Distributed Optimal Load Frequency Control and Balancing Authority Area Coordination

Dimitra Apostolopoulou, Peter W. Sauer, and Alejandro D. Domínguez-García

Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801
Email: {apostol2, psauer, aledan}@illinois.edu

***Abstract*—This paper proposes a distributed optimal load frequency control (OLFC) system that imitates the behavior of the economic dispatch process in a multi-area power system. Power systems are divided in balancing authority (BA) areas that exercise their own load frequency control to restore the system frequency and real power interchange to the desired values. In this paper, we use a simplified model to describe the power system dynamic behavior and propose a coordination scheme between BA areas in an interconnected power system that decreases the regulation amount needed as well as the associated costs by formulating a distributed OLFC system. The proposed control scheme is compared with traditional automatic generation control (AGC) systems, as well as the economic dispatch process. The proposed distributed OLFC system is illustrated through the Western Electricity Coordination Council (WECC) 9-bus 3-machine system.**

I. Introduction

The US bulk power system is divided into several organizations that implement various control processes in different timescales to maintain reliability in an economic way. More specifically, independent system operators (ISOs) are federally regulated organizations, formed at the recommendation of FERC (Orders Nos. 888/889), that, among other things, are responsible for implementing the economic dispatch process, which is used to allocate the total generation among the committed generating units at minimum cost (see, e.g., [1]). ISOs are further divided into balancing authority (BA) areas that are responsible (i) for maintaining load-interchange-generation balance within each BA area, and (ii) for supporting interconnection frequency in real time. When generation and load are not balanced, frequency is not at its nominal value. Thus, BA areas exercise load frequency control, such as automatic generation control (AGC), to restore the system frequency to the nominal value. The AGC system sends out command signals to the generating units, participating in regulation, to shift their generation output to restore the system frequency and the real power interchange to desired values (see, e.g., [1]). However, BA areas are obliged to purchase expensive ancillary services to accommodate variability in load and generation, which is intensified due to, e.g., deepening penetration of renewable-based generation. Thus, a method that improves the efficiency of the load frequency control system would be beneficial for all BA areas.

Next, we discuss some relevant works in the literature which have also looked at the design of load frequency control systems to improve their efficiency. In [2], the authors propose a real-time optimal power flow that takes into account changes in system load, tie-line flows, and generator limits, by successive execution of a security dispatch and a constrained economic dispatch. The authors of [3] propose a load frequency controller for multi-area power system with unknown parameters, based on fuzzy logic systems, that approximate governor dead band and generation rate constraint nonlinearities. In [4], the author presents an efficient load frequency control system with the critical load level-based adaptive participation factor, derived from security constrained economic dispatch solutions.

In this paper, we propose a distributed optimal load frequency control (OLFC), which replicates the behavior of the economic dispatch process and restores the system frequency to the desired value. The real power interchange between BA areas may be different than that scheduled, since the BA areas coordinate to reach the optimal solution, in terms of cost, of restoring the system frequency to the nominal value. BA areas do not wish to exchange any cost information of their regulating units. In the proposed distributed OLFC the only information BA areas exchange is the total mismatch of generation and load. In this regard, we formulate an optimization problem whose objective function includes a cost function and the frequency deviation, and we use the saddle point dynamics to develop its distributed solution. Next, we compare the developed OLFC system with the traditional AGC system and the economic dispatch implemented at the union of BA areas. The advantages of the proposed method are that the regulation amount and the cost are less than the traditional AGC system implemented at the BA area level. In addition, such a distributed algorithm is scalable, and may be used in a power system with a large number of small distributed generators. The response of the distributed OLFC is automatic to variations in the load or generation, which makes it suitable for the modern electric grid where there is a deepening penetration of renewable-based resources. In this paper, we numerically verified that the proposed distributed OLFC provides good results and we have compared with the traditional AGC system and economic dispatch process.

II. PRELIMINARIES

In this section, we provide models for the economic dispatch process, the BA area dynamic behavior, and the AGC system dynamics.

A. Economic Dispatch

We consider a power system with N buses indexed by $\mathscr{N} = \{1, \ldots, N\}$, L lines indexed by $\mathscr{L} = \{l_1, \ldots, l_L\}$, and G synchronous generating units indexed by $\mathscr{G} = \{1, 2, ..., G\}$; the power system $(\mathscr{N}, \mathscr{L})$ is under the jurisdiction of an ISO. We denote the total system load by P_D, the net interchange P_I, the power losses by P_{losses}, the output of the i^{th} unit by P_{g_i} and the i^{th} generation cost function by $c_i(\cdot)$. Usually, the cost function of a generator is modeled as quadratic function: $c_i(P_{g_i}) = a_i P_{g_i}^2 + \beta_i P_{g_i} + \gamma_i$ (see, e.g., [1]). We assume a linear change of the system losses due to a bus load change [2]. Let ρ be the sensitivity of the losses with respect to the system load, then the system losses are given by $P_{\text{losses}} = \rho P_D$. A simple formulation of the ED process is given by

$$\begin{aligned} &\underset{P_{g_i}}{\text{minimize}} \ \sum_{i \in \mathscr{G}} c_i(P_{g_i}) \\ &\text{such that} \ \sum_{i \in \mathscr{G}} P_{g_i} = (1+\rho)P_D + P_I \longleftrightarrow \lambda, \end{aligned} \tag{1}$$

where λ is the Lagrangian multiplier or dual variable associated with the constraint, which shows the marginal cost of supplying one more MW of generation. The outcome of the economic dispatch is the optimal output of each generator, denoted by P_{ed_i}, for all $i \in \mathscr{G}$.

B. BA Area Dynamic Model

Let us assume that we have M BA areas within the interconnected $(\mathscr{N}, \mathscr{L})$ system, denoted by $\mathscr{A} = \{1, \ldots, M\}$. For each $m \in \mathscr{A}$, we denote by $\mathscr{A}_m \subset \mathscr{A}$ the set of BA areas that have lines with the BA area m, and by $\mathscr{G}_m = \{1, \ldots, G_m\}$ the set of G_m generators; we have that $\mathscr{G} = \cup_{m \in \mathscr{A}} \mathscr{G}_m$. We denote the deviation of the center of inertia speed from the synchronous speed ω_s by $\Delta\omega_m$, the electrical output of BA area m by P_{G_m}, where $P_{G_m} = \sum_{i \in \mathscr{G}_m} P_{g_i}$, the BA area load by P_{D_m}, z_m the AGC command at BA area m, $P_{I_{m,\text{sch}}}$ the interchange schedule between BA areas m and $m' \in \mathscr{A}_m$, and ρ_m the sensitivity of the losses with respect to the BA area load. The dynamic behavior of a BA area $m \in \mathscr{A}$ is given by

$$\begin{aligned} M_m \frac{d\Delta\omega_m}{dt} &= P_{G_m} - (1+\rho_m)P_{D_m} - P_{I_{m,\text{sch}}} \\ &\quad - D_m \Delta\omega_m, \end{aligned} \tag{2}$$

$$T_{G_m} \frac{dP_{G_m}}{dt} = -P_{G_m} + z_m - \frac{1}{R_{D_m}} \Delta\omega_m, \tag{3}$$

where $M_m = \frac{2H_m}{\omega_s}$, with H_m being the system inertia constant; D_m is the load damping; R_{D_m} is the droop; and T_{G_m} is some time constant (see, e.g., [5]).

Let $\mathscr{B}_{mm'}$ be the set of nodes in BA area m with tie lines to nodes in BA area m'. Then, for small deviations from the nominal trajectory the dynamic behavior of the deviation of the real power interchange between BA areas m and m', $\Delta P_{mm'}$, is given by

$$\frac{d\Delta P_{mm'}}{dt} = B_{mm'}(\Delta\omega_{m'} - \Delta\omega_m), \tag{4}$$

where $B_{mm'} = \sum_{k \in \mathscr{B}_{mm'}, j \in \mathscr{B}_{m'm}} \frac{3V_k V_j cos(\theta_k^\star - \theta_j^\star)}{x_{kj}}$; V_k is the voltage magnitude and θ_k is the voltage angle at bus k; and x_{kj} is reactance of line (k, j) (see, e.g., [1]).

C. Automatic Generation Control Dynamic Model

For each BA area m we introduce a new variable z_m, which is the AGC command, and represents the shift in generation needed to restore the system frequency and real power interchange to the desired values [6]. The dynamic behavior of the z_m is given by

$$\frac{dz_m}{dt} = -z_m - K_{1_m}\Delta\omega_m - K_{2_m}\Delta P_{\text{tie}_m} + P_{G_m}, \tag{5}$$

where $\Delta P_{\text{tie}_m} = \sum_{m' \in \mathscr{A}_m} \Delta P_{mm'}$, with $\Delta P_{mm'}$ the deviation of the actual interchange from the scheduled; and K_{1_m} and K_{2_m} some positive control gains. The total regulation amount needed is $z_m - P_{ED_m}$, with P_{ED_m} the sum of the economic dispatch signal for all generators in BA area m, i.e., $P_{ED_m} = \sum_{i \in \mathscr{G}_m} P_{ed_i}$, and the AGC command signal to each generator i is

$$P_{c_i} = \kappa_{m_i} z_m, \forall i \in \mathscr{G}_m, \tag{6}$$

where κ_{m_i} is the so-called participation factor, with $\sum_{i \in \mathscr{G}_m} \kappa_{m_i} = 1$.

III. OPTIMAL LOAD FREQUENCY CONTROL (OLFC)

In this section, we develop the OLFC, and compare it with the economic dispatch process. Next, we derive the partial-dual algorithm for the OLFC and compare the proposed OLFC system with the traditional AGC system.

A. Formulation of Optimal Load Frequency Control (OLFC)

In order to formulate the load frequency control, we first need to define a set of response requirements the system must satisfy: (i) the static frequency deviation following a step-load change must be zero, and (ii) meet regulation at minimum cost. We design an optimal frequency control system that mimics the results of an economic dispatch at the entire set of BA areas $\mathscr{A}$, i.e., the power system $(\mathscr{N}, \mathscr{L})$. Let us assume that in (1), $P_I = 0$, i.e., there is no real power interchange with other power systems. We denote by $\Delta\omega$ the deviation of the speed of center of inertia from the synchronous speed. Then, the OLFC is given by

$$\begin{aligned} &\underset{P_{g_i}, \Delta\omega}{\text{minimize}} \sum_{i \in \mathscr{G}} c_i(P_{g_i}) + \frac{D}{2}\Delta\omega^2 \\ &\text{such that} \sum_{i \in \mathscr{I}} P_{g_i} - (1+\rho)P_D - D\Delta\omega = 0 \longleftrightarrow \mu, \\ &\qquad \sum_{i \in \mathscr{G}} P_{g_i} = (1+\rho)P_D \longleftrightarrow \nu, \end{aligned} \tag{7}$$

where μ and ν are the Lagrangian multipliers or dual variables associated with the corresponding constraints; D is the load damping for the power system $(\mathscr{N},\mathscr{L})$. Usually, we approximate D by $D=\sum_{m\in\mathscr{A}} D_m$. By using the Karush-Kuhn-Tucker (KKT) optimality conditions the solution of (7), with $c_i(P_{g_i})=a_iP_{g_i}^2+\beta_iP_{g_i}+\gamma_i$, for $i\in\mathscr{G}$, is:

$$
\begin{aligned}
P_{g_i}^{\star} &= \frac{\frac{(1+\rho)P_D+\sum_{i\in\mathscr{G}}\frac{\beta_i}{2a_i}}{\sum_{i\in\mathscr{G}}\frac{1}{2a_i}}-\beta_i}{2a_i}, &(8)\\
\Delta\omega^{\star} &= 0, &(9)\\
\mu^{\star} &= 0, &(10)\\
\nu^{\star} &= -\frac{(1+\rho)P_D+\sum_{i\in\mathscr{G}}\frac{\beta_i}{2a_i}}{\sum_{i\in\mathscr{G}}\frac{1}{2a_i}}. &(11)
\end{aligned}
$$

The total generation P_G in power system $(\mathscr{N},\mathscr{L})$ is $P_G=\sum_{i\in\mathscr{G}}P_{g_i}=(1+\rho)P_D$.

Now, we investigate the relationship of the OLFC given in (7) with the economic dispatch process. The solution of the economic dispatch of a power system, as given in (1), with $P_I=0$ is: $P_{g_i}^{\star}=\frac{\frac{(1+\rho)P_D+\sum_{i\in\mathscr{G}}\frac{\beta_i}{2a_i}}{\sum_{i\in\mathscr{G}}\frac{1}{2a_i}}-\beta_i}{2a_i}$, whose value is the same as in (8), which is the solution of (7).

B. Partial-dual algorithm for OLFC

Besides using the KKT conditions to find the solution of (7), we may utilize the partial-dual algorithm introduced in [7]. This algorithm is based on the saddle point dynamics of (7) (see, e.g., [8]). The partial-dual algorithm of (7) follows the saddle point dynamics for all variables besides $\Delta\omega$. Thus, we have

$$
\begin{aligned}
\frac{dP_{g_i}}{dt} &= -\eta_{g_i}(2a_iP_{g_i}+\beta_i+\mu+\nu), &(12)\\
\Delta\omega &= \mu, &(13)\\
\frac{d\mu}{dt} &= \gamma_\mu\Big(\sum_{i\in\mathscr{G}}P_{g_i}-(1+\rho)P_D-D\Delta\omega\Big), &(14)\\
\frac{d\nu}{dt} &= \gamma_\nu\Big(\sum_{i\in\mathscr{G}}P_{g_i}-(1+\rho)P_D\Big). &(15)
\end{aligned}
$$

where η_{g_i}, γ_μ, and γ_ν are some positive stepsizes.

In order to prove convergence of the algorithm given in (12)-(15), we need to define a Lyapunov function. We define $x=[P_{g_1},\ldots,P_{g_G},\Delta\omega]^T$, $y=[\mu,\nu]^T$, $w=[x^T,y^T]^T$, $\eta=[\eta_{g_1},\ldots,\eta_{g_G}]^T$, $\gamma=[\gamma_\mu,\gamma_\nu]^T$, $H=diag(\eta)$, and $\Gamma=diag(\gamma)$. Then, based on the Krasovskii's method of constructing Lyapunov functions we have that (see, e.g., [9])

$$
V(w)=\frac{dw}{dt}^TQ\frac{dw}{dt},\ \text{with}\ Q=\frac{1}{2}\begin{bmatrix}H^{-1} & 0\\ 0 & \Gamma^{-1}\end{bmatrix}. \qquad (16)
$$

We may write (12)-(15) as $\frac{dw}{dt}=f(w)$.Then, we have $\frac{dV}{dt}=\frac{dw}{dt}^T(\frac{\partial f}{\partial w}^TQ+Q\frac{\partial f}{\partial w})\frac{dw}{dt}$. We need to show that the term in the brackets is negative semidefinite at every w. We have that

$$
\frac{\partial f}{\partial w}^TQ+Q\frac{\partial f}{\partial w}=\begin{bmatrix}-\frac{\partial^2\mathcal{L}}{\partial x^2} & 0\\ 0 & \frac{\partial^2\mathcal{L}}{\partial y^2}\end{bmatrix}\le 0, \qquad (17)
$$

since the Lagrangian $\mathcal{L}$ is convex in x and concave in y. Thus, $\frac{dV}{dt}\le 0$, and $\frac{dV}{dt}=0$ only for the optimal point $(x^\star,y^\star)$.

C. OLFC vs AGC

We wish derive the relationship between the set of differential equations given in (12)-(15), and the power system dynamics with the traditional AGC system for the entire power system $(\mathscr{N},\mathscr{L})$. To this end, we introduce a new variable ζ, which is equal to

$$
\begin{aligned}
\zeta &= -\sum_{i\in\mathscr{G}}\frac{\eta_{g_i}}{1-2\eta_{g_i}a_i}\frac{\gamma_\nu}{\gamma_\mu}\mu+\sum_{i\in\mathscr{G}}\frac{\eta_{g_i}}{1-2\eta_{g_i}a_i}\nu\\
&\quad+\sum_{i\in\mathscr{G}}\frac{1}{1-2\eta_{g_i}a_i}P_{g_i}. \qquad (18)
\end{aligned}
$$

If we take the derivative of (18), we obtain

$$
\begin{aligned}
\frac{d\zeta}{dt} &= -\zeta-\Big(-\sum_{i\in\mathscr{G}}\frac{\eta_{g_i}}{1-2\eta_{g_i}a_i}\big(\frac{\gamma_\nu}{\gamma_\mu}+1-D\gamma_\nu\big)\Big)\Delta\omega\\
&\quad-\sum_{i\in\mathscr{G}}P_{g_i} \qquad (19)
\end{aligned}
$$

with $\eta_{g_i}>\frac{1}{2a_i}$, and $\frac{\gamma_\nu}{\gamma_\mu}+1-D\gamma_\nu>0$.

Now, we develop the traditional AGC of the entire power system $(\mathscr{N},\mathscr{L})$, in the scenario where the whole interconnected system is assumed to be operated by a single BA area, and $c_i(P_{g_i})=a_iP_{g_i}^2$. If we have one BA area then in (5), $\Delta P_{\text{tie}}=0$, and $\frac{dz}{dt}=-z-K_1\Delta\omega-P_G$, where P_G is the total generation at the interconnected system, i.e., $P_G=\sum_{i\in\mathscr{G}}P_{g_i}$. So we have $\frac{dz}{dt}=-z-K_1\Delta\omega+\sum_{i\in\mathscr{G}}P_{g_i}$, and if we set $K_1=-\sum_{i\in\mathscr{G}}\frac{\eta_{g_i}}{1-2\eta_{g_i}a_i}\Big(\frac{\gamma_\nu}{\gamma_\mu}+1-D\gamma_\nu\Big)$, then it totally matches (19).

In addition, the set of differential equations in (12)-(15), and (19) describes the dynamic behavior of the power system $(\mathscr{N},\mathscr{L})$, if we set: $\eta_{g_i}=\frac{\frac{1}{T_G}+1}{2a_i}$, $\forall i\in\mathscr{G}$; $\gamma_\mu=\frac{1}{M}$, and $\gamma_\nu=\frac{(\frac{1}{R_D}-\sum_{i\in\mathscr{G}}\eta_{g_i})\gamma_\mu}{\sum_{i\in\mathscr{G}}\eta_{g_i}}$. By substituting the aforementioned in (12)-(15), and (19), and by setting $\zeta=z$, we obtain the set of differential equations that describe the dynamic behavior of $(\mathscr{N},\mathscr{L})$:

$$
\begin{aligned}
M\frac{d\Delta\omega}{dt} &= P_G-(1+\rho)P_D-D\Delta\omega, &(20)\\
T_G\frac{dP_G}{dt} &= -P_G+z-\frac{1}{R_D}\Delta\omega, &(21)\\
\frac{dz}{dt} &= -z-K_1\Delta\omega+P_G, &(22)
\end{aligned}
$$

The traditional AGC system as given in (5), and its allocation given in (6) provides the same results as the economic dispatch process if we select appropriately the AGC participation factors. If we set in (6) the participation factors to be

$$
\kappa_i=\frac{1}{2a_i}\frac{1}{\sum_{i\in\mathscr{G}}\frac{1}{2a_i}},\forall i\in\mathscr{G}, \qquad (23)
$$

then, each generator output P_{g_i} coincides with the outcome of an economic dispatch. Thus, by setting the participation factor to the values given in (23) we operate the grid in an economic efficient.

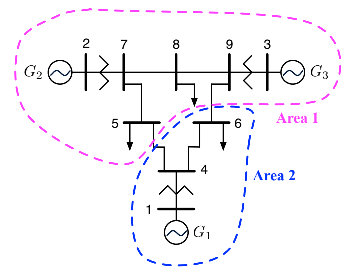

Fig. 1: One-line diagram of the 3-machine 9-bus power system.

IV. Distributed Optimal Load Frequency Control

However, in order to implement the OFLC, given in (7), or the traditional AGC, given in (5) in the power system $(\mathscr{N}, \mathscr{L})$, as one BA area, there is a need for a centralized entity. This would be impractical since each power system is divided into sub-control areas, the so-called BA areas, who implement their own load frequency control system. In this regard, we propose a distributed solution of a problem equivalent to (7). The optimization problem stated in (1), with $P_I = 0$, has the same solution in terms of P_{g_i} as (7). Thus, the distributed OLFC may be based on a distributed algorithm for (1), with $P_I = 0$. Such an algorithm restores the system frequency to the nominal value in the most economic way. More specifically, the control signal sent to each generator P_{c_i}, is evolved based on the saddle point dynamics of (1), and is described by

$$\frac{dP_{c_i}}{dt} = -\sigma_{g_i}(2a_iP_{c_i} + \xi), \quad (24)$$

$$\frac{d\xi}{dt} = \sigma_\xi(\sum_{i\in\mathscr{G}} P_{c_i} - (1+\rho)P_D), \quad (25)$$

with σ_{g_i}, for all $i \in \mathscr{G}$, and σ_ξ some positive stepsizes. The optimal generator output is $P^\star_{c_i} = (\rho+1)P_D \frac{1}{2a_i \sum_{i\in\mathscr{G}} \frac{1}{2a_i}}$.

However, in real time operations load frequency control is discrete. Thus, in order to write (24)-(25) in a discrete form we use Euler's method [10], and develop a discrete distributed OLFC algorithm. We assume that the control mechanism is exercised every 1 sec, thus $h = 1$ sec. For every step k, referring to time instant $t = kh$, we have P_{c_i} the OLFC signal to generator i to be:

$$\begin{aligned} P_{c_i}[k] &= P_{c_i}[k-1] \\ &\quad -\sigma_{g_i}(2a_iP_{c_i}[k-1] + \xi[k-1]), \forall i \in \mathscr{G} \quad (26) \\ \xi[k] &= \xi[k-1] + \sigma_\xi(\sum_{i\in\mathscr{G}} P_{c_i}[k-1] - (1+\rho)P_D). \quad (27) \end{aligned}$$

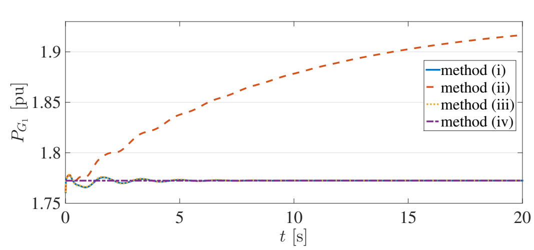

Fig. 2: Generator 1 output with the four methods.

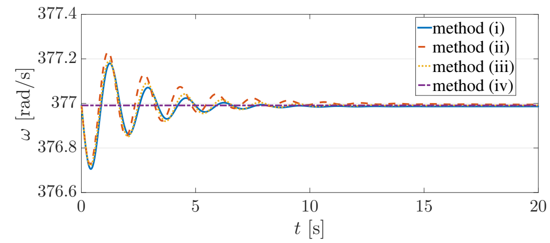

Fig. 3: Speed of center of inertia for all methods.

We define $\phi[k] = [P_{c_1}[k], \dots, P_{c_G}[k], \xi[k]]^T$, and have $\phi[k] = A\phi[k-1] + b$, with the matrix A, and vector b defined as

$$A = \begin{bmatrix} 1-2a_1\sigma_{g_1} & 0 & \dots & 0 & -\sigma_{g_1} \\ 0 & 1-2a_2\sigma_{g_2} & \dots & 0 & -\sigma_{g_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_\xi & \sigma_\xi & \dots & \sigma_\xi & 1 \end{bmatrix}, \quad (28)$$

$$b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -\sigma_\xi(1+\rho)P_D \end{bmatrix}. \quad (29)$$

The unique steady state equilibrium is $\phi^\star = (I-A)^{-1}b$, with I the identity matrix, and $|I - A| \neq 0$ [11]. By the substituting A, b above, we obtain

$$P^\star_{c_i} = (\rho+1)P_D \frac{1}{2a_i \sum_{i\in\mathscr{G}} \frac{1}{2a_i}}, \forall i \in \mathscr{G}, \quad (30)$$

$$\xi^\star = -\frac{(\rho+1)P_D}{\sum_{i\in\mathscr{G}} \frac{1}{2a_i}}, \quad (31)$$

which is the same as (8), and (11), with $c_i(P_{g_i}) = a_iP^2_{g_i}$.

V. Numerical Examples

In this section, we illustrate the proposed control system with the standard 3-machine 9-bus Western Electricity Coordination Council (WECC) power system model (see Fig. 1), which contains three synchronous generating units, and is denoted by $(\mathscr{N}, \mathscr{L})$. The machine, network and load parameter values may be found in [5, pp. 170-182]. Unless otherwise noted, all quantities in this section are expressed in per unit (pu) with respect to 100 MVA as base power. The cost functions for the generators are: $c_1(P_{g_1}) = 5P^2_{g_1}$, $c_2(P_{g_2}) = 10P^2_{g_2}$, and $c_3(P_{g_3}) = 15P^2_{g_3}$, in \$/pu. We solve the economic dispatch such that the load in bus 5 is $P_{d_5} + jQ_{d_5} = 1.25 + j0.50$, in bus 6 is $P_{d_6} + jQ_{d_6} = 0.90 + j0.30$ and in bus 8 is $P_{d_8} + jQ_{d_8} = 1.50 + j0.35$. The economic dispatch at $(\mathscr{N}, \mathscr{L})$ results in $P_{ed_1} = 1.7609$ pu, $P_{ed_2} = 0.8805$ pu, and $P_{ed_3} = 0.587$ pu. The total power system generation is $P_{ED} = 3.2284$ pu, and the total load is $P_D = 3.1500$ pu, thus the losses are $P_{\text{losses}} = 0.0784$ pu. In this regard the sensitivity of the losses with respect to the system load is $\rho = 0.0249$. We divide $(\mathscr{N}, \mathscr{L})$ into two BA areas, as depicted in Fig. 1. The load dampings for the areas are: $D_1 = 0.0116$ pu, and $D_2 = 0.0125$ pu, and the droops of the generators at each BA area are: $R_{D_1} = 0.1$ pu, and $R_{D_2} = 0.05$ pu. The interchange schedule from BA area 1 to 2 is -0.8328 pu.

We modify the load at bus 5, which belongs in BA area 1, as follows $P_{d_5} = 1.1$ pu, and the load at bus 6, which belongs in BA area 2, as $P_{d_6} = 1.07$ pu. We model the system response using fours methods: (i) the proposed distributed OLFC, (ii) the traditional AGC system in a BA area level with participation factors for BA area 1, $\kappa_{1_2} = 0.5$. $\kappa_{1_3} = 0.5$, and for BA area 2, $\kappa_{2_1} = 1$, (iii) the traditional AGC system for the entire power system $(\mathscr{N}, \mathscr{L})$ with participation factors proportional to the outcome of the most recent economic dispatch that is $\kappa_1' = 0.5454$, $\kappa_2' = 0.2727$, and $\kappa_3' = 0.1818$, and (iv) the economic dispatch process with the modified system load as input. In case (iv) it is assumed that the economic dispatch process is exercised at time $t = 0$ sec to meet the load.

In Fig. 2, we depict the generation output over time of Generator 1 for all the methods, assuming that the load modification occurred at time $t = 0$ s. We notice that the proposed control system, given in (i), converges to the solution of the economic dispatch process, given in (iv). We also notice that if we exercise the traditional AGC system in the 9-bus 3-machine system as one BA area and the participation factors are chosen based on (23), then the system converges to the solution of the economic dispatch process. However, in order to implement the AGC system in the entire system there is a need for a centralized entity that controls the entire system. We may see that method (ii) deviates more from the solution of (iv), and requires a larger regulation amount. The reason is that the AGC system at each BA area does not take advantage of the diversity of the load changes in the BA areas, and that the interchange schedule needs to remain unchanged. More specifically, the real power interchange between BA areas 1 and 2 is -0.6724 pu with methods (i), (ii), and (iv) and as scheduled, that is -0.8328 pu, with method (ii).

The speed of center of inertia ω is depicted in Fig. 3, where we notice that with all methods it converges to the nominal value ω_s. We may also see that the oscillations are higher with method (ii), and that the time it is needed for ω to converge to ω_s is smaller for methods (i), (iii). In Fig. 4, we may see that the cost associated with method (ii) is the highest and that the cost for the methods (i), and (iii) totally match. The total cost over the 20 sec period is $57,576$ \$/pu for methods (i), (iii), and (iv), and $57,849$ \$/pu for method (ii).

VI. Concluding Remarks

In this paper, we proposed an OLFC system that restores system frequency at minimum cost by BA area coordination. We show the relationship of the proposed control system with the economic dispatch process, and the traditional AGC system in the case where all BA areas are considered to be one single BA area. In addition, we demonstrate the relation of the OLFC and the power system dynamics including the AGC system. Next, we propose a discrete distributed solution of the OLFC so that BA areas do not exchange any cost information of their regulating units, and only the mismatch of generation and load needs to be known. We demonstrated via numerical examples how the proposed method works, and compared its performance with other three methods. We also showed that the proposed BA coordination scheme provides the same solution as the economic dispatch process and the traditional AGC system implemented in the union of the BA areas. Thus, reliability is met at minimum cost.

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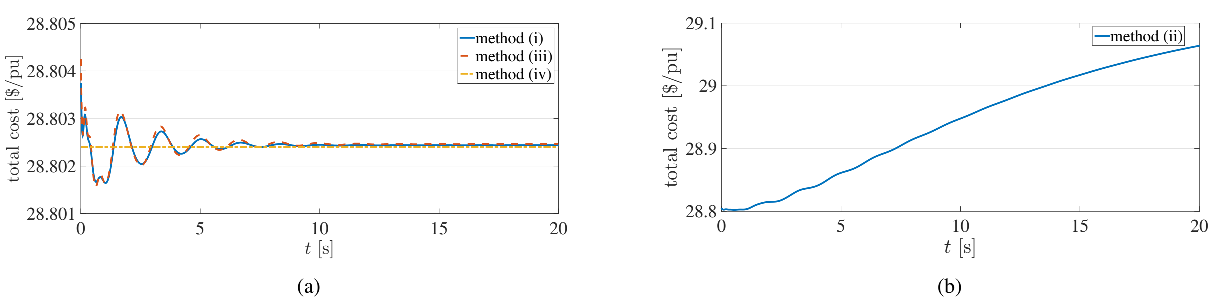

Fig. 4: Total cost of regulation with the four methods