A Framework to Determine the Probability Density Function for the Output Power of Wind Farms

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***Abstract*—This paper proposes a numerical framework to propagate wind-speed uncertainty through to the power output of a wind farm while factoring in the availability of the wind turbines in the farm. In particular, given a probability density function (pdf) that describes wind speed, and a statistical availability model for the wind turbines, we propose a method to determine the wind-farm power output pdf. The proposed framework offers several advantages over conventional methods, e.g., it is agnostic to the wind-speed distribution and easily incorporates wind farm availability. Case studies compare the wind farm power distributions computed using the proposed framework with field data for an actual wind farm. In addition, we demonstrate how the framework allows the computation of common wind generation indices such as the expected available wind energy, expected generated wind energy, and capacity factor.**

***Index Terms*—Availability, wind farms, random variable transformations.**

I. Introduction

Quantifying the impact of wind-speed uncertainty on wind-farm power output is critical in determining if the installed generating capacity can accommodate the system load at an agreeable reliability level [1], [2]. Time-domain simulations, e.g., sequential Monte Carlo methods [3], can be utilized in such studies. However, while these methods preserve chronological aspects, they are not only computationally expensive but also unsuitable when historical wind-speed time-series data is unavailable [3], [4]. To address these concerns, probabilistic methods have been widely applied in generation adequacy as well as bulk power system reliability studies [5]–[8].

This work proposes an improved numerical framework to propagate wind-speed uncertainty to the wind-farm power output—a critical step in any probabilistic method for system-level reliability assessment. Inputs to the framework are the wind-speed pdf and a statistical availability model for the wind turbines in the wind farm. These inputs are propagated through a mathematical model that describes the power output of the wind farm to obtain a pdf for the power produced by the wind farm. This is accomplished following well known random-variable transformation methods.

The proposed method has several advantages. First, the wind-turbine power curve is modeled by a continuous function up to the furling (cut-out) wind speed. In related works (see, e.g., [4], [9], [10]), the turbine output is generally assumed to be zero up to a cut-in wind speed and a constant between rated and furling speeds. On applying random-variable transformations to this model, two impulses are obtained in the power distribution—at zero and rated power. However, this is perhaps not a realistic model for real-world conditions (as is illustrated in the case studies). Second, wind-turbine availability is incorporated in a unified way into the analysis. Additionally, the proposed method is agnostic to the distribution adopted to model the wind speed. Finally, the framework easily allows computation of common wind generation indices such as the expected available wind energy, expected generated wind energy, and capacity factor [11], [12].

The remainder of this paper is organized as follows. In Section II we provide a brief overview of the proposed framework, formulate the wind turbine (and farm) power-wind speed characteristic, and describe the method based on random-variable transformation to obtain the pdf of the wind-farm power output. Then, in Section III, we demonstrate the validity of the proposed framework to infer wind farm power statistics by comparing results with field data from an actual wind farm. Finally, concluding remarks and directions for future work are discussed in Section IV.

II. Methodology

We begin this section with a brief overview of the proposed framework. Then, we describe the piecewise continuous functions utilized to model the power output of the wind turbines and the wind farm. Next, the statistical availability model is factored into the framework. Finally, we demonstrate how the uncertainty in wind speed can be propagated to the power output of the wind farm. This section also includes pseudocode suitable for computer implementation to implement the proposed method.

A. Overview of Proposed Framework

The building blocks of the proposed framework are illustrated in the block diagram shown in Fig. 1. The first input to the framework is wind speed, modeled as a continuous random variable V, with pdf $f_V(v)$, which is assumed to be known. The framework also requires a statistical availability model for the wind farm, i.e., for a wind farm comprised of n turbines, the probabilities π_m, $m = 0, 1, \ldots, n$, where π_m is the probability that m turbines are operational. These inputs

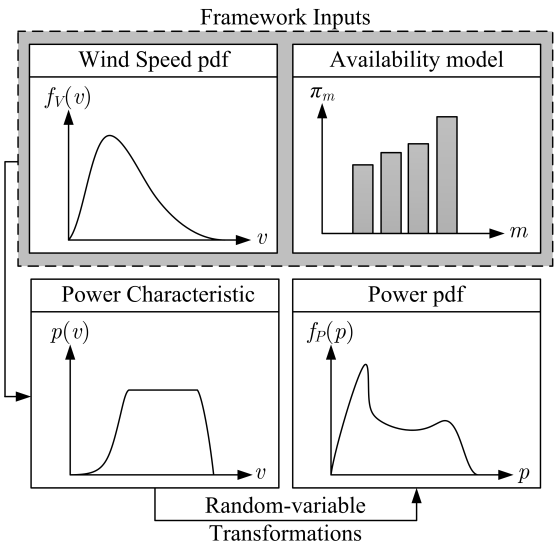

Figure 1. Block diagram that illustrates proposed framework.

are propagated through a mathematical model which captures the relation between power and wind-speed (referred as $p-v$ characteristic subsequently) of the wind farm to obtain the pdf of the wind-farm power—denoted by $f_P(p)$, where P is the random variable that describes the power.

B. Wind Turbine $p-v$ Characteristic

Following standard terminology (see e.g., [13]), denote the cut-in, rated, and furling wind speed of a wind turbine by v_c, v_r, and v_f, respectively. Further, denote the rated output power of the turbine by P_r. Let us consider the following piecewise continuous function to model the power output of the wind turbine p, as a function of wind speed v:

$$p(v) = \begin{cases} p_1(v) & \text{for} \quad 0 \le v \le v_{lim}, \\ p_2(v) & \text{for} \quad v_{lim} < v \le v_f, \end{cases} \tag{1}$$

where

$$p_1(v) = P_r - P_r \left[1 + \exp\left(\frac{v - v_{mid}}{c} \right) \right]^{-1}, \tag{2}$$

$$p_2(v) = P_r - P_r \alpha \left(v - v_{lim} \right)^q, \tag{3}$$

and where v_{mid}, c, α, and q are parameters that determine the shape of the functions [14]. We provide a brief note below on how the different parameters can be determined.

The parameter v_{mid} is the wind speed at which the power output of the turbine is half the rated value, i.e., $p_1(v_{mid}) = P_r/2$. Hence it can be determined by inspecting wind-turbine data sheets. The parameter c can be tuned to ensure that the characteristic is within some predetermined percentage of the rated power, P_r at the rated wind speed, v_r. For instance, if we require $p_1(v_r) = \beta \cdot P_r$ (typically $\beta \ge 0.99$ to ensure $p_1(v_r) \approx P_r$), it is easy to show

$$c = (v_r - v_{mid}) \left[\log\left(\frac{\beta}{1-\beta} \right) \right]^{-1}. \tag{4}$$

Table I
WIND-TURBINE PARAMETERS

Symbol	Parameter	Value
P_r	Rated power	2 MW
v_c	Cut-in wind speed	4 m/s
v_r	Rated wind speed	12 m/s
v_{lim}	Limiting wind speed	24 m/s
v_f	Furling wind speed	25 m/s
c	Shape parameter that ensures $p_1(v_r) = \beta \cdot P_r$	0.7617 m/s
v_{mid}	Wind speed such that $p_1(v_{mid}) = P_r/2$	8.5 m/s
q	Order of drop off for $v > v_{lim}$	2
α	Parameter that ensures $p_2(v_f) = 0$	1 (m/s)$^{-2}$

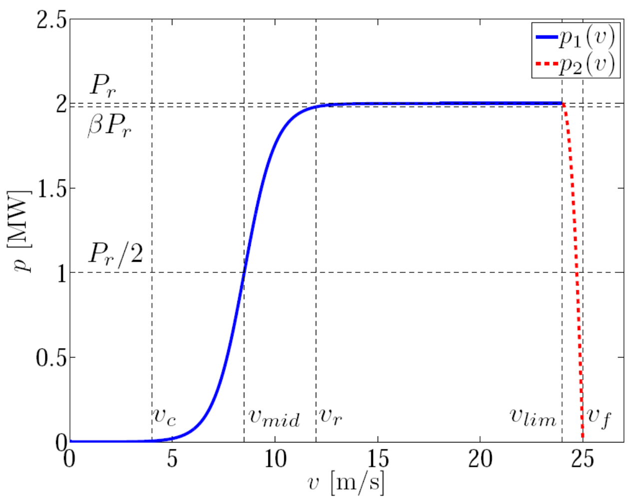

Figure 2. Sample $p-v$ characteristic for a representative turbine.

Finally, the parameter α is given by

$$\alpha = \left(v_f - v_{lim} \right)^{-q}, \tag{5}$$

which ensures that $p_2(v_f) = 0$. The function $p_2(v)$ is formulated to model a q-order drop-off in power output for wind speeds greater than a limiting wind speed, v_{lim}. The limiting wind speed can be chosen to be arbitrarily close to the furling speed. As will be shown in the case studies in Section III, a quadratic drop off ($q = 2$) models the output of an actual operating wind farm better than conventional models in which $v_{lim} = v_f$ so that $p_2(v) = 0 \; \forall v \ge v_f$. Figure 2 illustrates the $p-v$ characteristic for the Vestas V90-2.0 MW turbine [15]. Relevant turbine specifications and model parameters—extracted following the approach outlined above—are listed in Table I.

C. Wind Farm $p-v$ Characteristic

Let us consider a wind farm comprising n identical wind turbines. Recall that the $p-v$ characteristic of each turbine can be modeled by (1)-(3). If the turbines are staggered appropriately, interference effects can be minimized [13], and the $p-v$ characteristic of the farm is given by[1]

$$p(v) = \begin{cases} p_1(v) & \text{for} \quad 0 \le v \le v_{lim}, \\ p_2(v) & \text{for} \quad v_{lim} < v \le v_f, \end{cases} \tag{6}$$

where

$$p_1(v) = P_r^\xi - P_r^\xi \left[1 + \exp\left(\frac{v - v_{mid}}{c} \right) \right]^{-1}, \tag{7}$$

$$p_2(v) = P_r^\xi - P_r^\xi \alpha \left(v - v_{lim} \right)^q. \tag{8}$$

The rated power of the farm, denoted by P_r^ξ, is defined as

$$P_r^\xi = \xi P_r, \tag{9}$$

where ξ is the expected number of operational turbines over the period of study, and P_r is the rated power output of a single turbine. The expected number of turbines is given by

$$\xi = \pi_0 \cdot 0 + \pi_1 \cdot 1 + \ldots + \pi_n \cdot n, \tag{10}$$

where π_m is the probability that m turbines are operational. The probabilities π_i, $i = 0, 1, \ldots, n$, can be obtained from a Markov availability model [11], [12]. Alternately, they can be obtained from field data. We demonstrate this approach in the case studies.

D. Propagating Wind-Speed Uncertainty to the Wind-Farm Power Output

Suppose the wind speed is described by a random variable V, with pdf $f_V(v)$, which is assumed to be known. Applying random-variable transformations to (6), we get

$$f_P(p) = \frac{f_V(v_1)}{|p_1'(v_1)|} + \frac{f_V(v_2)}{|p_2'(v_2)|}, \tag{11}$$

where v_1 and v_2 can be obtained by inverting the $p_1(v)$, $p_2(v)$ characteristics in (7)-(8) as follows

$$v_1 = v_{mid} + c \cdot \log\left(\frac{p}{P_r^\xi - p} \right), \tag{12}$$

$$v_2 = v_{lim} + \left(\frac{1}{\alpha} \frac{P_r^\xi - p}{P_r^\xi} \right)^{1/q}. \tag{13}$$

The expressions for $p_1'(v)$ and $p_2'(v)$ in (11) can be obtained by differentiating (7)-(8) which results in

$$p_1'(v) = \frac{P_r^\xi}{c} \exp\left(\frac{v - v_{mid}}{c} \right) \left[1 + \exp\left(\frac{v - v_{mid}}{c} \right) \right]^{-2}, \tag{14}$$

$$p_2'(v) = -2\alpha P_r^\xi (v - v_{lim}). \tag{15}$$

The equation in (11) follows from well-known random-variable transformation methods [16]. In the context of propagating wind-speed uncertainty, these methods require inverting the $p-v$ characteristic for each value of p—which is easy given the form of $p(v)$ in (7)-(8).

[1] We abuse notation and denote the $p-v$ characteristic of the wind farm by $p(v)$. However, as expressed in (6)-(8), the rated power is different from that in (1)-(3). Subsequently, we will only be dealing with the $p-v$ characteristic of the wind farm as described in (6)-(8).

Wind speed pdf $f_V(v)$, can be obtained by fitting field data with standard distributions. For instance the Weibull distribution

$$f_V(v) = \frac{b}{a^b} v^{b-1} \exp\left(-\frac{v}{a} \right)^b, \tag{16}$$

where b is called the *shape parameter* and a is called the *scale parameter*, has been widely adopted to model wind-speed statistics [13], [17]. While we utilize the Weibull distribution in the case studies that follow, note that the method proposed above is agnostic to the wind-speed distribution.

We want to point out that the above method is accurate when the maximum wind speed at the location is less than the cut-out speed. As part of future work, we will augment the $p-v$ characteristic to model the extenuating case when the maximum wind speed at the location is expected to be significantly higher than the cut-out speed. If the power output is assumed to be zero beyond this value, mixed distributions will be obtained.

E. Computer Implementation

Algorithm 1 provides the pseudocode for computer implementation of the method outlined in (11)-(15) to compute $f_P(p)$. Since (11) has to be evaluated pointwise, p is appropriately discretized between p_{min} and p_{max} in steps of dp to obtain the vector $\bar{p} = [p_{min} : dp : p_{max}]$. In the for loop, $f_P(p)$ is evaluated point wise for each entry of $\bar{p}$, which we denote by $\hat{p}$. This involves computing v_1 and v_2 through (12)-(13), $p_1'(v_1)$ and $p_1'(v_1)$ through (14)-(15), and then applying (11).

Algorithm 1 Computation of $f_P(p)$

define $\bar{p} = [p_{min} : dp : p_{max}]$
  model parameters: v_c, v_r, v_{lim}, v_f, α, β, c, q, P_r
  Expected number of operational wind turbines, ξ
  Wind-speed pdf, $f_V(v)$
for $\hat{p} = p_{min} : dp : p_{max}$ **do**
  compute $v_1 = v_{mid} + c \cdot \log\left(\frac{\hat{p}}{P_r^\xi - \hat{p}} \right)$
    $v_2 = v_{lim} + \left(\frac{1}{\alpha} \frac{P_r^\xi - \hat{p}}{P_r^\xi} \right)^{1/q}$
  compute $p_1'(v_1) = \frac{P_r^\xi}{c} \exp\left(\frac{v_1 - v_{mid}}{c} \right) \left[1 + \exp\left(\frac{v_1 - v_{mid}}{c} \right) \right]^{-2}$
    $p_2'(v_2) = -2\alpha P_r^\xi (v_2 - v_{lim})$.
  compute $f_P(\hat{p}) = \frac{f_V(v_1)}{|p_1'(v_1)|} + \frac{f_V(v_2)}{|p_2'(v_2)|}$
end for

F. Wind Generation Indices

The proposed framework can be utilized to compute common wind generation indices that gauge the reliability/performability of wind farms [11], [12]. We introduce some metrics of interest below and explain how they can be computed.

1) Installed Wind Power, IWP: The installed wind power is the sum of nominal rated power of all turbines

$$\text{IWP} = n \times P_r \text{ [MW]}. \tag{17}$$

2) Installed Wind Energy, IWE : The installed wind energy is the maximum possible energy that can be extracted in one year from the wind farm

$$\text{IWE} = \text{IWP}\,[\text{MW}] \times 8760 \left[\frac{\text{hr}}{\text{yr}}\right]. \tag{18}$$

3) Expected Available Wind Energy, EAWE: Energy expected to be generated by the wind farm in one year without considering wind-turbine failure

$$\text{EAWE} = \int_p p \cdot f_P(p)|_{\xi=n}\, dp\, [\text{MW}] \times 8760 \left[\frac{\text{hr}}{\text{yr}}\right]. \tag{19}$$

To compute this index, we need the expected power output of the wind-farm without considering wind-turbine failure. The pdf of the power output without any failures, $f_P(p)|_{\xi=n}$, can be computed following the procedure in Section II-D, with $\xi = n \Rightarrow P_r^{\xi} = n \cdot P_r$.

4) Expected Generated Wind Energy, EGWE: Energy expected to be generated by the wind farm in one year considering wind-turbine failures and repairs

$$\text{EGWE} = \int_p p \cdot f_P\,(p)\, dp\, [\text{MW}] \times 8760 \left[\frac{\text{hr}}{yr}\right]. \tag{20}$$

To compute this index, we need the expected power output of the wind-farm while accommodating failures and repairs through an availability model.

5) Capacity Factor, CF: Capacity factor of the wind farm without considering wind-turbine failure

$$\text{CF} = \frac{\text{EAWE}}{\text{IWE}}. \tag{21}$$

6) Wind Generation Availability Factor, WGAF: Capacity factor of wind farm considering failures and repairs

$$\text{WGAF} = \frac{\text{EGWE}}{\text{IWE}}. \tag{22}$$

This is the capacity factor of the the wind farm while factoring in failures and repairs of the wind turbines.

III. Case Studies

In this section, we present two case studies. First, we validate the proposed approach by comparing the wind farm power pdf with actual field data. In the second, we demonstrate how the generation indices in Section II-F are computed for a hypothetical wind farm.

A. Model Validation with Field Data

This case study demonstrates how the proposed method can be applied to study the power distribution of a real wind farm. We have access to 1-second data for number of operational turbines, wind speed, and wind-farm power output recorded at a wind farm comprising $n = 75$ turbines over a period of two months. The goal of this case study is to demonstrate that the method proposed in Algorithm 1 can be used to accurately extract the pdf of the wind-farm power output by comparing the results with the field data.

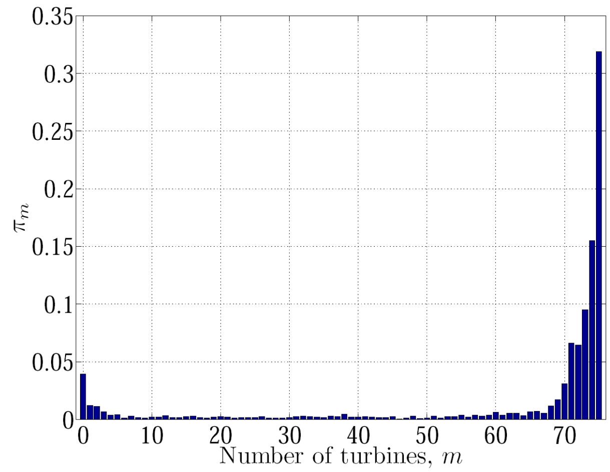

Figure 3. Availability of turbines in the wind farm.

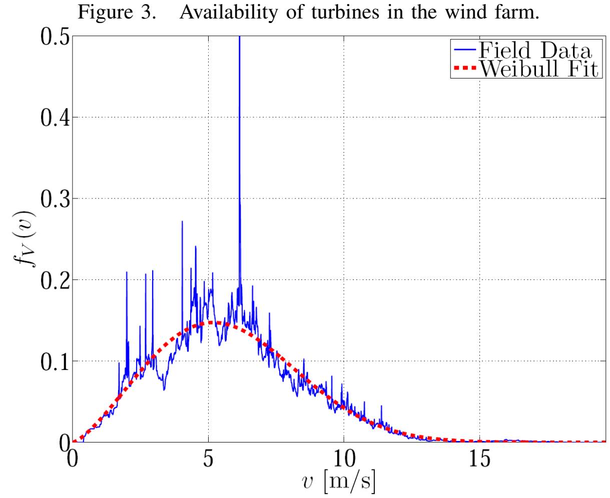

Figure 4. Weibull fit to distribution of wind speeds collected in the field.

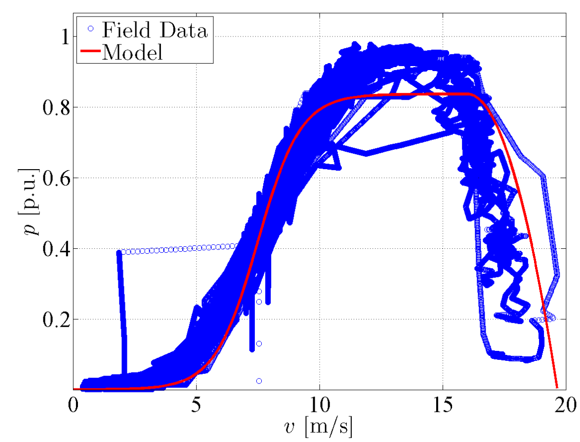

Figure 5. Field data compared to proposed $p - v$ characteristic.

The methodology comprises the following steps: i) obtain a statistical availability model of the turbines in the farm, ii) fit the pdf of recorded wind speed at the site with a Weibull distribution, iii) formulate the $p - v$ characteristic of the wind farm through (6)-(8), iv) propagate wind speed uncertainty through (11)-(15). These steps follow along Algorithm 1, and

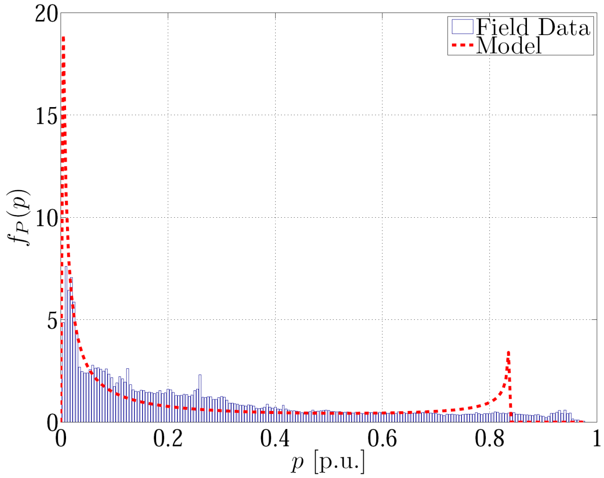

Figure 6. Distribution for power computed from Algorithm 1 compared with field data.

they are explained in detail subsequently.

As stated above, we first determine a statistical availability model for the wind farm. In the context of the discussion in Section II-C, this involves determining the probabilities π_m, $m = 0, 1, \ldots, n$, where π_m is the probability that the wind farm has m operational turbines. From the available data on number of wind turbines, π_m can be computed as

$$\pi_m = \frac{N_m}{N}, \tag{23}$$

where N_m is the number of seconds with m operational turbines, and $N = 2 \cdot 30 \cdot 24 \cdot 60 \cdot 60$ s, the number of seconds in 2 months—which is the duration of the observed period. The results are plotted in Fig. 3, and from (10), we get the expected number of operational turbines, $\xi = 62.7461$.

In the next step, a Weibull distribution is fitted to the distribution of wind speed utilizing MATLAB®'s *dfittool* function. The resulting scale and shape parameters are $a = 6.54818$ m/s, and $b = 2.35952$, respectively. Figure 4 plots the Weibull fit and the distribution computed from the raw wind speeds. The results show excellent agreement.

Next, we determine the $p - v$ characteristic of the wind farm. Figure 5 plots the $p - v$ characteristic of the wind farm computed from (6)-(8) and it also plots the raw power data as a function of wind speed. Finally, we follow along the steps in Algorithm 1 to determine the distribution $f_P(p)$ and compare the results with the field data. Results plotted in Fig. 6 show good agreement over a wide power range. The mean power from the field data is equal to 0.2673 p.u., while that computed using the proposed method is 0.2374 p.u.

B. Computing Wind Generation Indices

In this case study, we demonstrate the applicability of the proposed framework in computing the wind generation indices listed in Section II-F. Consider a wind farm comprised of $n = 50$ wind turbines with specifications in Table I. We will investigate wind-generation indices for wind regimes characterized by the Weibull distributions in Fig. 7. The installed wind power can be computed from (17) as IWP $= n \cdot P_r = 50 \times 2 = 100$ MW. Similarly, the installed wind energy can be computed from (18) as IWE = IWP $\times\, 8760 = 8.76 \times 10^5$ MWhr.

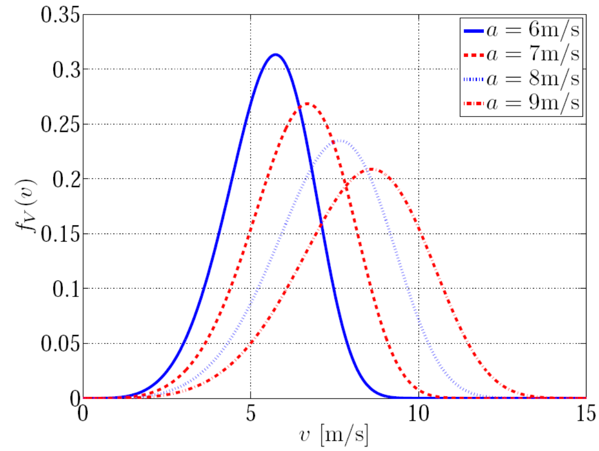

Figure 7. Representative Weibull distributions (a varied, $b = 5$) to model wind speeds.

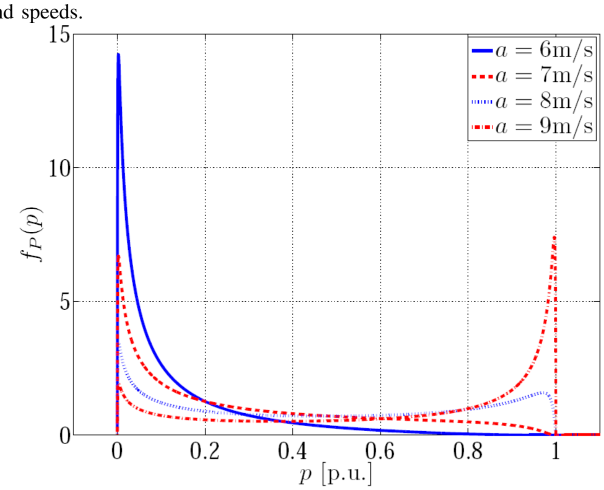

Figure 8. Power distribution computed for the wind-speed distributions in Fig. 7

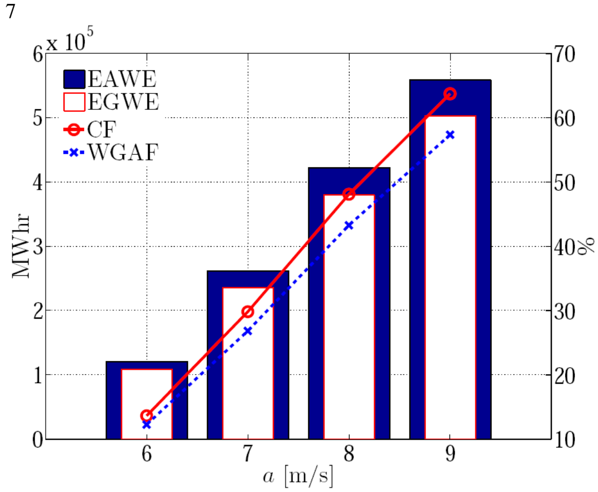

Figure 9. Wind generation indices as a function of the scale parameter.

Figure 8 depicts the pdfs $f_P(p)$ computed for wind speeds modeled by Weibull distributions in Fig. 7 assuming no wind-turbine failures. The base power corresponding to 1 p.u. is 100 MW, the IWP. Figure 9 depicts the expected available wind energy (EAWE), expected generated wind energy (EGWE), capacity factor (CF), and wind generation availability factor (WGAF) as a function of the scale parameter a, assuming $\xi = 0.9 \times n$, i.e., 45 turbines are expected to operate on average. These indices are computed from (19)-(22).

IV. Concluding Remarks and Future Work

This work proposes a numerical method to obtain pdfs for wind-farm output power, given the pdf of wind speed at the site. The main advantage of the proposed method is that it provides a unified way to integrate wind-farm availability, and is agnostic to the distribution adopted to model the wind speed. The suggested method can be applied in generation capacity adequacy and other similar power-system reliability studies. Additionally, the proposed method could be utilized by planners/developers to explore the impact of different repair strategies, wind turbines, and locations on the power output of the wind farm. Future work could investigate a Markov reliability model to capture the availability of the turbines in the farm. Interference effects, wind direction, etc., could also be incorporated into the framework.

Acknowledgments

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