Generalized Injection Shift Factors and Application to Estimation of Power Flow Transients

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***Abstract*—This paper proposes a method to estimate transmission line flows in a power system during the transient period following a loss of generation or increase in load contingency by using linear sensitivity injection shift factors (ISFs). Traditionally, ISFs are computed from an offline power flow model of the system with the slack bus defined. The proposed method, however, relies on generalized ISFs estimated via the solution of a system of linear equations that arise from high-frequency synchronized measurements obtained from phasor measurement units. Even though the generalized ISFs are obtained at the pre-disturbance steady-state operating point, by leveraging inertial and governor power flows during appropriate time-scales, they can be manipulated to predict active transmission line flows during the post-contingency transient period.**

I. Introduction

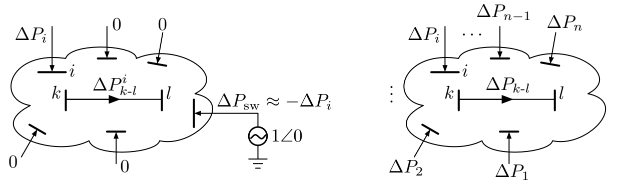
Power system operational reliability is monitored and maintained via online static and dynamic security assessment tools. By using these, operators can ensure the system is capable of withstanding a wide variety of disturbances, such as sudden loss of a generator or a transmission line. For example, dynamic security assessment may include consideration for post-fault system stability. On the other hand, static security assessment involves real-time N-1 contingency analysis, in which operators determine whether or not the system will meet operational reliability requirements in case of outage in any one particular asset and, in turn, any corrective actions required to ensure operation in a secure state [1].

With an up-to-date model of the system, operators can perform the N-1 security analysis by repeatedly solving the nonlinear power flow equations. However, for a large power system with many contingencies to consider, this process could take prohibitively long periods of time. One way to gain computational speed in contingency analysis is to use an estimate of the current operating point together with linear distribution factors (DFs), such as power transfer distribution factors (PTDFs), and line outage distribution factors (LODFs), obtained from an approximate power flow model of the system (see, e.g., [2]). These DFs can all be derived from the so-called injection shift factor (ISF), which approximates the change in active power flow across a transmission line due to a change in generation or load at a particular bus. Conventionally, the derivation of these ISFs requires an accurate model of the system that reflect current operating conditions; hence we refer to these as "model-based" or "conventional" ISFs.

In [3], we proposed a method to estimate ISFs in near real-time without relying on a power flow model of the system. The core idea behind this method is to find the solution of a system of linear equations formulated using active power bus injection and line flow data obtained from phasor measurement units (PMUs). In [3], we assumed an overdetermined system, with more equations than unknown ISFs, and obtained the solution via linear least-squares errors (LSE) estimation. The method is shown to be adaptable to undetected system topology and operating point changes, and thus represents significant improvement over the traditional one that relies on an accurate power flow model of the system. Using ISFs obtained via the measurement-based estimation method in [3], we can accurately predict transmission line flows throughout the system under the new steady-state operating point, following a contingency involving, e.g., loss of generation or increase in load.

Static security assessment tools are, in general, concerned with the system in steady-state operation, i.e., whether or not the system remains operationally reliable once it reaches steady-state under the new operating point following a disturbance. On the other hand, dynamic security assessment tools are used to determine whether or not the system is able to withstand the transients caused by a disturbance prior to reaching steady-state operation at the new operating point [4]. Traditionally, DFs have been used to verify system operational reliability in steady-state operation only (see, e.g., [2], [5]–[7]). To assess whether or not the system can withstand a contingency, however, it is also important to determine the distribution of power injections on transmission lines throughout the system during the transient post-disturbance period.

In this paper, we use ISFs obtained via the measurement-based method from [3] to describe transient phenomena associated with loss of generation or increase in load contingencies. Our analysis is based on the propagation of disturbance over time-scales for which inertial and governor power flows are valid (see, e.g., [8]). By leveraging these power flow problem formulation variants, we show that the measurement-based ISFs are general in the sense that they can be used to predict the active power flowing through transmission lines during the transient period between the pre- and post-disturbance steady-state operating points. Thus, in the remainder of this paper, we refer to these as "generalized" ISFs.



(a) Conventional ISF. (b) Generalized ISF.

Fig. 1: Conventional and generalized ISF conceptualization.

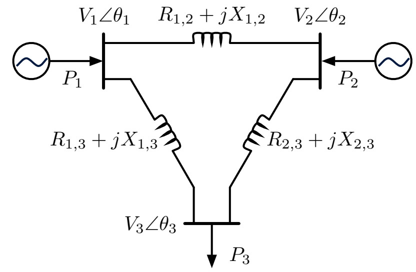


Fig. 2: Network topology for 3-bus system.

II. Conventional Injection Shift Factors

Distribution factors are linearized sensitivities used in contingency analysis and remedial action schemes [2]. A key distribution factor is the injection shift factor (ISF), which quantifies the redistribution of power through each transmission line in a power system following a change in generation or load on a particular bus in the system. In essence, the ISF captures the sensitivity of the flow through a line with respect to changes in generation or load. In this section, we outline the conventional ISF definition that relies on designating a slack bus, describe the model-based approach to compute them, and motivate the need for a measurement-based approach.

A. Conventional ISF Definition

Consider a power system with n buses. Conventionally, the ISF of line $L_{k\text{-}l}$ (assume positive real power flow from bus k to l) with respect to bus i, denoted by $\Psi^i_{k\text{-}l}$, is a linear approximation of the sensitivity of the active power flow in line $L_{k\text{-}l}$ with respect to the active power injection at node i, *with the slack bus defined* and all other quantities constant. Suppose P_i varies by a small amount ΔP_i, and denote by $\Delta P^i_{k\text{-}l}$ the change in active power flow in line $L_{k\text{-}l}$ resulting from ΔP_i. Then, it follows that

$$\Psi^i_{k\text{-}l} := \frac{\partial P_{k\text{-}l}}{\partial P_i} \approx \frac{\Delta P^i_{k\text{-}l}}{\Delta P_i}. \tag{1}$$

The approximation in (1) is shown conceptually in Fig. 1a. Traditionally, ISFs, along with other DFs, have been computed offline based on a power flow model of the system, including its topology and pertinent parameters, and with the slack bus defined. Next, we describe this model-based approach to compute conventional ISFs.

B. Model-Based Computation Approach

Let V_i and θ_i, respectively, denote the voltage magnitude and angle at bus i; additionally, let P_i and Q_i, respectively, denote the active and reactive power injection (generation or load) at bus i. Then, the static behavior of a power system can be described by the power flow equations, which we write compactly as

$$g(x, P, Q) = 0, \tag{2}$$

where $x = [\theta_1, \dots, \theta_n, V_1, \dots, V_n]^T$, $P = [P_1, \dots, P_n]^T$, $Q = [Q_1, \dots, Q_n]^T$, and $g : \mathbb{R}^{2n} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{2n}$. In (2), the dependence on network parameters, such as line series and shunt impedances, is implicitly considered in the function $g(\cdot)$.

Suppose a solution to (2) is obtained at (x_0, P_0, Q_0), i.e., $g(x_0, P_0, Q_0) = 0$, and assume $g(\cdot)$ is continuously differentiable with respect to x and P at (x_0, P_0, Q_0). Let $x = x_0 + \Delta x$ and $P = P_0 + \Delta P$; then, by assuming that ΔP and Δx are sufficiently small, we can approximate $g(x, P, Q_0)$ as

$$g(x, P, Q_0) \approx g(x_0, P_0, Q_0) + J\Delta x + D\Delta P, \tag{3}$$

where

$$J = \frac{\partial g}{\partial x}\Big|_{(x_0,P_0,Q_0)} \text{ and } D = \frac{\partial g}{\partial P}\Big|_{(x_0,P_0,Q_0)}.$$

Since $g(x_0, P_0, Q_0) = 0$, and Δx and ΔP are assumed to be small, we have that $g(x, P, Q_0) \approx 0$. Then, it follows from (3) that

$$0 \approx J\Delta x + D\Delta P. \tag{4}$$

Further, since J is the Jacobian of the power flow equations, which we assume to be invertible around (x_0, P_0, Q_0), we can rearrange (4) to obtain

$$\Delta x \approx -J^{-1} D\Delta P. \tag{5}$$

Next, we consider the active power flow through line $L_{k\text{-}l}$ as $P_{k\text{-}l} = h_{k\text{-}l}(x)$, where $h_{k\text{-}l} : \mathbb{R}^{2n} \to \mathbb{R}$. Under the same assumption of Δx being small, we can obtain an expression for small variations $\Delta P_{k\text{-}l}$ due to Δx as follows:

$$\Delta P_{k\text{-}l} = c\Delta x, \tag{6}$$

where $c = \frac{\partial h_{k\text{-}l}}{\partial x}\big|_{x_0}$. Substituting (5) into (6), it follows that

$$\Delta P_{k\text{-}l} \approx -cJ^{-1} D\Delta P. \tag{7}$$

Next, we show how to obtain the ISFs for a 3-bus system using the method outlined above.

Example 1 (3-bus Example): In this example, we consider a 3-bus system, the one-line diagram of which is shown in Fig. 2, and the parameters of which are listed in Table I. In this system, bus 1 is set as the slack bus. Let $\Psi_{k\text{-}l} = [\Psi^1_{k\text{-}l}, \Psi^2_{k\text{-}l}, \Psi^3_{k\text{-}l}]$. We compute the conventional ISFs for this system using (7) and obtain

$$\begin{aligned}
\Psi_{1\text{-}2} &= \begin{bmatrix} 0 & -0.7523 & -0.2712 \end{bmatrix}, \\
\Psi_{2\text{-}3} &= \begin{bmatrix} 0 & 0.2480 & -0.2710 \end{bmatrix}, \\
\Psi_{1\text{-}3} &= \begin{bmatrix} 0 & -0.2480 & -0.7290 \end{bmatrix}.
\end{aligned}$$

Note that, by definition, the ISFs with respect to the slack bus are 0. ■

TABLE I: Parameter values for 3-bus system shown in Fig. 2. All quantities are in p.u. unless otherwise noted.

V_1	V_2	P_1	P_2	P_3	Q_3	H_1 [s]	H_2 [s]
1.04	1.025	1.55931	0.7910	2.35	0.5	8	3.01
$R_{1,2}$	$R_{2,3}$	$R_{1,3}$	$X_{1,2}$	$X_{2,3}$	$X_{1,3}$	$1/R_1$	$1/R_2$
0.01	0	0	0.085	0.1610	0.0920	25	25

C. Need for a Measurement-Based Computation Approach

Since the derivation of the sensitivity vector in (7) relies on the linearization of the power flow equations, they are valid only for the system during steady-state operation. Next, we discuss a method to compute generalized ISFs using only PMU measurements obtained in near real-time without relying on a power flow model of the system. We show that they can be used to predict line flows during transients between two steady-state operating points.

III. Generalized Injection Shift Factors

In [3], a measurement-based ISF estimation method is proposed. This method relies only on inherent fluctuations in measurements of load and generation and does not employ a power flow model. The method is shown to be adaptive to operating point and topology changes, and is an improvement over the model-based approach described in Section II-B. In this section, we define the generalized ISF and show how the measurement-based ISF estimation approach proposed in [3] can be used to obtain these.

A. Generalized ISF Definition

Consider the same power system described in Section II-A, with the following exception. A real power system does not operate with a single slack bus that absorbs all power imbalances in the system. To this end, we define the concept of *generalized ISFs*, which as we show in Section IV, they can be used to recover the traditional definition of the ISF given in (1). Suppose P_i varies by a small amount ΔP_i and denote by $\Delta P^i_{k\text{-}l}$ the change in active power flow in line $L_{k\text{-}l}$ (measured at bus k) resulting from ΔP_i. Then, we define the generalized ISF of line $L_{k\text{-}l}$ with respect to bus i as

$$\Gamma^i_{k\text{-}l} := \frac{\Delta P^i_{k\text{-}l}}{\Delta P_i}. \qquad (8)$$

The definition in (8) is shown conceptually in Fig. 1b. Next, we describe the method used in [3] to estimate generalized ISFs from real-time measurements.

B. Measurement-Based Estimation of Generalized ISFs

Let $P_i(t)$ and $P_i(t+\Delta t)$, respectively, denote the active power injection at bus i at times t and $t+\Delta t$, $\Delta t > 0$ and small. Define $\Delta P_i(t) = P_i(t+\Delta t) - P_i(t)$ and denote the change in active power flow in line $L_{k\text{-}l}$ resulting from $\Delta P_i(t)$ by $\Delta P^i_{k\text{-}l}(t)$. Then, according to the approximation in (1), we need $\Delta P^i_{k\text{-}l}(t)$, which is not readily available from PMU measurements. We assume that the net variation in active power through line $L_{k\text{-}l}$, denoted by $\Delta P_{k\text{-}l}(t)$, however, is available from PMU measurements. We express this net variation as the sum of active power variations in line $L_{k\text{-}l}$ due to active power injection variations at each bus i:

$$\Delta P_{k\text{-}l}(t) = \Delta P^1_{k\text{-}l}(t) + \cdots + \Delta P^n_{k\text{-}l}(t). \qquad (9)$$

By substituting (8) into each term in (9), we can rewrite (9) as

$$\Delta P_{k\text{-}l}(t) \approx \Delta P_1(t)\Gamma^1_{k\text{-}l} + \cdots + \Delta P_n(t)\Gamma^n_{k\text{-}l}, \qquad (10)$$

where $\Gamma^i_{k\text{-}l} \approx \frac{\Delta P^i_{k\text{-}l}}{\Delta P_i}$, $i = 1, \dots, n$. Suppose $m+1$ sets of synchronized measurements are available. Let

$$\Delta P_i[j] = P_i((j+1)\Delta t) - P_i(j\Delta t),$$
$$\Delta P_{k\text{-}l}[j] = P_{k\text{-}l}((j+1)\Delta t) - P_{k\text{-}l}(j\Delta t),$$

$j = 1, \dots, m$; and define

$$\Delta P_{k\text{-}l} = \begin{bmatrix}\Delta P_{k\text{-}l}[1] & \cdots & \Delta P_{k\text{-}l}[j] & \cdots & \Delta P_{k\text{-}l}[m]\end{bmatrix}^T,$$
$$\Delta P_i = \begin{bmatrix}\Delta P_i[1] & \cdots & \Delta P_i[j] & \cdots & \Delta P_i[m]\end{bmatrix}^T.$$

Let $\Gamma_{k\text{-}l} = [\Gamma^1_{k\text{-}l}, \dots, \Gamma^i_{k\text{-}l}, \dots, \Gamma^n_{k\text{-}l}]$; then, it follows that

$$\Delta P_{k\text{-}l} = \begin{bmatrix}\Delta P_1 & \cdots & \Delta P_i & \cdots & \Delta P_n\end{bmatrix}\Gamma^T_{k\text{-}l}. \qquad (11)$$

For ease of notation, let ΔP represent the $m \times n$ matrix $[\Delta P_1, \dots, \Delta P_i, \dots, \Delta P_n]$; then, the system in (11) becomes

$$\Delta P_{k\text{-}l} = \Delta P \Gamma^T_{k\text{-}l}. \qquad (12)$$

If $m \geq n$, then (12) is an overdetermined system. Further, assuming the ISFs are approximately constant over the $m+1$ measurements, then we can solve for $\Gamma_{k\text{-}l}$ via least-squares errors (LSE) estimation as follows [3]:

$$\hat{\Gamma}^T_{k\text{-}l} = (\Delta P^T \Delta P)^{-1} \Delta P^T \Delta P_{k\text{-}l}. \qquad (13)$$

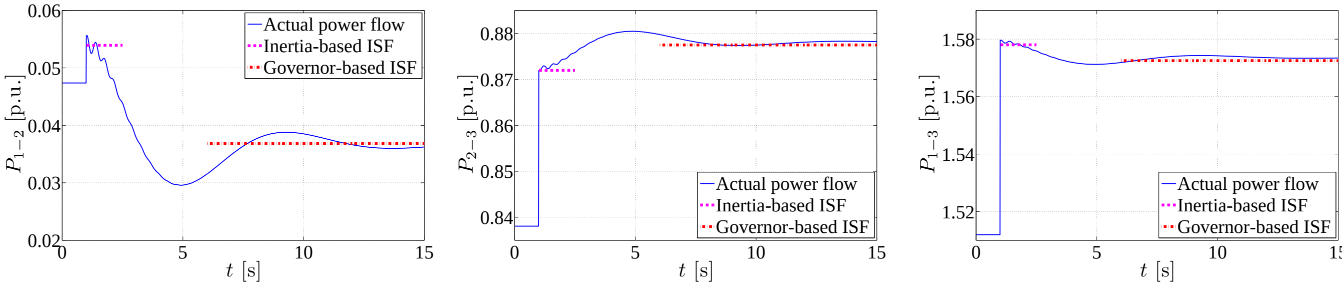
Next, we estimate the generalized ISFs for a 3-bus system.

Example 2 (3-bus System): In this example, we consider again the 3-bus system in Fig. 2. The synchronous generators at buses 1 and 2 are modeled with the subtransient machine dynamic model equipped with dc exciter and turbine governor (see, e.g., [4]). In order to simulate fluctuations in active power demand, we generate random time-series data as

$$P_i[j] = P^0_i[j] + \sigma\nu,$$

where $P^0_i[j]$ is the nominal active load at bus i and $\sigma\nu$ is a pseudorandom value drawn from a normal distribution with 0-mean and standard deviation σ. In this example, we generate random load data for only bus 3, with $\sigma = 0.03$ p.u. The dynamic simulation tool Power System Toolbox [9] is used throughout to obtain relevant transmission line flow measurements from synthetic power injections profiles. Using (13) in conjunction with 60 line flow and power injection measurements (collected at 30 samples per second), the generalized ISFs are estimated as

$$\hat{\Gamma}_{1\text{-}2} = \begin{bmatrix}-2.6408 & -3.3967 & -2.9131\end{bmatrix},$$
$$\hat{\Gamma}_{2\text{-}3} = \begin{bmatrix}-3.6408 & -3.3967 & -3.9131\end{bmatrix},$$
$$\hat{\Gamma}_{1\text{-}3} = \begin{bmatrix}3.6408 & 3.3967 & 2.9131\end{bmatrix}.$$



(a) Active power flow across line $L_{1\text{-}2}$. (b) Active power flow across line $L_{2\text{-}3}$. (c) Active power flow across line $L_{1\text{-}3}$.

Fig. 3: Line flows in 3-bus system due to 0.1 p.u. increase in active power demand at bus 3.

Note that $\hat{\Gamma}_{k\text{-}l}$'s above are distinctly different from the conventional ISFs $\Psi_{k\text{-}l}$'s computed in Example 1. Next, we show how the conventional ISFs can be recovered from the generalized ISFs. ∎

IV. Application of Generalized ISFs

The generalized ISFs obtained via the measurement-based method described in Section III can be immediately utilized to estimate the active transmission line flows at a new steady-state operating point, following a loss of generation or increase in load contingency. In this section, we first describe how conventional ISFs can be recovered from generalized ones. We also describe the application of generalized ISFs estimate line flows during the post-disturbance transient period.

A. Obtaining Conventional ISFs

To motivate the recovery of other ISFs from generalized ISFs, we introduce the power transfer distribution factor (PTDF). The PTDF, denoted by $\Phi^{ij}_{k\text{-}l}$, approximates the sensitivity of the active power flow in line $L_{k\text{-}l}$ with respect to an active power transfer of a given amount of power, ΔP_t, injected at bus i and withdrawn at bus j [10]. Thus, we have

$$\Delta P_{k\text{-}l} \approx \Phi^{ij}_{k\text{-}l} \Delta P_t, \tag{14}$$

where the PTDF $\Phi^{ij}_{k\text{-}l} = \Gamma^{i}_{k\text{-}l} - \Gamma^{j}_{k\text{-}l}$.

Based on the PTDF described in (14), we note that the conventional ISF is simply the sensitivity of the active power flow across line $L_{k\text{-}l}$ with respect to a power transfer from bus i to the slack bus. Thus, we can recover conventional ISFs from generalized ones as follows:

$$\hat{\Psi}^{i}_{k\text{-}l} = \hat{\Gamma}^{i}_{k\text{-}l} - \hat{\Gamma}^{s}_{k\text{-}l}, \tag{15}$$

where $\hat{\Gamma}^{s}_{k\text{-}l}$ denotes the generalized ISF of line $L_{k\text{-}l}$ with respect to the designated slack bus.

Example 3 (3-bus System): In this example, we verify the validity of (15) for the 3-bus system in Fig. 2 by applying it to the $\hat{\Gamma}_{k\text{-}l}$'s from Example 2, as follows:

$$\begin{aligned}
\hat{\Psi}_{1\text{-}2} &= \begin{bmatrix} 0 & -0.7559 & -0.2722 \end{bmatrix}, \\
\hat{\Psi}_{2\text{-}3} &= \begin{bmatrix} 0 & 0.2441 & -0.2722 \end{bmatrix}, \\
\hat{\Psi}_{1\text{-}3} &= \begin{bmatrix} 0 & -0.2441 & -0.7278 \end{bmatrix}.
\end{aligned}$$

We note that, indeed, the $\hat{\Psi}_{k\text{-}l}$'s are very similar to the $\Psi_{k\text{-}l}$'s obtained in Example 1. ∎

B. Obtaining Participation Factor-Based ISFs

In reality, unlike the case described in (14), there may be multiple simultaneous power transfers throughout the system. Thus, in general, the change in active power flow through line $L_{k\text{-}l}$ is due to multiple injections and can be expressed as

$$\Delta P_{k\text{-}l} \approx \Gamma^{1}_{k\text{-}l} \Delta P_1 + \cdots + \Gamma^{n}_{k\text{-}l} \Delta P_n. \tag{16}$$

In the context of contingency analysis, multiple generators may respond to a loss in generation or increase in load. Thus, suppose we are interested in the sensitivity of the active power flow across line $L_{k\text{-}l}$ with respect to a particular injection at bus i, denoted as ΔP_i, and this injection is balanced by some linear combination of injections at other buses, i.e.,

$$\Delta P_j = -\alpha_j \Delta P_i, \; j \neq i, \; \sum_{j \neq i} \alpha_j = 1. \tag{17}$$

The participation factors α_j's can arise from numerous factors, such as economic dispatch, governor participation, and synchronous generator inertia. For example, generator inertia-based participation factors are obtained with

$$\alpha_j = \frac{H_j}{\sum_j H_j}, \tag{18}$$

where H_j denotes the inertia of the synchronous generator j in the system. Similarly, governor participation factors are obtained as

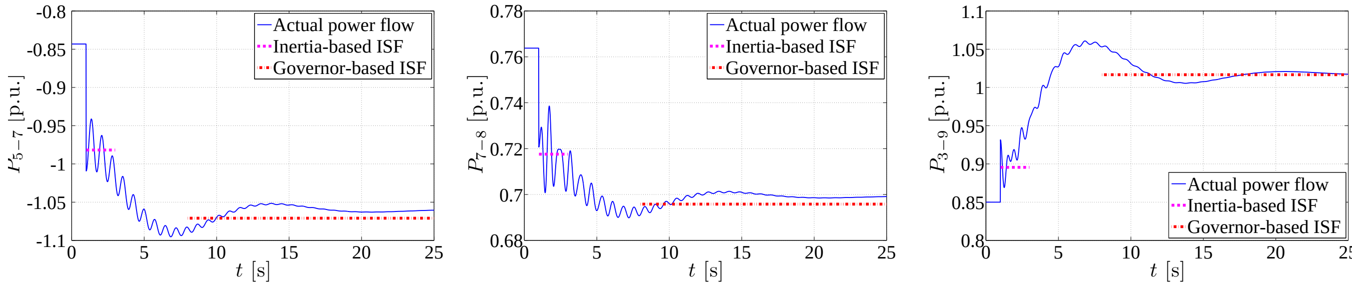
$$\alpha_j = \frac{1/R_j}{\sum_j 1/R_j}, \tag{19}$$

where $1/R_j$ represents the steady-state governor gain for generator j.

Denote by $\hat{\Psi}^{i}_{k\text{-}l}$ the sensitivity of $P_{k\text{-}l}$ with respect to P_i with consideration for generator participation factors. Substituting (17) into (16), we obtain that $\Delta P_{k\text{-}l} \approx \hat{\Psi}^{i}_{k\text{-}l} \Delta P_i$, where

$$\hat{\Psi}^{i}_{k\text{-}l} = \hat{\Gamma}^{i}_{k\text{-}l} - \sum_{j \neq i} \hat{\Gamma}^{j}_{k\text{-}l} \alpha_j. \tag{20}$$

We note that the case of the conventional ISF is a special case of (20) with α_s, the participation factor corresponding to the slack bus generator, equal to 1 and all other $\alpha_j = 0$, $j \neq i, s$.



(a) Active power flow across line $L_{5\text{-}7}$.

(b) Active power flow across line $L_{7\text{-}8}$.

(c) Active power flow across line $L_{3\text{-}9}$.

Fig. 4: Line flows in WECC 3-machine 9-bus system due to 0.5 p.u. increase in active power demand at bus 5.

The participation factor-based ISFs in (20) can be used to predict the transient active power flowing through transmission lines following a disturbance. Since the inertial response is faster than that of the governor, we expect the inertia-based ISFs obtained using (20) with (18) to be valid for a short time after the disturbance. Following this time-scale, we expect that the governor-based ISFs obtained using (20) with (19) to be valid until participation factors arising from economic dispatch become appropriate.

V. Case Studies

In this section, we illustrate the concepts described above with the 3-bus system and the Western Electricity Coordinating Council (WECC) 3-machine, 9-bus system.

A. 3-Bus System

After estimating generalized ISFs for the 3-bus system in Example 2 by relying on fluctuations in the load at bus 3, we predict the active power flow across all three lines using (20), given a 0.1 p.u. increase in the load at bus 3. Based on the information provided in Table I, the inertia-based participation factors for the two synchronous generators in the system are $\alpha_1 = 8/11.01$ and $\alpha_2 = 3.01/11.01$. Using these factors with (20), we obtain the change in active power flow through the three lines as $\Delta P_{1\text{-}2} = 0.0066$ p.u., $\Delta P_{2\text{-}3} = 0.0339$ p.u., and $\Delta P_{1\text{-}3} = 0.0661$ p.u. Similarly, the governor-based participation factors are $\alpha_1 = \alpha_2 = 1/2$, and the corresponding change in power flow are $\Delta P_{1\text{-}2} = -0.0106$ p.u., $\Delta P_{2\text{-}3} = 0.0394$ p.u., and $\Delta P_{1\text{-}3} = 0.0606$ p.u. We use Power System Toolbox [9] to obtain relevant transmission line flow measurements due to the change in load at bus 3, shown as the solid trace in Fig. 3. Superimposed onto the actual active line flows, in Fig. 3, we also plot the line flows predicted by the inertia-based ISFs and governor-based ISFs in dash and dash-dot traces, respectively. Indeed, we observe that the inertia-based ISFs provide a good approximation to the line flows immediately after the load increase, while the governor-based ISFs provide a good approximation for longer time-scales.

B. WECC 3-Machine 9-Bus System

As in Example 2, we estimate the generalized ISFs for the 3-machine 9-bus system and obtain inertia- and governor-based ISFs. We validate the predicted line flows against actual power flows obtained in dynamic simulations and plot results for three lines in Fig. 4.

VI. Concluding Remarks

In this paper, we introduce the concept of generalized ISF, which can be obtained via a measurement-based method. Generalized ISFs can be used to estimate the active power flow through transmission lines over time-scales for which inertial and governor power flows are valid. Even though the generalized ISFs are obtained at the pre-disturbance steady-state operating point, we show, through numerical examples, that they can be easily manipulated to predict transmission line flow during the post-disturbance transient period.

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