flow change in line L_{k-l} due to the outage of line $k'-l'$ as a percentage of pre-outage active power flow through $k'-l'$ [7]. Suppose line L_{k-l} connects bus k to l , while line $k'-l'$ connects bus k' to l' . In this case, $\Xi_{k-l}^{k'-l'}$ is expressed as

$$\Xi_{k-l}^{k'-l'} = \frac{\Phi_{k-l}^{k'l'}}{1 - \Phi_{k'-l'}^{k'l'}} = \frac{\Psi_{k-l}^{k'} - \Psi_{k-l}^{l'}}{1 - (\Psi_{k'-l'}^{k'} - \Psi_{k'-l'}^{l'})}.$$

Example 2 (3-Machine 9-Bus System): Consider again the system in Fig. 1; in this example, we examine the scenario with outage in line L_{8-9} . Particularly, we compare the change in actual power flowing across each remaining line due to the line outage to the corresponding quantities computed from the estimated ISFs and the model-based approximate ISFs. In general, as shown in Table II, the measurement-based approach outperforms the model-based one. ■

III. CASE STUDY

With a model of the system, power system operators obtain the DFs offline and use them in online N-1 contingency analysis. In particular, operators must ensure that the power system remains operable with an outage in any single power system asset. For example, LODFs indicate the portion of pre-outage flow in a line, after its outage, that is redistributed onto remaining lines. Using current active power line flows and LODFs, we can estimate the flow through all other lines if there were an outage on one line. If no line constraints are violated with any single line outage, we conclude the

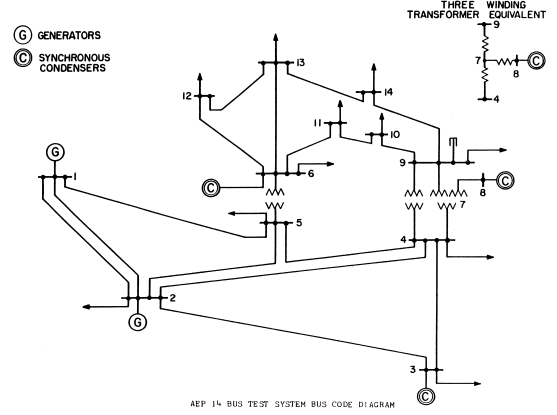


Fig. 2: Network topology for IEEE 14-bus system.

system is N-1 secure with respect to line outages. The pre-calculated model-based LODFs may not, however, be accurate if the system operating point and network topology deviate sufficiently far away from those at which the sensitivity factors were computed.

In this section, we apply the ideas presented previously in contingency analysis for the IEEE 14-bus system, the network topology of which is shown in Fig. 2. In this case study, we compute the LODFs offline using the original model and compare the accuracy of these compared to the DFs estimated online in contingency analysis. Next, we assume that line L_{10-11} fails in an open-circuit fashion but is undetected by system operators. This scenario is realistic since operators may not have full knowledge of current conditions in neighboring control areas, one of which could contain L_{10-11} . For these studies, we construct net active power injection “measurements” as in Example 1, again with $\sigma_1 = \sigma_2 = 0.1$. And line flows are inferred from power flows computed for each set of injections.

A. Comparison of DFs in base case contingency analysis

In Table III, we present contingency analysis results for only the hypothetical case that line L_{4-5} fails. The line flow predictions obtained using the model-based approximation

TABLE III: Contingency analysis on base case system.

Line	Actual (p.u.)	Model-based (p.u.)	LSE (p.u.)
ΔP_{1-2}	0.2019	0.1758	0.1962
ΔP_{1-5}	-0.1762	-0.1758	-0.1643
ΔP_{2-3}	0.1530	0.1465	0.1494
ΔP_{2-4}	0.3185	0.3047	0.3075
ΔP_{2-5}	-0.2816	-0.2754	-0.2698
ΔP_{3-4}	0.1423	0.1465	0.1409
ΔP_{4-7}	-0.0991	-0.0910	-0.1000
ΔP_{4-9}	-0.0566	-0.0531	-0.0571
ΔP_{5-6}	0.1615	0.1441	0.1535
ΔP_{6-11}	0.0981	0.0882	0.0938
ΔP_{6-12}	0.0127	0.0099	0.0120
ΔP_{6-13}	0.0507	0.0459	0.0477
ΔP_{7-8}	0.0	0.0	0.0
ΔP_{7-9}	-0.0991	-0.0910	-0.1000
ΔP_{9-10}	-0.0947	-0.0882	-0.0952
ΔP_{9-14}	-0.0610	-0.0559	-0.0619
ΔP_{10-11}	-0.0948	-0.0882	-0.0939
ΔP_{12-13}	0.0125	0.0099	0.0126
ΔP_{13-14}	0.0619	0.0559	0.0602

TABLE IV: Contingency analysis on modified system.

Line	Pre-contingency		Post-contingency	
	Actual (p.u.)	Model-based (p.u.)	Actual (p.u.)	LSE (p.u.)
P_{1-2}	1.5684	1.8004	1.7492	1.8084
P_{1-5}	0.7582	0.5610	0.5774	0.5769
P_{2-3}	0.7295	0.9065	0.8803	0.9052
P_{2-4}	0.5617	0.9268	0.8751	0.9218
P_{2-5}	0.4172	0.0933	0.1339	0.1146
P_{3-4}	-0.2358	-0.0717	-0.0850	-0.0700
P_{4-5}	-0.6124	0.0	0.0	0.0
P_{4-7}	0.2801	0.2107	0.1864	0.2148
P_{4-9}	0.1597	0.1201	0.1051	0.1225
P_{5-6}	0.4452	0.5626	0.5935	0.5604
P_{6-11}	0.0351	0.0351	0.1259	0.0403
P_{6-12}	0.0887	0.1125	0.0989	0.1109
P_{6-13}	0.2094	0.3029	0.2567	0.2972
P_{7-8}	0.0	0.0	0.0	0.0
P_{7-9}	0.2801	0.2107	0.1864	0.2148
P_{9-10}	0.0903	0.0904	-0.0004	0.0899
P_{9-14}	0.0544	-0.0545	-0.0031	-0.0477
P_{12-13}	0.0268	0.0501	0.0370	0.0489
P_{13-14}	0.0976	0.2116	0.1551	0.2053

and the measurement-based estimation appear almost equally effective when compared to the exact solution. Actually, for contingency under consideration here, the average deviation away from the exact power flow solution is 0.0069 p.u. using the model-based approximation method, while the average error is 0.0034 p.u. using the measurement-based estimation method, about half that obtained by the former traditional method. In this case, the accuracy of the traditional approach seems comparable to the LSE method. However, the proposed LSE method, due to its adaptability to changing operating points and network topology, is especially advantageous over the traditional method for a case in which the system no longer matches the model that was used to compute the DFs, as we illustrate next.

B. Comparison of DFs with undetected line outage

Suppose a line outage occurs in L_{10-11} , unbeknownst to system operators, perhaps because it is located in a neighboring control area. Contingency analysis continues to be conducted on the system using the LODFs computed based on the system model, which is no longer accurate due to the undetected line outage. For the revised system with line outage, in Table IV, we present contingency analysis results in the hypothetical case in which line L_{4-5} fails. More specifically, we compare between pre- and post-contingency (of L_{4-5}) actual line flows, model-based computed line flows, and measurement-based estimated line flows. A rough visual inspection of the post-contingency line flows reveals that the LSE prediction (column 5), which is updated by taking up-to-date measurements of bus injection and line flow incremental changes, is much closer to the actual post-contingency flow (column 3) than the model-based approximations (column 4). In fact, for the L_{4-5} contingency under consideration, with the model-based approximation approach, the average deviation away from the exact power flow solution is 0.0346 p.u., while, with the measurement-based approach, the average error is 0.0052 p.u., almost an order of magnitude smaller.

Further, suppose that the thermal limit of lines L_{2-3} and L_{13-14} are 0.9 p.u. and 0.2 p.u., respectively. We note that the actual post-contingency flow on these lines would be

0.9065 p.u. and 0.2107 p.u., both violating their respective thermal limits. While our measurement-based method captures these overloads, the model-based LODFs are out-of-date and do not alarm operators to the potential problem if the contingency on L_{4-5} were to occur. On the other hand, suppose the thermal line limit on L_{6-11} is 0.1 p.u. The post-contingency flow predicted by the model-based method is 0.1259 p.u., over the prescribed limit, while the actual flow is only 0.0351 p.u. In this case, the model-based LODFs causes a misdetection, while using the measurement-based ones, we obtain a much more accurate estimate of the actual post-contingency flow.

IV. CONCLUDING REMARKS

In this paper, we presented a method to estimate DFs by employing PMU measurements collected in real-time that does not rely on the system power flow model. Beyond eliminating the reliance on the system model, as shown in the examples and the case study in Sections II and III, the proposed measurement-based approach provides more accurate results than conventional model-based approximations and can adapt to unexpected system topology and operating point changes.

Further work includes accurate estimation of DFs in the presence of corrupted measurements or the availability of only a subset of measurements. Also, the measurement-based method necessitates an over-determined system. Hence, an avenue for future work would be to devise algorithms that estimate the DFs accurately using fewer measurements.

REFERENCES

- [1] FERC and NERC. (2012, Apr.) Arizona-southern california outages on september 8, 2011: Causes and recommendations. [Online]. Available: <http://www.ferc.gov/legal/staff-reports/04-27-2012-ferc-nerc-report.pdf>
- [2] U.S.-Canada Power System Outage Task Force. (2004, Apr.) Final report on the august 14th blackout in the united states and canada: causes and recommendations. [Online]. Available: <https://reports.energy.gov>
- [3] Z. Dong and P. Zhang, *Emerging Techniques in Power System Analysis*. Springer-Verlag, 2010.
- [4] United States DOE & FERC. (2006, Feb.) Steps to establish a real-time transmission monitoring system for transmission owners and operators within the eastern and western interconnections. [Online]. Available: <http://energy.gov>
- [5] A. Wood and B. Wollenberg, *Power Generation, Operation and Control*. New York: Wiley, 1996.
- [6] P. Sauer, "On the formulation of power distribution factors for linear load flow methods," *IEEE Transactions on Power App. Syst.*, vol. PAS-100, pp. 764–779, feb 1981.
- [7] B. Stott, J. Jardim, and O. Alsac, "Dc power flow revisited," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1290 – 1300, 2009.
- [8] T. Güler, G. Gross, and M. Liu, "Generalized line outage distribution factors," *Power Systems, IEEE Transactions on*, vol. 22, no. 2, pp. 879–881, 2007.
- [9] J. Guo, Y. Fu, Z. Li, and M. Shahidehpour, "Direct calculation of line outage distribution factors," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1633 – 1634, 2009.
- [10] J. E. Tate and T. J. Overbye, "Line outage detection using phasor angle measurements," *IEEE Transactions on Power Systems*, vol. 23, no. 4, pp. 1644 – 1652, 2008.
- [11] H. Zhu and G. B. Giannakis, "Sparse overcomplete representations for efficient identification of power line outages," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 2215 – 2224, 2012.
- [12] E. B. Makram, M. C. Vutsinas, A. A. Girgis, and Z. Zhao, "Contingency analysis using synchrophasor measurements," *Electric Power Systems Research*, vol. 88, pp. 64 – 68, 2012.
- [13] J. De La Ree, V. Centeno, J. Thorp, and A. Phadke, "Synchronized phasor measurement applications in power systems," *IEEE Transactions on Smart Grid*, vol. 1, no. 1, pp. 20 –27, june 2010.