

Price-Based Distributed Control for Networked Plug-in Electric Vehicles

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Abstract—We introduce a framework for controlling the charging and discharging processes of plug-in electric vehicles (PEVs) via pricing strategies. Our framework consists of a hierarchical decision-making setting with two layers, which we refer to as *aggregator layer* and *retail market layer*. In the aggregator layer, there is a set of aggregators that are requested (and will be compensated for) to provide certain amount of energy over a period of time. In the retail market layer, the aggregator offers some price for the energy that PEVs may provide; the objective is to choose a pricing strategy to incentivize the PEVs so as they collectively provide the amount of energy that the aggregator has been asked for. The focus of this paper is on the decision-making process that takes places in the retail market layer, where we assume that each individual PEV is a price-anticipating decision-maker. We cast this decision-making process as a game, and provide conditions on the pricing strategy of the aggregator under which this game has a unique Nash equilibrium. We propose a distributed consensus-based iterative algorithm through which the PEVs can seek for this Nash equilibrium. Numerical simulations are included to illustrate our results.

I. INTRODUCTION

Advanced communication and control systems, renewable-based electricity generation resources, and storage-capable loads such as plug-in electric vehicles (PEVs), will bring new opportunities for a more flexible and efficient operation of electric power systems. For instance, PEVs can be utilized to provide active power for up and down regulation services, e.g., energy peak-shaving during peak hours and load-leveling at night [1], [2]. However, in order to enable the added functionality that these technologies may provide, it is necessary to develop appropriate control mechanisms. In this paper, we address this problem in the context of PEVs; specifically, we consider a competitive scenario in which individual PEVs are decision makers, and develop a framework for controlling their charging and discharging via pricing strategies.

In our setting, we consider a two-layer decision-making structure. In the first layer—the aggregator layer—there is a set of aggregators that, through some market-clearing mechanism, are requested (and will be compensated for) to provide certain amount of energy over some predetermined period of time. In the second layer—the retail market layer—each aggregator offers a price for the energy that PEVs may provide (positive if charging and negative if discharging); then, the objective is for the aggregator to design a pricing strategy in order to incentivize PEVs to charge and/or discharge so they collectively provide the amount of energy that the aggregator has been asked for. The focus

of this paper is on the retail market layer. We assume that each PEV is a price-anticipating decision-maker (i.e., it is aware of the aggregator pricing strategy), and in order to inform its decision making, it exchanges some information with neighboring PEVs, with the objective of estimating the average charge capacity collectively available in its immediate neighborhood. Then each PEV uses this estimate together with the aggregator’s pricing strategy information to decide the amount of energy that it will sell or buy. We cast this decision-making process as a game, and provide conditions on the pricing strategy of the aggregator under which this game has a unique Nash equilibrium. We then propose a distributed consensus-based iterative algorithm through which the PEVs seek for this Nash equilibrium.

The work in this paper has connections with the literature on distributed sensing and control in energy systems and game-theoretic modeling in energy markets. The importance of distributed sensing and control in future grids has been mentioned in several recent papers; examples include [1], [3], and [4]. The distributed strategies introduced in this paper are closely related to distributed optimization algorithms for the optimization of a sum of convex functions (see e.g. [5], [6], [7], [8], [9], [10]). These works build on consensus-based dynamics to find the solutions of the optimization problem in a variety of scenarios and are typically designed in discrete time, with possible exception of [8] and [9].

Game-theoretic models have been used recently for studying energy markets (see, e.g., [11], [12], [13], [14]). The game-theoretic aspects of our work are related to noncooperative resource allocation problems, see for example [15], [16], [17], where under appropriate concavity assumptions, the existence of Nash equilibrium in pure strategies is guaranteed using the results in [18]. In [11], a game-theoretic model is introduced for studying the charging and discharging processes of PEVs. In addition to the fact that the model does not take into account the original available charge of PEVs for participating in the game, the considered PEVs are not price anticipating, i.e., they do not take into account the fact that the prices are set based on the average available charge. Also the fact that future PEVs are decision-makers and have personal utility functions are not taken into account in this model. The process of charging in our work is related to the work in [14] and [19]; however, in our setting, we deal with a scenario in which the PEVs are individual decision-makers and seek for the Nash equilibrium using the information available from their neighboring PEVs (along with the price set by the aggregator). With respect to this, a focus of our work is on the key role played by the PEVs’ network structure. Also, our setting allows for scenarios in which PEVs are capable of both charging and discharging.

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The contributions of this paper are as follows. The first one is the introduction of a framework for controlling the charging and discharging of PEVs via pricing strategies. Our second contribution is to cast this market scenario as a multi-stage game and provide conditions on the pricing strategy of the aggregator under which this game has a unique Nash equilibrium. Our third contribution is the design of a distributed consensus-based iterative algorithm through which the PEVs seek the Nash equilibrium (when unique) of the game describing the decision-making process that takes place in the retail market layer. We establish the asymptotic convergence property of this dynamic algorithm, when the payoff functions are locally Lipschitz (i.e., not necessarily differentiable) and concave, and the underlying PEVs' network is undirected and connected. As a by-product, our distributed scheme can be used for inducing the Nash equilibrium, when unique, for other locally Lipschitz concave games on undirected graphs with no shared constraints.

II. MATHEMATICAL PRELIMINARIES

We start with some notational conventions. Let \mathbb{R} , $\mathbb{R}_{\geq 0}$, \mathbb{Z} , and $\mathbb{Z}_{\geq 1}$ denote the set of real, nonnegative real, integer, and positive integer numbers, respectively. We denote by $\mathcal{B}(X)$ the space of bounded real-valued functions on a set $X \subset \mathbb{R}^d$, $d \in \mathbb{Z}_{\geq 1}$; we use $\mathcal{B}^0(X)$ when the functions are, additionally, continuous. We also denote by $\text{co}(X)$ the convex hull of X . We use the short-hand notation $\mathbf{1}_d = (1, \dots, 1)^T \in \mathbb{R}^d$ and $\mathbf{0}_d = (0, \dots, 0)^T \in \mathbb{R}^d$.

A. Discrete Set-valued Analysis

Here, we provide a brief exposition of useful concepts from discrete-time set-valued dynamical systems following [20]. For $X \subseteq \mathbb{R}^d$, let $F : X \rightrightarrows X$ denote a set-valued map that takes a point in X to a subset $F(x)$ of X . The map F is nonempty if $F(x) \neq \emptyset$, for all $x \in X$. A point $x^* \in X$ is a *fixed point* of F if $x^* \in F(x^*)$. An *evolution* of F on X is any trajectory $\gamma : \mathbb{Z}_{\geq 0} \rightarrow X$ such that

$$\gamma(k+1) \in F(\gamma(k)), \quad \text{for all } k \in \mathbb{Z}_{\geq 0}.$$

The set-valued map F is *upper semicontinuous* at $x \in X$ if, for any two convergent sequences $\{x_k\}_{k=0}^{\infty}$ and $\{y_k\}_{k=0}^{\infty}$, with $\lim_{k \rightarrow \infty} x_k = x$, $\lim_{k \rightarrow \infty} y_k = y$, and $y_k \in F(x_k)$, for all $k \in \mathbb{Z}_{\geq 0}$, we have $y \in F(x)$. The map F is upper semicontinuous on X if it is upper semicontinuous at x , for all $x \in X$. A set $W \subset X$ is *weakly positively invariant* with respect to F if for any $x \in W$ there exists $y \in W$ such that $y \in F(x)$ and *strongly positively invariant* with respect to F if $F(x) \subset W$, for all $x \in W$. Finally, a continuous function $V : X \rightarrow \mathbb{R}$ is called *non-increasing* along F in $W \subset X$ if $V(y) \leq V(x)$, for all $x \in W$ and $y \in F(x)$. Equipped with these tools, one can formulate the following set-valued version of the LaSalle invariance principle [21], [22], which will be most useful in the developments later.

Theorem 2.1: (LaSalle invariance principle for discrete-time set-valued dynamical systems): Let $F : X \rightrightarrows X$ be an upper semicontinuous set-valued map on $X \subset \mathbb{R}^d$ and let $W \subset X$ be strongly positively invariant with respect to F . Suppose F is nonempty

on W and all evolutions of F with initial condition in W are bounded. Let $V : X \rightarrow \mathbb{R}$ be continuous and non-increasing function along F on W . Then, any evolution of F with initial condition in W approaches a set of the form $S \cap V^{-1}(c)$, where $c \in \mathbb{R}$ and S is the largest weakly positively invariant set contained in $\{x \in W \mid \text{there exists } y \in F(x) \text{ such that } V(x) = V(y)\}$.

B. Nonsmooth Analysis

We recall some notions from nonsmooth analysis [23]. A function $f : \mathbb{R}^d \rightarrow \mathbb{R}$ is *locally Lipschitz* at $x \in \mathbb{R}^d$ if there exists a neighborhood \mathcal{U} of x and $C_x \in \mathbb{R}_{\geq 0}$ such that $|f(y) - f(z)| \leq C_x \|y - z\|$, for $y, z \in \mathcal{U}$; f is locally Lipschitz on \mathbb{R}^d if it is locally Lipschitz at x for all $x \in \mathbb{R}^d$. Locally Lipschitz functions are differentiable almost everywhere. The *generalized gradient* of f is

$$\partial f(x) = \text{co} \left\{ \lim_{k \rightarrow \infty} \nabla f(x_k) \mid x_k \rightarrow x, x_k \notin \Omega_f \cup S \right\},$$

where Ω_f is the set of points where f fails to be differentiable and S is any set of measure zero. We recall the following properties of generalized gradients [23].

Lemma 2.2: (Continuity of the generalized gradient map): Let $f : \mathbb{R}^d \rightarrow \mathbb{R}$ be a locally Lipschitz function at $x \in \mathbb{R}^d$. Then the set-valued map $\partial f : \mathbb{R}^d \rightrightarrows \mathbb{R}^d$ is upper semicontinuous and locally bounded at $x \in \mathbb{R}^d$ and moreover, $\partial f(x)$ is nonempty, compact, and convex.

For $f : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R}$ and $z \in \mathbb{R}^d$, we let $\partial_x f(x, z)$ denote the generalized gradient of $x \mapsto f(x, z)$. Similarly, for $x \in \mathbb{R}^d$, we let $\partial_z f(x, z)$ denote the generalized gradient of $z \mapsto f(x, z)$. A point $x \in \mathbb{R}^d$ with $\mathbf{0}_d \in \partial f(x)$ is a *critical point* of f . A function $f : \mathbb{R}^d \rightarrow \mathbb{R}$ is *regular* at $x \in \mathbb{R}^d$ if for all $v \in \mathbb{R}^d$ the right directional derivative of f , in the direction of v , exists at x and coincides with the generalized directional derivative of f at x in the direction of v . We refer the reader to [23] for definitions of these notions. A convex and locally Lipschitz function at x is regular [23, Proposition 2.3.6]. The notion of regularity plays an important role when considering sums of Lipschitz functions as the next result shows.

Lemma 2.3: (Finite sum of locally Lipschitz functions): Let $\{f^i\}_{i=1}^n$ be locally Lipschitz at $x \in \mathbb{R}^d$. Then $\partial(\sum_{i=1}^n f^i)(x) \subseteq \sum_{i=1}^n \partial f^i(x)$, and equality holds if f^i is regular for $i \in \{1, \dots, n\}$.

Here the summation on the lefthand-side of the inequality should be understood in the sense described in [23]. A locally Lipschitz and convex function f satisfies, for all $x, x' \in \mathbb{R}^d$ and $\xi \in \partial f(x)$, the *first-order condition* of convexity, $f(x') - f(x) \geq \xi \cdot (x' - x)$.

C. Graph Theory

A *directed graph*, or simply *digraph*, is a pair $\mathcal{G} = (V, E)$, where V is a finite set called the vertex set and $E \subseteq V \times V$ is the edge set. When E is unordered, we call \mathcal{G} an *undirected graph* or simply a *graph*. In this paper, we only deal with undirected graphs. Given an edge $(u, v) \in E$, we call u and v neighbors and denote the set of neighbors of v by $\mathcal{N}_{\mathcal{G}}(v)$. A graph is called *connected* if there exists a path between any two vertices. A *weighted graph* is a triplet $\mathcal{G} = (V, E, A)$, where (V, E) is a graph and $A \in \mathbb{R}_{\geq 0}^{n \times n}$ is the

adjacency matrix of \mathcal{G} . The adjacency matrix has the property that $a_{ij} > 0$ if $(v_i, v_j) \in E$ and $a_{ij} = 0$, otherwise. The weighted degree v_i , $i \in \{1, \dots, n\}$ is $d^w(v_i) = \sum_{j=1}^n a_{ij}$. The weighted degree matrix D is the diagonal matrix defined by $(D)_{ii} = d^w(v_i)$, for all $i \in \{1, \dots, n\}$. The Laplacian is $L = D - A$. For an undirected graph, $L\mathbf{1}_n = \mathbf{1}_n^T L = 0$ and $L = L^T$ and is positive semidefinite [24]. When \mathcal{G} is connected, the zero eigenvalue is simple.

D. Game Theory

We recall the class of concave games in the absence of shared constraints from [18]. A *concave game* (with unshared constraints) is a triplet $\mathbf{G} = (V, S, \{f_i\}_{i=1}^n)$, where

- V is a group of $n \in \mathbb{Z}_{\geq 1}$ players,
- $S = S_1 \times S_2 \times \dots \times S_n$ is the strategy set, $S_i \subset \mathbb{R}^{d_i}$, $d_i \in \mathbb{Z}_{\geq 1}$ is nonempty, convex and compact, and
- $f_i : S \rightarrow \mathbb{R}$, the payoff for player $i \in \{1, \dots, n\}$, is a locally Lipschitz concave mapping.

A point $x^* \in S$ is called the *Nash equilibrium* of G if and only if, for all $i \in V$,

$$f_i(x^*) = \max_{y_i} \{f_i(x_1^*, \dots, x_{i-1}^*, y_i, x_{i+1}^*, \dots, x_n^*) \mid y_i \in S_i\}.$$

In other words, when the game is at x^* , no player can improve its payoff by unilaterally deviating from this point. A celebrated theorem by Rosen guarantees the existence of Nash equilibrium for this class of games [18]. A uniqueness result can also be obtained under the so-called diagonally strict concavity assumption, along with differentiability (see [18, Theorem 4]), when one considers another suitable notion of equilibrium (the so-called normalized or variational equilibrium), see [25]. When the constraints are not shared, as it is the case in this paper, these notions of equilibria match, yielding an applicable uniqueness result.

In many applications, including the one in this paper, the differentiability assumption does not hold. Furthermore, the convergence proof of the gradient flow procedure for seeking this Nash equilibrium [18, Theorem 7] is no longer valid; however, the results are still valid, see [26].

III. PROBLEM STATEMENT

We consider a set of aggregators, denoted by $\{v_1^{\text{agg}}, \dots, v_N^{\text{agg}}\}$, $N \in \mathbb{Z}_{\geq 1}$, that, through some market-clearing mechanism, are requested to provide certain amount of energy over a predetermined period of time. Each aggregator v_i^{agg} , $i \in \{1, \dots, N\}$, is assumed to have a backup energy storage device such that if the PEVs do not provide the requested amount of energy (or they provide more than the request), the storage device can be used to provide (or store) the difference. Alternatively, one can assume that the aggregator contracts some insurance with a third party that will provide the difference. Then, v_i^{agg} is responsible for controlling the charging/discharging processes of a group of $n_i \in \mathbb{Z}_{\geq 1}$ PEVs, which we call the v_i^{agg} -group, by offering them a pricing strategy.

Each PEV is a decision maker and can freely choose to participate after receiving a request from its aggregator. The PEVs' actions include remaining idle, charging, or discharging. The decision that each PEV is faced with, among

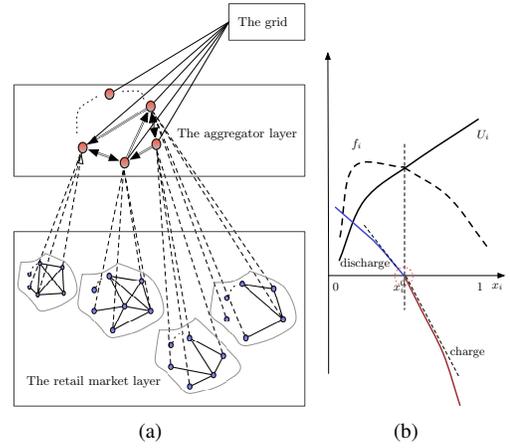


Fig. 1. In (a), networks of PEVs and their corresponding aggregators are shown. In each network, each PEV can communicate with some neighbors. PEVs can also communicate with their corresponding aggregator. In (b), given a pricing strategy and $x_{-i} \in [0, 1]^{n-1}$, the payoff function of the PEV v_i is shown (for fixed x_{-i}). The blue curve assigns to each point $x_i \in [0, x_i^0)$ the value of $(x_i - x_i^0)P_{\text{discharge}}(x_{\text{ave}})$; similarly, the red curve assigns to each point $x_i \in (x_i^0, 1]$ the values of $-(x_i - x_i^0)P_{\text{charge}}(x_{\text{ave}})$. In the scenario shown in the figure the PEV v_i is more likely to discharge.

other things, depends on its own utility function, along with the pricing strategy designed by the aggregator. The PEVs considered in this paper are price anticipating, in the sense that they are aware that the pricing is designed by the aggregator with respect to the average charge available in the v_i^{agg} -group. We also assume that each PEV, in order to make its decision, is able to collect information from neighboring PEVs with which it can exchange information. Specifically, the exchange of information among all PEVs is described by a connected undirected graph, denoted by $\mathcal{G}_{\text{PEVs}}$. The collection of all v_i^{agg} -group builds a new layer, which we term the *retail market layer*. The concepts described above are illustrated in Figure 1(a). In this paper, we focus on the retail market layer.

A. The Retail Market Layer

Let $Q \in \mathbb{R}_{<0}$ ($Q \in \mathbb{R}_{>0}$) be the amount of energy that the aggregator has contracted to provide (absorb) over some period of time. Thus, when $Q \in \mathbb{R}_{<0}$, the aggregator needs to encourage the PEVs to discharge the extra charge in their batteries. Conversely, the aggregator needs to encourage the PEVs to charge their batteries whenever $Q \in \mathbb{R}_{>0}$.

Let us formalize the statement of the problem after introducing some notions. We denote by $V = \{v_1, \dots, v_n\}$, $n \in \mathbb{Z}_{\geq 1}$, the set of PEVs in v_i^{agg} -group, where the available energy of each v_i , $i \in \{1, \dots, n\}$, at time $t \in \mathbb{R}_{\geq 0}$ is denoted by $x_i(t) \in [0, 1]$; thus, one can think of $x_i(t)$ as the state of charge of the i^{th} PEV battery. Without loss of generality, we assume that the PEV is willing to participate in the process of charging and discharging in the range $[0, 1]$ (in practice each PEV insures that $x_i(t) > x_{\min}$, for some $x_{\min} \in \mathbb{R}_{>0}$). The mappings $P_{\text{charge}} : [0, 1] \rightarrow \mathbb{R}_{\geq 0}$ and $P_{\text{discharge}} : [0, 1] \rightarrow \mathbb{R}_{\geq 0}$ given by $P_{\text{charge}}(x_{\text{ave}}(t))$ and $P_{\text{discharge}}(x_{\text{ave}}(t))$, $x_{\text{ave}}(t) = \frac{1}{n} \sum_{i=1}^n x_i(t)$, denote, respectively, the price per unit of

energy that the PEVs pay and receive when charging and discharging their batteries. Also, it is reasonable to assume that the PEVs are making their decisions on the total amount of energy to be charged/discharged for a certain fixed period of time. [The PEVs decide at the beginning of this period what their charge level will be at the end of the period.] We also need to define a set of utility functions $U_i : [0, 1] \rightarrow \mathbb{R}_{\geq 0}$, with values $U_i(x_i)$, $i \in \{1, \dots, n\}$. This function is an increasing function of the available charge, i.e., at no cost, it is beneficial for each PEV to keep its battery charged.

Let us next describe the decision-making process that each PEV is faced with. Similar to other scenarios of resource allocation problems (see, e.g., [15]), each PEV wishes to maximize a payoff function $f_i : \mathbb{R}^n \times X_{\text{agg}} \rightarrow \mathbb{R}$, where $X_{\text{agg}} = \mathcal{B}^0([0, 1]) \times \mathcal{B}^0([0, 1])$, is given by

$$f_i(x_i, x_{-i}, P_{\text{charge}}, P_{\text{discharge}}) = \begin{cases} U_i(x_i) - (x_i - x_i^0)P_{\text{charge}}(x_{\text{ave}}), & x_i > x_i^0, \\ U_i(x_i) - (x_i - x_i^0)P_{\text{discharge}}(x_{\text{ave}}), & x_i \leq x_i^0, \end{cases} \quad (1)$$

where $(x_i^0, x_{-i}^0) \in X$, $X = [0, 1]^n$, denotes the initial state of charge of the PEV batteries. The aggregator's goal is ensure that the PEVs collectively provide $Q \in \mathbb{R}$ units of energy; thus it wishes to maximize

$$f_{\text{agg}}(x, P_{\text{charge}}, P_{\text{discharge}}) = -|Q - \sum_{i=1}^n \alpha_i(x_i - x_i^0)|,$$

where $\alpha_i \in \mathbb{R}_{>0}$, for all $i \in \{1, \dots, n\}$.

Based on the description given above, the aggregator and the PEVs define a game, which we call the *retail market game*,

$$\mathbf{G}_{\text{PEVs-AGG}} = (V \cup \{v^{\text{agg}}\}, X \times X_{\text{agg}}, f_1 \times \dots \times f_n \times f_{\text{agg}}),$$

where players wish to maximize their objective functions. We are now ready to formulate the problems of interest:

- (*Existence of equilibria*): given the pricing strategies of the aggregator $P_{\text{charge}}, P_{\text{discharge}} \in \mathcal{B}^0([0, 1])$, does there exist a Nash equilibrium solution to the retail market game? If so, is the equilibrium unique?
- (*Distributed equilibria seeking*): if the answers to both parts of (a) are positive, can the PEVs use a (distributed) strategy to seek the Nash equilibrium, after the pricing strategy is fixed?
- (*Optimal pricing*): if the answer to the existence part of the previous question is positive, does there exist pricing strategies $P_{\text{charge}}, P_{\text{discharge}} \in \mathcal{B}^0([0, 1])$ such that

$$x^* \in \{z \in X \mid z = \operatorname{argmax}_x f_{\text{agg}}(x, P_{\text{charge}}, P_{\text{discharge}})\},$$

where x^* is the retail market game Nash equilibrium?

The main focus of this paper is to provide answers to (a) and (b); the study of optimal pricing is left for future work.

IV. EXISTENCE AND UNIQUENESS OF EQUILIBRIUM POINTS

In this section, we characterize the properties of the retail market game. We start by stating some assumptions on the payoff functions of players.

Assumption 4.1: (Properties of the payoff functions): We assume that

- the function U_i is concave, nondecreasing, and continuously differentiable, for all $i \in \{1, \dots, n\}$,
- the function P_{charge} is convex, twice differentiable, and nondecreasing,
- the function $P_{\text{discharge}}$ is concave, twice differentiable, nondecreasing, and
- $P_{\text{charge}}(x_{\text{ave}}) > P_{\text{discharge}}(x_{\text{ave}})$, for all $x_{\text{ave}} \in [0, 1]$.

Assumption (i) means that without any incentive for discharging, PEVs would prefer to have full charge at all times. The nondecreasing parts of assumptions (ii) and (iii), respectively, ensure that when the average value x_{ave} is high (meaning that PEVs are storing a large amount of energy), the aggregator increases the charging price and when the x_{ave} is low (meaning that PEVs do not have enough energy stored) the aggregator increases the discharging price. These assumptions are reasonable, when the request Q matches a realistic operating scenario, where most PEVs are willing to charge overnight and discharge during the daytime. The convexity and concavity assumptions of (ii) and (iii) are technical and ensure the concavity of the payoff function of each player (see Proposition 4.2 bellow). Finally, the last assumption prevents PEVs from simultaneously trading power for increasing their payoff and ensures concavity of the pricing strategy. We have the following result.

Proposition 4.2: (Properties of the payoff functions): Under Assumption 4.1, the payoff function of each PEV, given by (1), is concave in its first argument.

Figure 1(b) shows a payoff function which satisfies the conditions of Proposition 4.2. Using this result, and in view of the fact that the strategy sets are convex, the existence of a Nash equilibrium is guaranteed for the problem at hand [18].

Theorem 4.3: (Existence of solutions for $\mathbf{G}_{\text{PEVs-AGG}}$): Under Assumption 4.1, $\mathbf{G}_{\text{PEVs-AGG}}$ has a Nash equilibrium.

An extension of [18, Theorem 4] to nonsmooth functions, see [26], can now be applied to guarantee uniqueness, under the assumption of the *diagonally strict concave*, see [18].

V. DISCRETE-TIME DISTRIBUTED STRATEGIES FOR SEEKING THE NASH EQUILIBRIUM

We now design a strategy, distributed in a sense that will be described shortly, which allows for seeking the Nash equilibrium, when it is unique. The strategy can be thought of as the distributed version of the gradient-flow procedure [18, Theorem 7] for seeking the Nash equilibrium, extended to include nonsmooth payoff functions. It is discrete-time and consensus-based and is motivated by the distributed optimization protocols in [8], [9], and the Nash-seeking strategies for noncooperative games in [27]. Although we derive our results by considering $\mathbf{G}_{\text{PEVs-AGG}}$, when the strategy sets are convex and compact, and the constraints are not shared, they are readily extendable to include other concave games with unique Nash equilibrium.

Each PEV can only communicate with its neighboring PEVs; the exchange of information among all PEVs is described by a connected undirected graph, denoted by $\mathcal{G}_{\text{PEVs}}$. Each player has only access to its own payoff function, which

is assumed to be concave in all its arguments but not necessarily differentiable. To ensure uniqueness, we additionally assume that the diagonally strict concave condition of [26] holds. We denote this unique Nash equilibrium by $x^* \in X$, $X = [0, 1]^n$. We assume that each player makes an estimate of what this Nash equilibrium should be; we denote the estimate of v_i by $x^i \in \mathbb{R}^n$. We let $\mathbf{x}^T = (x^1, \dots, x^n) \in X^n$. Let $\Psi_\delta : X^n \times Z^n \rightrightarrows X^n \times Z^n$, $Z = \mathbb{R}^n$, be a mapping given by

$$\Psi_\delta(\mathbf{x}, \mathbf{z}) = \{(P(\mathbf{x} - \delta(\mathbf{L}\mathbf{x} + \mathbf{L}\mathbf{z} - \mathbf{s}_\mathbf{x})), \mathbf{z} + \delta\mathbf{L}\mathbf{x}) \mid \mathbf{s}_\mathbf{x} \in \mathcal{D} = \{u \in \mathbb{R}^{n^2} \mid u = \underbrace{(\eta_1, 0, \dots, 0}_{\text{computed by } v_1}, \dots, 0, \dots, 0, \eta_n)^T, \eta_i \in \partial_{x_i} f_i(x^i)\}\},$$

where $\delta \in \mathbb{R}_{>0}$, $\mathbf{L} = \mathbf{L} \otimes \mathbf{I}_n \in \mathbb{R}^{n^2 \times n^2}$, \mathbf{L} is the Laplacian of $\mathcal{G}_{\text{PEVs}}$, and $P = \prod_{i=1}^n P_i$, where $P_i : \mathbb{R} \rightarrow [0, 1]$, $i \in \{1, \dots, n^2\}$, is the natural projection map onto $[0, 1]$.

The mapping Ψ_δ has the following key properties:

- (i) by Lemma 2.2, it is nonempty and upper semicontinuous;
- (ii) the projections of their fixed points to its first argument are given by $\mathbf{1}_n \otimes x^*$, where $x^* \in X$ is the Nash equilibrium of $\mathbf{G}_{\text{PEVs-AGG}}$, as we establish next.

Lemma 5.1: (Fixed points of Ψ_δ): When $\mathcal{G}_{\text{PEVs}}$ is undirected and connected, Ψ_δ has at least one fixed point. Moreover, $(\mathbf{x}^*, \mathbf{z}^*)$ is a fixed point of Ψ_δ if and only if $\mathbf{x}^* = \mathbf{1}_n \otimes x^*$, where $x^* \in X$ is the Nash equilibrium of $\mathbf{G}_{\text{PEVs-AGG}}$.

Consider now the discrete-time set-valued dynamical system defined on $X^n \times Z^n$ as

$$(\mathbf{x}(k+1), \mathbf{z}(k+1)) \in \Psi_\delta(\mathbf{x}(k), \mathbf{z}(k)); \quad (2)$$

in what follows, we will occasionally refer to these system as the *concave Nash seeking dynamics*. Note that (2) is clearly distributed over the network $\mathcal{G}_{\text{PEVs}}$ and each player only uses the information about its own payoff function. Our primary goal in this section is to characterize the convergence properties of these dynamics.

Theorem 5.2: (Asymptotic convergence of (2)): When $\mathcal{G}_{\text{PEVs}}$ is undirected and connected, the dynamics in (2) is asymptotically convergent for $\delta \in \mathbb{R}_{>0}$ small enough.

VI. NUMERICAL SIMULATIONS

Consider a group of PEVs connected to an aggregator; for illustration purposes, we have only selected six PEVs $\{v_1, \dots, v_6\}$. The PEVs can obtain information from each other via a network described by a connected and undirected graph with adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

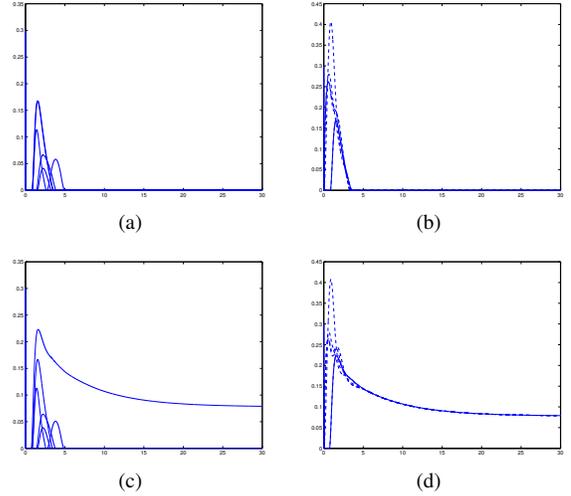


Fig. 2. (a) and (c) show the evolutions of (2) for each PEV in **Case-1** and **Case-2**, respectively. In (b) and (d), we have shown, as an example, the consensus between the groups of PEVs on the value of charge in v_5 ; in these two figures the dashed lines show the perception of other PEVs about the state value of v_5 .

Each PEV's utility function $U_i : [0, 1] \rightarrow \mathbb{R}_{\geq 0}$, $i \in \{1, \dots, 6\}$, is given by

$$U_i(x) = u_i^1 \log(1+x) + u_i^2 x,$$

where U_i is normalized so that $u_i^1, u_i^2 \in (0, 1]$. Note that U_i is increasing and strictly concave and thus satisfies Assumption 4.1(i). Let us assume that the aggregator has linear pricing strategies given by

$$\begin{aligned} P_{\text{charge}}(x_{\text{ave}}) &= c_1 x_{\text{ave}} + c_2, \\ P_{\text{discharge}}(x_{\text{ave}}) &= d_1 x_{\text{ave}} + d_2, \end{aligned}$$

where these functions are normalized so that $c_1, d_1 \in (0, 1]$ and $c_2, d_2 \in [0, 1]$. The payoff functions of v_i , $i \in \{1, \dots, 6\}$, are of the form in (1). Note that these pricing functions satisfy Assumption 4.1(ii-iii). Also, by assuming that $c_1 > d_1$ and $c_2 > d_2$, Assumption 4.1(iv) holds true and as a result, the payoff function of each PEV is concave (see Proposition 4.2). In fact, one can easily verify that each of these functions additionally satisfy the diagonally strict concavity assumption, and the Nash equilibrium of the $\mathbf{G}_{\text{PEVs-AGG}}$ with these pricing strategies is unique. We start our set of case studies with a scenario in which the aggregator needs to encourage the PEVs to discharge their batteries. For this reason, the aggregator chooses the pricing parameters as $c_1 = 0.9$, $c_2 = 0.9$, $d_1 = 0.8$, and $d_2 = 0.8$. The parameters associated with each PEV are given in Table I. We consider two cases.

- **Case-1:** all PEVs have low initial available charge and no incentive for charging;
- **Case-2:** all PEVs have low initial available charge; the only PEV with incentive of charging is v_5 .

Figure 2 shows the evolution of (2) for each PEV in these two cases; the value of the Nash equilibrium for each case, which all the PEVs have agreed on, is given in Table I. Unlike **Case-1**, in **Case-2**, PEV v_5 has a higher incentive to charge its battery (see the value of u_5 in Table I).

		PEV 1	PEV 2	PEV 3	PEV 4	PEV 5	PEV 6
Case-1	u^1	0.1602	0.1126	0.2502	0.2113	0.3216	0.1984
	u^2	0.1250	0.1457	0.1780	0.1078	0.1162	0.1942
	x^*	0	0	0	0	0	0
Case-2	u^1	0.1602	0.1126	0.2502	0.2113	0.8216	0.1984
	u^2	0.1250	0.1457	0.1780	0.1078	0.1162	0.1942
	x^*	0	0	0	0	0.0780	0

TABLE I

THE PROPERTIES OF EACH PEV IS GIVEN. THE PRICING PARAMETERS IS GIVEN BY $c_1 = 0.9, c_2 = 0.9, d_1 = 0.8, d_2 = 0.8$ FOR **CASE-1** AND **CASE-2**. $x_0 = (0.1, 0.2, 0.1, 0.2, 0.3, 0.2)^T$.

VII. CONCLUDING REMARKS

We have introduced a framework for controlling the charging and discharging processes of networked PEVs via pricing strategies set by an aggregator. We have formulated the overall scenario as a hierarchical decision-making problem in which the aggregator is the leader and sets the pricing strategies for charging and discharging. The PEVs are the followers and after receiving the pricing strategies, evaluate their next batteries' charges. After the pricing strategy is selected, we give conditions under which this game is a concave game and determine conditions under which it has a unique Nash equilibrium. Finally, we introduce a discrete-time set-valued dynamical system, distributed over the network of PEVs, which allows the PEVs to compute the Nash equilibrium, when unique.

Future work will focus on: (i) the characterization of optimal pricing strategies for the aggregator; (ii) extension of the convergence results to communication networks described by directed graphs, studying groups of aggregators and their interconnections with the retail market layer; and (iii) robustness and resilience in pricing strategies.

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