Control of Distributed Energy Resources for Reactive Power Support

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***Abstract*—This paper proposes a method to utilize distributed energy resources (DERs) to provide reactive power support for voltage control in electric power systems. Rather than controlling each of these resources directly, a distributed control algorithm is developed in which a leader node sends a request for reactive power to a few DERs that it can directly communicate with. Then through an iterative algorithm, the initial request is distributed among all DERs so that they collectively fulfill the leader node's request. A case study illustrating this method is presented.**

***Index Terms*—Distributed Iterative Algorithm, Reactive Power Support, Voltage Control**

I. Introduction

As the penetration of distributed energy resources (DERs) continues to increase, these devices have the potential to provide ancillary services to the grid that they are connected to [1]–[5]. For example, plug-in hybrid electric vehicles (PHEV) and other energy storage devices can be used to provide active power for up and down regulation for energy peak-shaving during the day and load-leveling at night [3]. Although the primary purpose of DERs is to produce active power, through the proper control of their power electronics grid interface, they can also be utilized to provide reactive power support, which is critical in electric power systems for voltage stability and control [6]. Thus, these devices could provide the necessary reactive power support to control certain bus voltages and keep the system operating within specifications, i.e., operational requirements constrain bus voltages within $\pm 5\%$ of their nominal voltage.

A solution to control DERs for reactive power support can be achieved through a centralized strategy, where DERs communicate directly to a central controller, or a hierarchical approach where the central controller communicates indirectly with each DER through other devices. This strategy has been proposed in [4], where the system is grouped into members, referred to as reactive support groups, that belong to a chain of command structure much like the Incident Command System (ICS) used by emergency personnel. Each DER is indirectly controlled by a centralized controller that prioritizes dispatch commands relative to the sensitivities of the system. Similarly, [2] proposes reactive power support in distribution feeders, and assumes that DERs have two-way communications with a central controller either directly or through other DERs.

This paper proposes an alternative strategy to control DERs for reactive power support. Rather than a centralized controller communicating with every device on the network, a leader node can send requests to a few of these devices, or nodes, that are part of a mesh network. Through an iterative process, DERs determine their power injections so that their collective contributions have the same effect as a centralized control strategy. The coordination controller can be part of a hierarchical or centralized strategy to interact with other areas, but local variables, i.e., bus voltage, can be maintained independently from the rest of the network. This control strategy offers many potential benefits over a centralized strategy: (i) it is more economical because it does not require a significant communication infrastructure overlay, (ii) the system is more resilient to faults or unpredictable node behavior, (iii) local information is sufficient to control the devices, and (iv) as new DERs are connected to the network, they can adopt a "plug and play" strategy for syncing with existing devices. This methodology tailors the distributed algorithm presented in [5] to handle resource allocation and couples it with a two-stage control strategy.

The proposed algorithm for DER control has similarities with consensus algorithms that have been studied extensively in the field of control (e.g., see [7] and the references therein). The purpose of consensus algorithms is for the agents in a network to agree upon the value of a desired variable given a specific set of initial conditions, e.g., a group of sensors measuring the temperature of a room. The coordination algorithms proposed in this paper differ in two key ways: (i) the nodes are working towards a common goal and each node has its own set of constraints, i.e., capacity limits, so the contribution of the nodes will vary across the network, and (ii) the resulting transition matrices describing the dynamics of the proposed coordination algorithms are column stochastic, thus the node sums remain constant at each step of the algorithm, whereas with consensus algorithms the node sums do not remain constant unless the algorithm solves the average-consensus problem [7].

The remainder of this paper is structured as follows. Section II presents a high-level overview of the proposed architecture and formulates the control algorithm. Section III introduces a case study to illustrate the utilization of the proposed control to provide reactive power support in an electric power system. Section IV presents concluding remarks and proposes directions for future work.

II. Control Architecture

Figure 1 illustrates the proposed control architecture including the coordination controller and the mesh network describing the exchange of information among DERs. The power system is comprised of a total of m electrical buses, with m generator buses and $(n-m)$ load buses, on the left and right sides of the Network block, respectively. This particular architecture uses bus voltage to determine the amount of reactive power that must be provided by the DERs. On load bus i, voltage V_i is measured, compared to reference voltage V_i^{ref}, and the error passed to the coordination controller, which computes ρ_d—the reactive power demand. This request command is sent to the leader node. Suppose that the leader can communicate with l nodes, then it will evenly split the demand ρ_d to each of these l neighboring nodes. Each node runs a distributed algorithm that converges asymptotically to the desired solution. The power injections of the DERs sum to $P_i^d + jQ_i^d$, and the perceived load by the rest of the network is $(P_i+P_i^d)+j(Q_i+Q_i^d)$. Assuming that the DERs can meet the coordination controller request ρ_d, Q_i^d is explicitly defined. In the context of this paper, the active power P_i^d provided by the DERs is not controlled. However, similarly to reactive power support for voltage control, active power can potentially be used to provide up and down regulation services for frequency control.

A. DER Communication Network

The network describing the exchange of information between DERs can be represented by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with $\mathcal{V} = \{1, 2, ..., n\}$ representing the set of vertices (nodes), and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ representing the set of directed edges, i.e., ordered pairs $(j, i) \in \mathcal{E}$ of nodes for which node j receives information from node i. We assume by convention that we do not allow self-loops in $\mathcal{E}$. Edges can either be directed (also known as arcs), i.e., $(j, i) \in \mathcal{E}$ does not imply that $(i, j) \in \mathcal{E}$; or they can be undirected, i.e., if $(j, i) \in \mathcal{E}$, then $(i, j) \in \mathcal{E}$. All nodes that can send information to node j are said to be neighbors of node j. The set of neighbors of j, denoted by $\mathcal{N}_j$, is given by

$$\mathcal{N}_j := \{i \in \mathcal{V} \,:\, (j, i) \in \mathcal{E}\}\,. \tag{1}$$

In a graph $\mathcal{G}$, the number of nodes that have j as a neighbor, or where j is the head of an arc, is the in-degree $\mathcal{D}_j^-$ of j. On the other hand, the number of nodes that j transmits to, or where j is the tail of an arc, is the out-degree $\mathcal{D}_j^+$ of j. Let $\mathcal{X} \subset \mathcal{V}$, then $\mathcal{G}$ is considered to be strongly connected if $\mathcal{D}_{\mathcal{X}}^- \neq 0$ and $\mathcal{D}_{\mathcal{X}}^+ \neq 0$ for every nonempty $\mathcal{X}$ that is a proper subset of $\mathcal{V}$. In other words, a graph is strongly connected if there exists a path of finite length from node i to node j $\forall i,\, j = 1,\, 2,\, \ldots,\, n$ [8].

B. Distributed Control Algorithm

The proposed distributed control algorithm takes advantage of the graph-theoretic notions and properties of nonnegative matrices. Each node j will maintain some value $\mu_j[k]$ and update it iteratively as follows

$$\mu_j\,[k+1] = \frac{1}{1+\mathcal{D}_j^+}\mu_j\,[k] + \sum_{i \in N_j} \frac{1}{1+\mathcal{D}_i^+}\mu_i\,[k]\,, \tag{2}$$

where $\mathcal{D}_j^+$ and $\mathcal{D}_i^+$ are the out-degree of nodes j and i. This linear set of equations can be rewritten in the form

$$\mu\,[k+1] = P\mu\,[k]\,. \tag{3}$$

The choice of the weights in (2) will ensure that $P \in \mathbb{R}^{n \times n}$ is column stochastic. Additionally, by assuming that the graph that describes the exchange of information is strongly connected, it follows that P is primitive [9]. These are sufficient conditions to ensure that the algorithm will converge to the solution $\mu^{ss} \in \mathbb{R}^n$, i.e., $\mu^{ss} = P\mu^{ss}$ such that $\mu^{ss} = \alpha\pi$ where $\pi \in \mathbb{R}^n$ is unique, $\sum_{i=1}^{n} \pi_i = 1$, and $\alpha = \sum_{i=1}^{n} \mu_i[0]$ [9].

Assume that each node i has a limit for the amount of reactive power that they can produce, which we denote by μ_i^{max}. Define $\mu^{max} \in \mathbb{R}^n$ as the vector of the maximum capacities for the n nodes. The reactive power support capacity of the DERs is $\chi = \sum_{i=1}^{n} \mu_i^{max}$. In the constrained case, there may exist some i such that $\rho_d\pi_i > \mu_i^{max}$, so the solution $\mu^{ss} = \rho_d\pi$ may no longer be valid. Assume that $\rho_d < \chi$, then a feasible solution is

$$\mu^{ss} = \frac{\rho_d}{\chi}\mu^{max}. \tag{4}$$

The ratio of the demanded reactive power to the network capacity determines the percentage of each resource's capacity that is required by the coordination controller. Since the nodes have access to only local information, ρ_d/χ has to be obtained iteratively by each node. If $\hat{\mu}[0] = \rho_d\mu_0$ where $\mu_0 = [1/l,\, \ldots,\, 1/l,\, 0,\, \ldots,\, 0]$, then the system will converge to $\hat{\mu}^{ss} = \rho_d\pi$. Similarly, if $\bar{\mu}[0] = \mu^{max}$, then the steady-state solution is $\bar{\mu}^{ss} = \chi\pi$. These solutions can be obtained by computing (3) twice in parallel with the appropriate choice of initial conditions. Therefore the distributed control algorithm that each node uses to compute the solution for the constrained case is

$$\mu_i\,[k] \quad = \quad \frac{\hat{\mu}_i\,[k]}{\bar{\mu}_i\,[k]}\mu_i^{max}, \tag{5}$$

which converges asymptotically:

$$\lim_{k \to \infty} \mu\,[k] = \frac{\rho_d}{\chi}\mu^{max}. \tag{6}$$

Note that $\bar{\mu}_i\,[k] > 0$ and $\hat{\mu}_i\,[k] \geq 0$ $\forall i, k$ because P is column stochastic and nonnegative. The algorithm described in (2) — (6) generalizes Algorithm 1 from [5] for the case when the communication network is described by a directed graph.

If $\rho_d \leq \chi$, then this solution guarantees that node capacities are not violated and is computed iteratively without global knowledge of the mesh network. The strength of this algorithm is its simplicity. As long as the second largest eigenvalue of P is small, the algorithm can compute a solution that accounts for node capacity quickly and without any modifications to the state transition matrix. If $\rho_d > \chi$, then the calculated μ_i^{ss} by each node will be larger than its capacity, so each node i will fix their contribution to be $\mu_i^{ss} = \mu_i^{max}$.

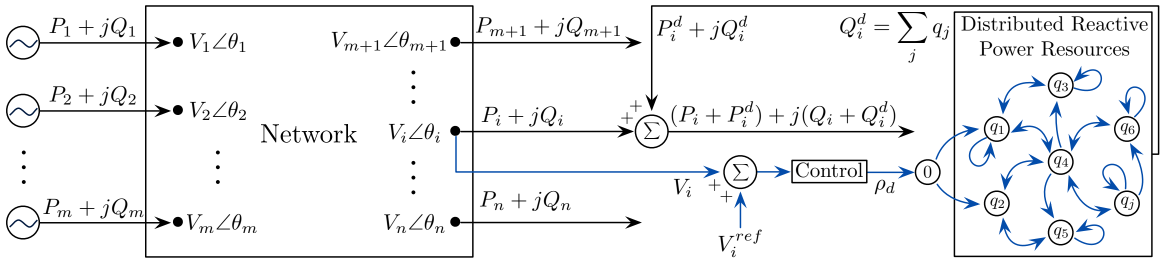


Figure 1: System Architecture for the Proposed Distributed Control

C. Coordination Controller

Figure 2 is the block diagram of the feedback control for the coordination controller design. The plant models the dynamics of the DERs from Fig. 1. The sensor on the feedback loop is equivalent to the voltage on bus i due to operating conditions in the power system, and thus V_i can be obtained through the power flow equations. For the purpose of designing the coordination controller, the following assumptions are made: i) the dynamics of the plant are significantly faster than the controller, so the plant is modeled as a constant, i.e., $G_{plant} = 1$, ii) the difference between phase angles of bus i and bus k is very small $\forall i, k = 1, \ldots, n$, iii) the voltage sensitivities with respect to changes in the operating point do not change much for different operating points, so the nominal values are used to design the controller. Since the dynamics of the machines were not modeled, a proportional-integral controller is sufficient to control this system.

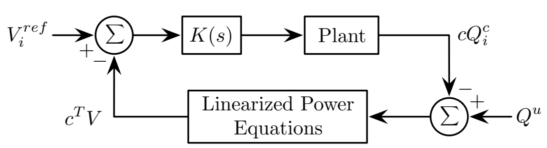


Figure 2: Block Diagram for the Coordination Controller Design

In an n bus power system, the complex power at bus i is computed by

$$S(x)_i = P_i(x) + jQ_i(x) = V_i \sum_{k \in N_k} V_k e^{j(\theta_i - \theta_k)} Y_{ik}^*, \quad (7)$$

where $N_k := \{j \,|\, i \text{ and } j \text{ are directly connected}\}$, $x = [\theta^T \; V^T]^T$ are the independent variables, and the transmission line admittance Y has the form $G + jB$ [10]. The linearized Taylor Series expansion of the power equations around an operating point x^* is

$$\begin{aligned} S(x) &= S(x^*) + \nabla S(x^*)(x - x^*) + h.o.t. \\ &\approx S(x^*) + \nabla S(x^*)(x - x^*). \end{aligned} \quad (8)$$

Let $V \in \mathbb{R}^n$ be the vector of voltage magnitudes for the n buses. It was assumed that $\theta_i \approx \theta_k$, which implies (8) is not dependent on θ, so $x = V$. Therefore, for some constant $\gamma \in \mathbb{R}^n$,

$$V = imag\left(\nabla^{-1} S(x^*)\right) Q + \gamma. \quad (9)$$

Let $c \in \mathbb{R}^n$ be a column vector of all zeros except for $c_i = 1$.

The discrete-time controller is obtained from the bilinear transformation

$$H_T(z) = H(s)|_{s = \frac{2}{T}\frac{z-1}{z+1}}, \quad (10)$$

where T is the sample period [11]. Let Q_i^c be the controllable reactive power injected by the DERs on bus i and $Q^u \in \mathbb{R}^n$ be the uncontrollable reactive power consumed or supplied on the buses. Since the DERs reduce the amount of reactive power consumed on load bus i, $c \times Q_i^c$ is subtracted from Q^u and the gains, K_p and K_i, are positive. The voltage on bus i is

$$\begin{aligned} V_i(s) &= c^T imag\left(\nabla^{-1} S(x^*)\right) \times \\ &\quad \left[Q^u - cK(s)(V_i^{ref} - V_i)\right] + c^T \gamma. \end{aligned} \quad (11)$$

Define the constants $\alpha := -c^T imag(\nabla^{-1} S(x^*))c > 0$ and $\beta := c^T imag(\nabla^{-1} S(x^*))Q^u + c^T \gamma$. Then (11) simplifies to

$$V_i(s) = \frac{\alpha K(s)}{1 + \alpha K(s)} V_i^{ref} + \frac{\beta}{1 + \alpha K(s)}. \quad (12)$$

The controller transfer function is $K(s) := K_p + K_i/s$. As long as the pole(s) for the system are stable, β will decay exponentially to zero. The pole of the system is $s = -\alpha K_i/(1 + \alpha K_p)$. For the input-output transfer function, the zero is at $s = -K_i/K_p$. To ensure that the system is stable and settles quickly, K_i and K_p are chosen such that $K_i \gg K_p > 0$. The sample period T needs to be selected such that the distributed algorithm has sufficient time to converge. For example, if each step of the distributed algorithm requires t seconds and no more than N steps to converge within a tolerance $\pm\epsilon$, then an appropriate choice for the sample period is $T > Nt$.

III. Case Study

A case study was performed with the Western System Coordinating Council (WSCC) standard 3 machine, 9 bus power system model [12]. The model was simplified to 6 buses by removing the transformers. Figure 3 describes the topology for the simplified WSCC system.

It was assumed that the system is lossless by setting the transmission line resistance to zero. Table I lists the values for the transmission lines. The values for the nominal power flow calculation prior to a contingency are listed in Table II, where bus 1 is the system slack bus. Under ideal conditions, generator 3 on bus 3 is operating at its full capacity.

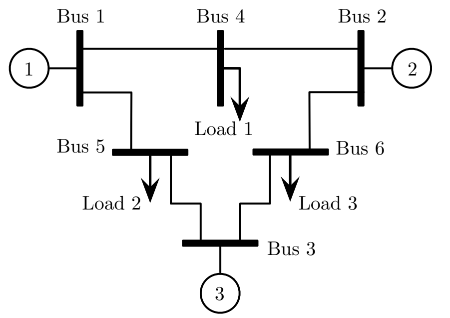


Figure 3: Simplified WSCC 3 Machine, 6 Bus System

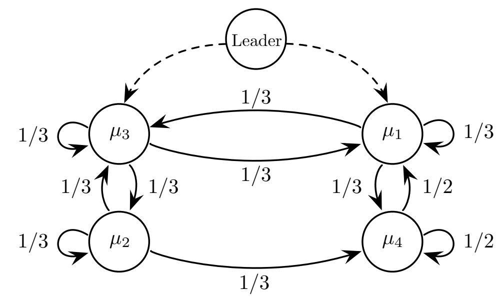


Figure 4: Four-Node Network Topology

At time $t = 5$ s, generator 2 is disconnected from the system and is switched from a PV bus to a PQ bus with zero load. For the system to be considered within acceptable operating conditions, the bus voltages are required to be within $\pm 5\%$ the nominal voltage of 1 pu, otherwise, a system failure is observed.

Two control architectures were used to test the ability of the DERs to provide reactive power support to stabilize and recover the bus voltages:

1) Only bus 6 was able to provide reactive power support through the coordination of the DERs.
2) All of the load buses are able to provide reactive power through the coordination of the DERs.

A. Distributed Control Algorithm

For simplicity, the four-node network in Fig. 4 was used for the network topology of the DERs on any given load bus. The number of DERs attached to each bus is irrelevant and the convergence time of the distributed control algorithm will not be hampered, e.g., a properly defined network of 10,000 nodes will not affect the simulation results.

The algorithm begins when the leader node splits the demand from the coordination controller to nodes 1 and 2, so the initial measure is $\mu[0] = \rho_d \times [0.5,\ 0.5,\ 0,\ 0]'$, where ρ_d is the demand from the coordination controller. Following (2), the state transition matrix that describes the algorithm is given by

$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 1/2 \\ 1/3 & 0 & 1/3 & 0 \\ 0 & 1/3 & 1/3 & 1/2 \end{bmatrix}. \tag{13}$$

The maximum capacities for the nodes in the constrained case are $\mu^{max} = [0.04,\ 0.02,\ 0.04,\ 0.01]'$, and the network capacity is $\chi = 0.11$. The invariant distribution of P is $\pi = [0.231,\ 0.346,\ 0.115,\ 0.308]'$. The solution to the non-adaptive fair splitting algorithm is $\mu^{ss} = \rho_d \times [0.364,\ 0.182,\ 0.364,\ 0.091]'$ while $\rho_d \le 0.11$. Otherwise, the solution is $\mu^{ss} = \mu^{max}$ for $\rho_d > 0.11$.

Table I: TRANSMISSION LINE VALUES

From	To	R	X
1	4	0	0.0720
2	4	0	0.1008
1	5	0	0.1610
2	6	0	0.1700
3	5	0	0.0850
3	6	0	0.0920

Table II: WSCC SYSTEM NOMINAL POWER FLOW VALUES

Bus	V	θ	P_g	Q_g	P_l	Q_l
1	1	0	1.5840	0.5388	0	0
2	1	-1.6406	0.8500	0.3458	0	0
3	0.9916	-5.9563	0.7160	0.5500	0	0
4	0.9841	-3.1300	0	0	1.00	0.35
5	0.9617	-8.0622	0	0	1.25	0.50
6	0.9740	-7.6144	0	0	0.90	0.30

B. Coordination Controller

The time for each iterative step of the distributed control algorithm was set to 10 ms. Let the initial distribution be $\mu[0] = [0.5,\ 0.5,\ 0,\ 0]'$ and $\epsilon = 0.001$. Then for $\mu[k+1] = P\mu[k]$, $\mu_i[k] \in (\mu_i^{ss} - \epsilon,\ \mu_i^{ss} + \epsilon)\ \forall i$ after 10 simulation steps, so the sample period of the discrete coordination controller has a lower limit of 1 ms. The sample period $T = 1$ s was selected and guarantees the solution converges within $\pm\epsilon$ of the limit.

For the simplified WSCC system, the value α from (12) varies between 0.065 and 0.2. The gains were assigned to be $K_i = 5.5$ s^{-1} and $K_p = 1$ and achieve settling times that are approximately 10 s.

C. Simulation Results

1) Base Case: No Reactive Power Control: Figure 5 shows the bus voltages from the base case. Generator 3 was already operating at its maximum capacity prior to the contingency, so it switches to a PQ bus at time $t = 5$ s. Therefore, generator 1 is required to produce the difference in real and reactive power caused due to the loss of generator 2. The contingency causes a voltage drop on every bus except for the slack bus. Based on the topology of the system, bus 6 is the load bus most sensitive to failures on bus 2 and bus 3, and experiences the most significant voltage drop after the contingency. The system is considered failed with bus 5 and bus 6 outside specifications at 0.9421 pu and 0.9399 pu, respectively.

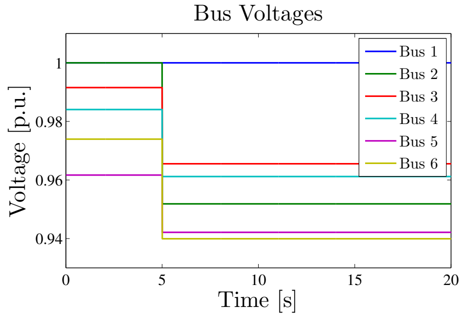


Figure 5: Base Case Without Control

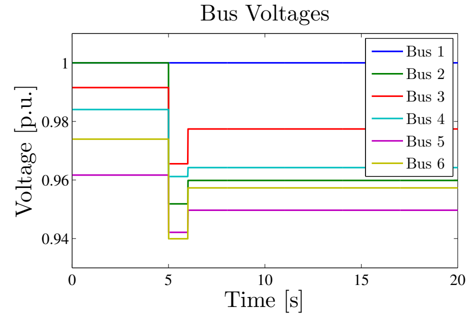
2) Reactive Power Control on Bus 6: Figure 6a shows the system response with the coordination of the DERs for voltage control on bus 6. The resources on bus 6 reach their network capacity of 0.11 pu immediately. The change in reactive power consumption on bus 6 proves to be enough to return all of the buses within specification. Although, bus 5 has a marginally acceptable voltage reading at 0.9497 pu due to its low sensitivity to bus 6. If the failure was more significant, bus 5 would not have recovered.

3) Reactive Power Control on All Load Buses: Figure 6b shows the system response with coordination on every load bus. Unlike the previous case, the additional control enabled every bus to operate well within its voltage specifications. Buses 4 and 6 have the highest sensitivities to failures on bus 2 and will operate at their capacities. Bus 5 injects more reactive power to compensate for buses 4 and 5 reaching their limits and increases its contribution to 0.0804 pu. Notice that bus 5 has the ability to produce more reactive power and has the lowest bus voltage. A possible method for future control strategies is that the reference voltage on this bus could be increased to reduce the voltage error on other buses with high sensitivities to it.

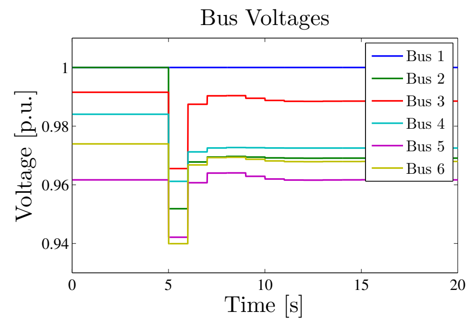
IV. Concluding Remarks

This paper develops and demonstrates a distributed control method to compute the reactive power injection of DERs to regulate bus voltages in an electrical network. The case study illustrates the potential of this technique to recover bus voltages during a contingency that would otherwise cause a system failure. These results should extend to active power control to regulate frequency, droop control, etc. The control strategy presented in this paper is ideal for resource allocation in constrained networks as it does not account for system values at each node beyond its capacity.

The effects of the implementation of these devices are relatively unknown. Consider the situation where a communication link between two nodes becomes severed, directional, or intermittent. In addition to physical component failure, faults associated with data computations and transmission could have serious repercussions on system stability and performance. A critical component of future work is to understand the possible failure modes of this methodology. A technique employing the matrix group inverse will be used to study the consequences of faults by computing node sensitivities to perturbations in the state transition matrix.



(a) Coordination Control on Bus 6



(b) Coordination Control on all Load Buses

Figure 6: System Responses

References

[1] G. Joos, B. T. Ooi, D. McGillis, F. D. Galiana, and R. Marceau, "The potential of distributed generation to provide ancillary services," in *Power Engineering Society Summer Meeting, 2000. IEEE*, vol. 3, 2000, pp. 1762–1767.

[2] M. E. Baran and I. M. El-Markabi, "A multiagent-based dispatching scheme for distributed generators for voltage support on distribution feeders," *IEEE Transactions on Power Systems*, vol. 22, no. 1, pp. 52–59, February 2007.

[3] C. Guille, "A conceptual framework for the vehicle-to-grid (V2G) implementation," Master's thesis, University of Illinois at Urbana-Champaign, Urbana, IL, February 2009.

[4] K. Rogers, R. Klump, H. Khurana, A. Aquino-Lugo, and T. Overbye, "An authenticated control framework for distributed voltage support on the smart grid," *IEEE Transactions on Smart Grid*, vol. 1, no. 1, pp. 40–47, June 2010.

[5] A. D. Dominguez-Garcia and C. N. Hadjicostis, "Coordination and control of distributed energy resources for provision of ancillary services," in *IEEE International Conference on Smart Grid Communications*, Gaithersburg, MD, October 2010, pp. 537–542.

[6] T. V. Cutsem and C. Vournas, *Voltage Stability of Electric Power Systems*. Norwell, MA: Kluwer Academic Publishers, 1998.

[7] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215 –233, January 2007.

[8] J. A. Bondy and U. S. R. Murty, *Graph Theory*, 2nd ed. New York, NY: Springer, 2008.

[9] R. A. Horn and C. R. Johnson, *Matrix Analysis*, 23rd ed. New York, NY: Cambridge University Press, 2010.

[10] J. D. Glover, M. S. Sarma, and T. J. Overbye, *Power System Analysis and Design*, 4th ed. Stamford, CT: Cengage Learning, 2008.

[11] G. F. Franklin, J. D. Powell, and M. Workman, *Digital Control of Dynamic Systems*, 3rd ed. Half Moon Bay, CA: Ellis-Kagle Press, 2006.

[12] P. W. Sauer and M. A. Pai, *Power System Dynamics and Stability*. Champaign, IL: Stipes Publishing, 2006.