Sensitivity-Based Line Outage Angle Factors

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***Abstract*—In this paper, we propose a model-based approach to the computation of line outage angle factors (LOAFs), which relies on the use of angle factors (AFs) and power transfer distribution factors (PTDFs). A LOAF provides the sensitivity of the voltage angle difference between the terminal buses of a transmission line in the event the line is outaged to the pre-outage active power flow on the line. Large angle differences between the terminal buses of an outaged line can prevent the successful reclosure of the line—such an event was a significant contributing factor to the 2011 San Diego blackout. The proposed model-based LOAFs, along with the AFs and injection shift factors (ISFs), enable the fast computation of the impact on the angle across lines of line outages and active power injections, and provide system operators a systematic mean by which to assess line outage angles and undertake the appropriate dispatch actions necessary to alleviate large phase angle differences. We demonstrate the effectiveness of the proposed LOAFs with a case study carried out on the IEEE 14-bus test system.**

***Index Terms*—Voltage Angle, Reliability, Sensitivity Analysis, Distribution Factors, Operations, SCED.**

I. Introduction

System operators have long known that a transmission line breaker that has tripped open can be safely reclosed only if the angle across the terminals of the outaged line, which we will refer to as the *line outage angle*, is sufficiently small (see, e.g., [1]). In fact, the failure to reclose of a breaker due to a large phase angle across the terminal buses of a tripped line was a contributing factor in the chain of events that ultimately resulted in the 2011 San Diego blackout [2]. Moreover, in the case of the San Diego blackout, the system operator did not have in place a systematic mean by which to quickly assess the necessary actions, e.g., generation redispatch, needed to bring the outage angle within a range that permitted the line breakers to be safely reclosed. Indeed, the after-the-fact assessment of the blackout event concluded that "underlying factors that contributed to the event... [included] not providing effective tools and operating instructions for use when reclosing lines with large phase angle differences across the reclosing breakers [2]." Further, the report recommended that "transmission operators should have: (1) the tools necessary to determine phase angle differences following the loss of lines; and (2) mitigation and operating plans for reclosing lines with large phase angle differences [2]." In this work, we introduce a sensitivity-based formulation of the line outage angle factors (LOAFs) that can be used to develop such tools.

A LOAF provides the sensitivity of the voltage angle difference between the terminal buses of a transmission line in the event the line is outaged with respect to the pre-outage active power flow on the line. The LOAF derivation and deployment relies on angle factors (AFs), which provide the sensitivity of the bus voltage angles to changes in the bus active power injections, assuming the injection changes are balanced by the slack bus, and injection shift factors (ISFs), which provide the sensitivity of line flows to bus active power injections assuming they are balanced by the slack bus. LOAFs, together with AFs and ISFs, can be used to develop tools that enable system operators to quickly compute the impact on line outage angles of line outages and active power injections. Further, they provide system operators with a systematic mean by which to assess the appropriate dispatch actions necessary to alleviate large line outage angles pre- or post-outage.

Power system sensitivities, e.g., PTDFs, which provide the sensitivity of the active power flow on a line to active power transactions between buses in the system, and ISFs, are a critically important component of real-time contingency analysis and the real-time security-constrained economic dispatch (SCED) (see, e.g., [3]). Conventional sensitivities and tools such as the SCED, however, have not effectively captured outage angle impacts. Further, formulations of angle-sensitivity-based dispatch tools go back to the 1980s (see, e.g., [4]) and the LOAFs were first proposed in [5]. However, previous angle-sensitivity-based dispatch tools focused on the control of phase-shifting transformers and did not address the issue of large line outage angles. Moreover, the derivation of the LOAFs in [5] requires the computation of the bus impedance matrix for each outage topology—a computationally burdensome requirement for online applications.

In this work, we derive the LOAFs from a linearized power flow model taking a sensitivity-focused approach based on the AFs and PTDFs. Our approach harnesses the concept of the flow-canceling transaction used in the formulation of other sensitivities commonly utilized in contingency analysis, e.g., the line outage distribution factors (LODFs) [6]. In doing so, we are able to compute the LOAF of any line from a single system topology—the outage-free topology. Our proposed LOAFs form the basis for effective and computationally efficient tools for use by system operators in addressing the challenge of line outage angle monitoring and control.

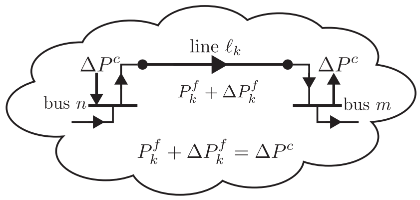


Fig. 1: The flow-canceling transaction.

The remainder of this paper is organized as follows. In Section II, we provide the definition of LOAF and the AFs and PTDFs upon which its computation is based. In Section III, we describe the key applications of the LOAFs. In Section IV, we present the results of a case study carried out to demonstrate the strengths of the proposed LOAFs and their applications. The final section reiterates the contributions of this work and points to some directions for future work.

II. Model-Based AF and LOAF Computation

In this section, we describe our approach to the model-based LOAF computation. We begin by deriving the AFs. Next, we derive the ISFs and PTDFs, which we then use along with the AFs to derive the LOAFs.

A. Model-Based AF Computation

The AFs are the sensitivities of the bus voltage angles with respect to the bus active power injections assuming the change is balanced by a pre-specified slack bus. Let θ_i be the voltage angle at bus i and let P_j be the net power injection at a bus j. Then, we define the AF for bus i with respect to net injections at bus j as

$$\Omega[i,j] = \frac{\partial \theta_i}{\partial P_j}.$$

In order to derive the model-based AF, we consider a system that consists of N buses indexed by $\mathcal{N} = \{1, \ldots, N\}$, and L lines indexed by $\mathcal{L} = \{\ell_1, \ldots, \ell_L\}$, where each ℓ_l is an ordered pair (n, m), $n, m \in \mathcal{N}$, representing a transmission line between buses n and m, with the convention that positive flow on such a line is in the direction *from n to m*. Moreover, assume that there are G generators indexed by $\mathcal{G} = \{1, \ldots, G\}$, and D loads indexed by $\mathcal{D} = \{1, \ldots, D\}$. Let $\mathcal{G}_n \subseteq \mathcal{G}$ be the subset of generators at bus $n \in \mathcal{N}$, and let $\mathcal{D}_m \subseteq \mathcal{D}$ be the subset of loads at bus $m \in \mathcal{N}$.

Let P_i^g be the output of generator $i \in \mathcal{G}$, with the convention that $P_i^g > 0$ if the generator injects active power into the system; and let P_j^d be the demand of load $j \in \mathcal{D}$, with the convention that $P_j^d > 0$ if the load withdraws active power from the system. Then, define the vectors of generation and demand as $P^g = [P_1^g, \ldots, P_G^g]^T$ and $P^d = \left[P_1^d, \ldots, P_D^d\right]^T$, respectively. With these quantities, we define the net injection at a bus $n \in \mathcal{N}$ as

$$P_n = \sum_{i \in \mathcal{G}_n} P_i^g - \sum_{j \in \mathcal{D}_n} P_j^d,$$

with the convention that $P_n > 0$ if active power is injected *into* the system. Then, define the vector of net injections at all buses as $P = [P_1, \ldots, P_N]^T$.

Let V_i and θ_i denote the voltage magnitude and angle, respectively, at a bus i and let bus 1 be the angle reference bus, i.e., $\theta_1 = 0$, and slack bus, i.e., the bus 1 active power injection compensates any system-wide active power mismatch due to losses and discrepancies between total generation and total load. Further, let $Q = [Q_1, \ldots, Q_N]^T$ be the N-dimensional column vector of net reactive power injections at each bus $i \in \mathcal{N}$. Then, the steady-state behavior of the power system can be described compactly by the power flow equations as

$$g(x, P, Q) = 0 \tag{1}$$

where $x = [\theta_1, \ldots, \theta_N, V_1, \ldots, V_N]^T$, and $g : \mathbb{R}^{2N} \times \mathbb{R}^N \times \mathbb{R}^N \to \mathbb{R}^{2N}$, which we assume to be continuously differentiable with respect to x, P, and Q.

Suppose the system defined above is operating at the point (x_0, P_0, Q_0), i.e., $g(x_0, P_0, Q_0) = 0$. Let $x = x_0 + \Delta x$, $P = P_0 + \Delta P$, and $Q = Q_0 + \Delta Q$. Assuming Δx, ΔP, and ΔQ are sufficiently small, we can approximate $g(x, P, Q)$ as

$$g(x, P, Q) \approx g(x_0, P_0, Q_0) + J\Delta x + C\Delta P + D\Delta Q, \tag{2}$$

with

$$J = \left.\frac{\partial g}{\partial x}\right|_{(x_0, P_0, Q_0)}$$

$$D = \frac{\partial g}{\partial P} = \begin{bmatrix} I_{N\times N} \\ 0_{N\times N} \end{bmatrix}, \quad C = \frac{\partial g}{\partial Q} = \begin{bmatrix} 0_{N\times N} \\ I_{N\times N} \end{bmatrix},$$

where $I_{N\times N}$ ($0_{N\times N}$) is the $N \times N$ identity (all-zeros) matrix.

By definition, $g(x_0, P_0, Q_0) = 0$ and $g(x, P, Q) = 0$. Thus, from (2) we have that

$$0 \approx J\Delta x + C\Delta P + D\Delta Q. \tag{3}$$

We assume the Jacobian of the power flow equations, J, is invertible around the solution (x_0, P_0, Q_0). Thus, we may solve (3) for Δx to arrive at

$$\Delta x \approx -J^{-1}C\Delta P - J^{-1}D\Delta Q. \tag{4}$$

Further, we partition the inverse Jacobian as follows:

$$-J^{-1} = \begin{bmatrix} E & F \\ K & H \end{bmatrix}, \tag{5}$$

where $E, F, K, H \in \mathbb{R}^{N\times N}$. From (4) and (5) we conclude that

$$\Delta\theta \approx E\Delta P + F\Delta Q. \tag{6}$$

In addition, from (4) we have that

$$\Delta V \approx K\Delta P + H\Delta Q. \tag{7}$$

Solving for ΔQ in (7) we obtain

$$\Delta Q \approx H^{-1}\Delta V - H^{-1}K\Delta P. \tag{8}$$

Then, substituting (8) into (6) and assuming, consistent with the loose coupling between ΔP and ΔV in the power flow Jacobian, $\Delta V \approx 0$, we arrive at

$$\Delta\theta \approx (E - FH^{-1}K)\Delta P. \tag{9}$$

Thus, from (9) we arrive at the $N \times N$ matrix of AFs

$$\Omega \approx (E - FH^{-1}K). \tag{10}$$

B. Model-Based LOAF Definition

The LOAF for line $\ell_k = (n, m)$ provides the sensitivity of the voltage angle difference between the terminal buses of line ℓ_k in the event of the outage of ℓ_k to the pre-outage flow on line ℓ_k. Let $\Delta\theta_{n\text{-}m} = \Delta\theta_n - \Delta\theta_m$ be the change in the voltage angle across line ℓ_k in response to the outage of line ℓ_k, which has pre-outage flow P_k^f. Then, define the LOAF for the outage of line ℓ_k as follows:

$$\Sigma_k = \frac{\Delta\theta_{n\text{-}m}}{P_k^f}.$$

Now, let $A = [a_1, \ldots, a_i, \ldots, a_L]$ denote the transmission network incidence matrix, where $a_i \in \mathbb{R}^N$, the jth entry of which is equal to 1 if bus j is the *from* bus of line i, -1 if bus j is the *to* bus of line i, and zero otherwise. Further, let $b \in \mathbb{R}^L$ denote the vector of branch susceptances, and define the diagonal $L \times L$ branch susceptance matrix as $B_b = \text{diag}\{b\}$, where $\text{diag}\{\cdot\}$ denotes a diagonal matrix such that $B_b[i,i] = b_i, \forall i$. Then, define the $(N-1) \times (N-1)$ reduced nodal susceptance matrix as $\tilde{B} = \tilde{A} B_b \tilde{A}^T$, where $\tilde{A}$ is the incidence matrix absent the row corresponding to the specified slack bus.

Let Ψ be the $L \times N$ linear flow sensitivity matrix, or ISF matrix, an entry of which, denoted by $\Psi[l, i]$, provides the sensitivity of the flow on line $\ell_l \in \mathcal{L}$ to an injection at bus i that is withdrawn at the slack bus. Under the dc assumptions,[1] Ψ can be calculated directly from the network connectivity and parameters as follows (see, e.g., [7]):

$$\Psi = B_b \tilde{A}^T \tilde{B}^{-1}. \tag{11}$$

We assume that Ψ has been augmented in the appropriate location with a column of zeros corresponding to the slack bus such that it is of dimensions $L \times N$.

Moreover, let Φ_k be the $N \times N$ matrix of PTDFs for a line $\ell_k = (n, m)$, an entry of which, denoted by $\Phi_k[n, m]$, gives us the proportion of an active power transaction injected at bus n and withdrawn at bus m that flows over line ℓ_k [7]. The PTDF for a line ℓ_k with respect to an active power transaction between buses n and m is calculated directly from the ISFs as follows (see, e.g., [7]):

$$\Phi_k[n, m] = \Psi[k, n] - \Psi[k, m]. \tag{12}$$

C. Model-Based LOAF Compuation

To derive LOAFs, we simulate the outage of line $\ell_k = (n, m)$ with an active power injection ΔP_k^c at bus n that is withdrawn at bus m and that results in zero net injection at each end of line ℓ_k—a so-called *flow-canceling transaction* [6], shown in Fig. 1. Such an injection can be written as

$$\Delta P_k^c = P_k^f + \Delta P_k^f, \tag{13}$$

[1](i) the system is lossless, (ii) the voltages are unity, and (iii) the differences between voltage angles are small [7].

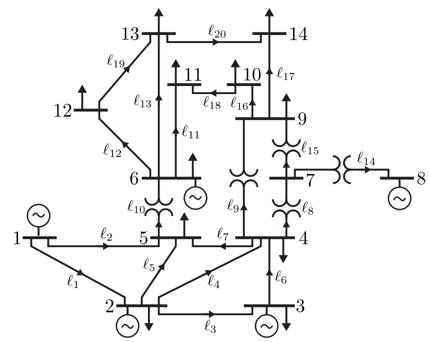


Fig. 2: The IEEE 14-bus test system.

where ΔP_k^f is the additional flow that appears on line ℓ_k in response to injection (withdrawal) ΔP_k^c at buses n (m). Under the dc assumptions, we can write ΔP_k^f as

$$\Delta P_k^f = \Phi_k[n, m] \Delta P_k^c. \tag{14}$$

Substituting (14) into (13) and rearranging we arrive at

$$\Delta P_k^c = \frac{P_k^f}{1 - \Phi_k[n, m]}. \tag{15}$$

The application of the flow canceling transaction, ΔP_k^c, approximates the impact on the system of the outage of line ℓ_k. Thus, using superposition, as was done in the approach in (12) to obtain the PTDFs from the ISFs, one can easily see that in order to obtain the change in the angle at any bus due to the flow canceling transaction we can use (10) and the ΔP_k^c injection (withdrawal) at bus n (m). In particular, we can approximate the change in the angles at the line ℓ_k terminal buses n and m, respectively, with respect to the outage of ℓ_k as

$$\Delta\theta_n \approx (\Omega[n, n] - \Omega[n, m]) \Delta P_k^c, \tag{16}$$

and

$$\Delta\theta_m \approx (\Omega[m, n] - \Omega[m, m]) \Delta P_k^c. \tag{17}$$

Substituting (15) into (16) and (17), we obtain expressions relating the line ℓ_k terminal bus voltage angles to its pre-outage flow

$$\Delta\theta_n \approx \frac{(\Omega[n, n] - \Omega[n, m])}{1 - \Phi_k[n, m]} P_k^f = T_n P_k^f, \tag{18}$$

and

$$\Delta\theta_m \approx \frac{(\Omega[m, n] - \Omega[m, m])}{1 - \Phi_k[n, m]} P_k^f = -T_m P_k^f. \tag{19}$$

Subtracting the angle change at each terminal node, given by (18) and (19), we arrive at

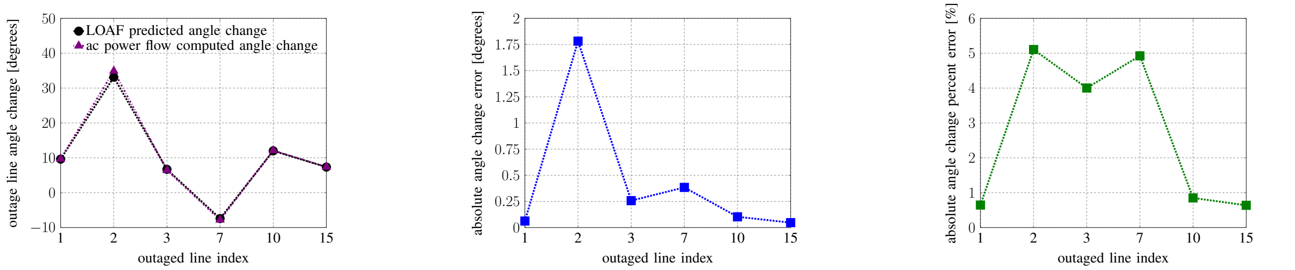
$$\Delta\theta_{n\text{-}m} = \Delta\theta_n - \Delta\theta_m = (T_n + T_m) P_k^f.$$

Thus, the LOAF for line ℓ_k is given by

$$\Sigma_k = T_n + T_m \tag{20}$$

III. LOAF Applications

In this section, we give an overview of two primary online LOAF applications.



(a) Outage angle comparison. (b) Outage angle absolute errors. (c) Outage angle percent absolute errors.

Fig. 3: IEEE 14-bus LOAF-predicted and ac power flow-computed outage angle values and errors for lines with outage angles greater than 5 degrees.

A. Outage Angle Monitoring

The AF and LOAFs can be used to alert operators of the existence of large line outage angles, such as that which contributed to the San Diego blackout [2]. To this end, the total outage angle differences between the terminal buses of a line $\ell_k = (n, m)$ can be written in terms of the bus angles and line flows (obtained from a state estimator), and the LOAFs as follows:

$$\theta^f_k = \theta_{n\text{-}m} + \Sigma_k P^f_k. \tag{21}$$

where $\theta_{n\text{-}m} = \theta_n - \theta_m$. When an angle is shown via (21) to exceed a pre-specified limit, the system operator can take actions to mitigate the angle using LOAF-based dispatch tools, which we describe next.

B. Angle-Constrained Dispatch

The line outage angles computed online with (21) alert operators to potential angle issues. In order to address identified angle issues, we propose extending the real-time SCED to include angle constraints.

The real-time SCED is a widely used operational tool that aims to determine the change in the generator dispatch targets, ΔP^g, required to economically meet the forecast change in the load, ΔP^d, plus losses, ΔP^ℓ, and ensure system physical constraints and reliability standards are met [7]. Let the system be operating with a load and dispatch (P^d_0, P^g_0) and corresponding line flows P^f_0. Then, the real-time SCED problem, which is typically formulated under the dc assumptions, can be stated as follows

$$\min_{\Delta P^g} \sum_{i \in \mathcal{G}} \mathscr{C}_i(\Delta P^g_i + P^g_{i,0}) \tag{22a}$$

$$s.t.$$

$$\mathbb{1}^T_G \Delta P^g - \mathbb{1}^T_D \Delta P^d - \Delta P^\ell = 0 \tag{22b}$$

$$\underline{P}^g \leq P^g_0 + \Delta P^g \leq \bar{P}^g \tag{22c}$$

$$\underline{P}^f \leq P^f_0 + \Psi \Delta P \leq \bar{P}^f, \tag{22d}$$

$$\underline{P}^s \leq P^f_0 + \Psi^s \Delta P \leq \bar{P}^s, \tag{22e}$$

where $\mathscr{C}_i(\cdot)$ is the cost function of generator i (typically a quadratic or piecewise-linear function); $P^g_{i,0}$ is ith element of P^g_0; $\mathbb{1}_G$ and $\mathbb{1}_D$ are G- and D-dimensional all-ones vectors, respectively; and ΔP^ℓ is the change in system-wide real power losses, which typically takes the form of a marginal loss model (see, e.g., [8]); $\underline{P}^g$ ($\bar{P}^g$) and $\underline{P}^f$ ($\bar{P}^f$) are the G- and L-dimensional vectors of generator and line flow lower (upper) limits, respectively; Ψ^s is the matrix of post-outage ISFs for lines at risk of overload due to an outage as determined from the results of contingency analysis (see, e.g., [7]); and $\underline{P}^s$ ($\bar{P}^s$) is the appropriately dimensioned vector of security constraint lower (upper) limits.

The primary outcomes of the real-time SCED in (22) are generator dispatch instructions, ΔP^g, that ensure $N - 1$ reliability, i.e., no piece of equipment is overloaded by a single outage. However, the conventional real-time SCED does not guard against large line outage angles.

To bring outage angle considerations into the SCED, let $\Delta\theta^f_k$ be the change in the outage angle across line ℓ_k due to active power injection changes, the vector of which we denote by ΔP. Then, we formulate outage angle-focused reliability constraint for each line ℓ_k of interest as follows:

$$-\bar{\theta}^f_k \leq \theta^f_{k,0} + (\Omega_n - \Omega_m)\Delta P + \Sigma_k \Psi_k \Delta P \leq \bar{\theta}^f_k, \tag{23}$$

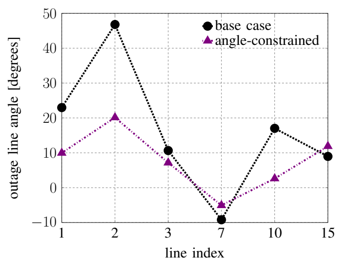
where Ω_n (Ω_m) are the nth (mth) rows of Ω, $\theta^f_{k,0}$ is the nominal line ℓ_k outage angle (computed with (21)) , and $\bar{\theta}^f_k$ is the maximum allowable line ℓ_k outage angle. Those lines deemed to be critical, e.g., lines that must be reclosed quickly in the event of a fault, can have their outage angle restricted in (22) by including a constraint (23) for each such line.

IV. Case Studies

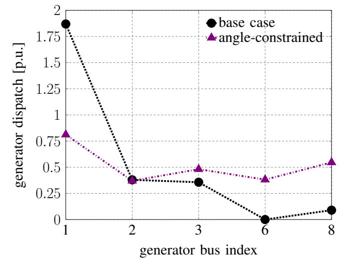
In this section, we present the results of a case study carried out using a modified version of the IEEE 5-generator, 20-line, 14-bus test system provided in the simulation package MATPOWER [9], shown in Fig. 2. The test system has been modified by condensing the parallel lines between buses 1 and 2 into a single equivalent line. Further, to push flow from ℓ_1 onto line ℓ_2 so as to more effectively illustrate the impacts of large outage angle mitigation, we increase the line ℓ_1 reactance, x_1, to 0.4438 p.u.

A. Outage Angle Prediction Accuracy

We first investigate the accuracy with which the LOAFs are capable of predicting each line's outage angle. We compute the AFs and LOAFs for the 14-bus system using (10) and



(a) Outage angles.



(b) Generator dispatch.

Fig. 4: IEEE 14-bus base case vs angle-constrained outage angles and generator dispatch.

(20), respectively, and compare, for each line ℓ_k, the outage angle predicted by the LOAFs around the nominal power flow solution to that computed with the full nonlinear ac power flow with line ℓ_k outaged.

Figure 3a presents the outage angles change for lines with outage angle changes greater than 5 degrees; whereas Figs. 3b and 3c, respectively, report the absolute angle and percent errors between the LOAF-predicted and ac power flow-computed outage angle changes for the same lines. As the figures show, LOAFs prove to be an accurate predictor of the outage angle. The error of any single line does not exceed 6% and the mean-squared error (MSE) of the LOAF vs the ac power flow-computed outage angel change across all lines is 1.845 degrees.

B. Real-Time Angle-Constrained SCED

Now, we investigate the application of the LOAFs in the SCED to preventatively dispatch the system to maintain the line outage angles below a pre-specified threshold—20 degrees in our case. To determine the base case dispatch, we solve the real-time SCED problem in (22) using the quadratic cost functions provided with the 14-bus test system. The angle-constrained dispatch is computed by solving (22) with the additional constraints provided in (23) for all lines except line ℓ_{14}, the outage of which results in the islanding of bus 8. We compute the model-based LOAFs around the base case dispatch and all required AC power flow solutions with bus 2 as the slack bus.

The goal of the angle-constrained SCED is to prevent the total outage angle of each line (i.e., the pre-outage angle between the terminal buses plus the angle change due to the outage of the line) within the bounds of the specified limit. Figure 4a provides the total outage angle of lines with outage angles above 5 degrees in the base and angle-constrained cases. Note that the line ℓ_2 outage angle is greater than 45 degrees in the base case. Such an angle may make it difficult to quickly reclose line ℓ_2 in the event of its outage. Furthermore, line reclosure with such a large angle difference may risk damage to nearby generators.

To address the large line ℓ_2 outage angle, we rerun the real-time SCED with angle constraints from (23) enforced. The dispatch in the base and angle-constrained cases is reported in Fig. 4b. In the angle-constrained dispatch, a significant portion of the output from the generator at bus 1 (a terminal bus of line ℓ_2) is shifted to other generators in the system. As shown in Fig. 4a, this redispatch of the generation brings the outage angle corresponding to ℓ_2 within the specified 20 degree limit. This redispatch, however, comes at a cost: the total dispatch cost in the base case is \$8159, while that in the angle-constrained case is \$8693, an increase of \$534 or 6.55%. Thus, there is a tradeoff between additional reliability and additional cost.

V. Concluding Remarks

In this paper, we introduced a formulation of the LOAFs based on the AFs and PTDFs. The proposed LOAFs, together with the AFs and ISFs, provide a means of assessing the impacts of line outage angles using only the non-outage system topology.

Furthermore, we proposed LOAF-based angle monitoring and dispatch tools, the need of which was identified as critically important by the 2011 San Diego blackout after-the-fact assessment. The angle-constrained SCED proposed in this work provides a systematic means of controlling outage angles and assessing the costs of maintaining outage angle reliability, as demonstrated in our case study.

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