

# POWER SYSTEM LINE OUTAGE DETECTION AND IDENTIFICATION — A QUICKEST CHANGE DETECTION APPROACH

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## ABSTRACT

A method to detect and isolate power system transmission line outages in near real-time is proposed. In particular, a linearized power system model is presented and a statistical model for line outage detection and isolation is developed using this model. To detect and isolate the line outage quickly, algorithms based on statistical quickest change detection are employed.

**Index Terms**— Power systems, line outage detection and identification, quickest change detection, CuSum.

## 1. INTRODUCTION

Existing tools for online power system operational reliability monitoring rely on a system model obtained offline, constructed from the transmission network, line parameters, and historical and forecasted power generation and demand [1, 2]. These online analyses generally include repeated computations of power flow solutions using a full nonlinear model or a linearized model. Thus, the validity of the study results rely on the accuracy of the system model used (including up-to-date network topology and parameters), which is heavily dependent on accurate records and telemetry data. Such deficiencies in situational awareness, including knowledge of transmission line statuses, have contributed to numerous major North American blackouts [1, 2]. For example, in the 2011 San Diego blackout, operators could not detect that certain lines were overloaded or close to being overloaded because the network model was not up-to-date [2]. Thus, there exists an impetus to develop efficient and robust online tools to detect and identify topology changes. With respect to this, phasor measurement units (PMUs) are an enabling technology for timely identification of transmission line outages and, in general, changes in network topology [3, 4, 5].

In this paper, we propose an approach based on the theory of quickest change detection (QCD) for line outage detection

and identification. In QCD, a decision maker observes a sequence of random variables, the distribution of which changes abruptly due to some event. The objective is to detect this change in distribution as quickly as possible subject to a constraint on the false alarm rate; see [6] for a survey. We model the incremental change in voltage angle and power injection measurements obtained from PMUs as random variables. A line outage changes the probability distribution of the incremental change in voltage angle measurements. By processing this data sequentially, we employ a QCD algorithm to detect this change in distribution. The algorithm can also be used to identify the location of the fault. To the best of our knowledge, the problem of line outage detection and identification has not been studied in the framework of QCD previously.

Existing approaches to line outage detection and/or identification consider the phase difference between two sets of PMU voltage measurements obtained pre- and post-event, and proceed via hypothesis testing [3], sparse vector estimation [4], or mixed-integer nonlinear optimization [5]. These papers either assume that the line outage instance is known (in which case the objective is isolation of the line outage), or propose a detection technique that does not exploit the fact that the line outage is persistent. In a quickest change detection algorithm, the fact that the fault persists after it has occurred is exploited to detect the outage more efficiently.

## 2. POWER SYSTEM MODEL

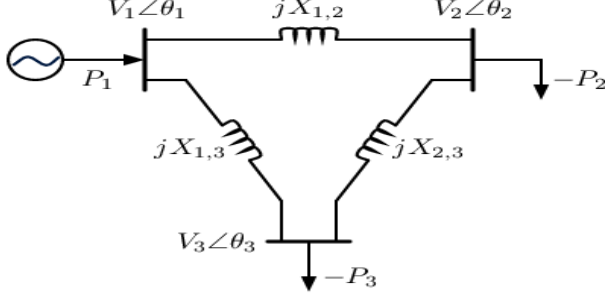
In this section, we describe the linearized power system model and the PMU sampling process adopted in this work, before and after a line outage. We also obtain statistical models that describe PMU measurements obtained before and after a line outage.

### 2.1. Linearization of the Power Flow Model

We consider a power system network represented by a graph, with  $N$  nodes denoted by  $\mathcal{V} = \{1, \dots, N\}$ , each one corresponding to a bus. The set of edges in the graph, denoted by  $\mathcal{E}$ , represent the grid of transmission lines in the power system, i.e., for  $n, m \in \mathcal{V}$ ,  $(n, m) \in \mathcal{E}$  if there exists a transmission line between buses  $n$  and  $m$ . We use  $|\mathcal{E}|$  to denote the number of edges or transmission lines in the graph. At time instant  $k$ , let  $V_n[k]$  and  $\theta_n[k]$ , respectively, denote the

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The work of Banerjee and Veeravalli is supported in part by the National Science Foundation (NSF) under grant DMS 12-22498. The work of Chen and Domínguez-García was supported in part by the National Science Foundation (NSF) under grant ECCS-CAR-0954420, and by the Consortium for Electric Reliability Technology Solutions. \* denotes equal contributions to the work.



**Fig. 1.** Network topology for 3-bus system.

voltage magnitude and angle at bus  $n$ ; additionally,  $P_n[k]$  and  $Q_n[k]$ , respectively, denote the net active and reactive power injection (generator or load) at bus  $n$ . Then, with<sup>1</sup>  $\theta[k] = [\theta_1[k], \dots, \theta_N[k]]^T$ ,  $V[k] = [V_1[k], \dots, V_N[k]]^T$ ,  $P[k] = [P_1[k], \dots, P_N[k]]^T$ , and  $Q[k] = [Q_1[k], \dots, Q_N[k]]^T$ , the static behavior of a power system can be described by the power flow equations, which can be compactly written as real power and reactive power balance components as

$$\begin{aligned} P[k] &= f_P(\theta[k], V[k]), \\ Q[k] &= f_Q(\theta[k], V[k]), \end{aligned} \quad (1)$$

where the dependence on network parameters, such as line series and shunt impedances, is implicitly considered in the functions  $f_P(\cdot)$  and  $f_Q(\cdot)$ .

Suppose, a solution exists at  $(\theta[k], V[k], P[k], Q[k])$ , i.e.,  $P[k] = f_P(\theta[k], V[k])$  and  $Q[k] = f_Q(\theta[k], V[k])$ , and assume  $f_P(\cdot)$  and  $f_Q(\cdot)$  are continuously differentiable with respect to  $\theta$  and  $V$  at  $(\theta[k], V[k], P[k], Q[k])$ . Define small variations in voltage magnitudes and angles between time instants  $k$  and  $k+1$  as  $\Delta V[k] = V[k+1] - V[k]$  and  $\Delta\theta[k] = \theta[k+1] - \theta[k]$ , respectively. Similarly, small variations in the active and reactive power injections are defined as  $\Delta P[k] = P[k+1] - P[k]$  and  $\Delta Q[k] = Q[k+1] - Q[k]$ , respectively. Then, assuming that  $\Delta P[k]$ ,  $\Delta Q[k]$ ,  $\Delta\theta[k]$ , and  $\Delta V[k]$  are sufficiently small, we can approximate (1) with a first-order Taylor series expansion as

$$\begin{bmatrix} P[k] + \Delta P[k] \\ Q[k] + \Delta Q[k] \end{bmatrix} \approx \begin{bmatrix} f_P(\theta[k], V[k]) \\ f_Q(\theta[k], V[k]) \end{bmatrix} + J[k] \begin{bmatrix} \Delta\theta[k] \\ \Delta V[k] \end{bmatrix},$$

where

$$J[k] = \begin{bmatrix} H[k] & N[k] \\ K[k] & L[k] \end{bmatrix} = \begin{bmatrix} \frac{\partial f_P}{\partial \theta} & \frac{\partial f_P}{\partial V} \\ \frac{\partial f_Q}{\partial \theta} & \frac{\partial f_Q}{\partial V} \end{bmatrix}_{(\theta[k], V[k])}.$$

Since  $P[k] = f_P(\theta[k], V[k])$  and  $Q[k] = f_Q(\theta[k], V[k])$ , we have

$$\begin{bmatrix} \Delta P[k] \\ \Delta Q[k] \end{bmatrix} \approx \begin{bmatrix} H[k] & N[k] \\ K[k] & L[k] \end{bmatrix} \begin{bmatrix} \Delta\theta[k] \\ \Delta V[k] \end{bmatrix}. \quad (2)$$

A standard assumption used in transmission systems analysis is that the entries of  $H$  and  $L$  in (2) are much larger

<sup>1</sup> $[\cdot]^T$  denotes the matrix transpose operation

**Table 1.** Parameter values for 3-bus system shown in Fig. 1.

$P_2$	$P_3$	$X_{1,2}$	$X_{2,3}$	$X_{1,3}$
-1	-0.9	0.0504	0.0372	0.0636

than those of  $N$  and  $K$  [7]. This effectively decouples (2) so that variations in active power injections primarily affect bus voltage angles, while variations in reactive power injections mainly affect bus voltage magnitudes. In this paper, we assume the decoupling assumptions holds and only consider  $\Delta P[k] \approx H[k]\Delta\theta[k]$ . Further, under the assumptions used to devise the so-called DC model (the system is lossless,  $V_n[k] = 1$  per unit (p.u.) for all  $n, k$ , and  $\theta_n[k] - \theta_m[k] \approx 0$  for all  $k$  and for  $n, m \in \mathcal{V}$ ), the matrix  $H[k]$  becomes simply the negative of the imaginary part of the network admittance matrix constructed while neglecting transmission line resistances [7]. Under these assumptions, then,  $H[k]$  becomes independent of the operating point, i.e.,  $H[k] = H$ , for all  $k$ . And we obtain

$$\Delta P[k] \approx H\Delta\theta[k]. \quad (3)$$

The matrix  $H$  has the same structure as the graph Laplacian for the graph representation underlying the power network. In this way, the network topology is encoded into  $H$ .

**Example 1 (3-Bus System)** *In this example, we illustrate the power system modeling concepts above with the lossless system shown in Fig. 1, where bus 1 is the so-called slack bus, i.e.,  $V_1\angle\theta_1 = 1\angle 0^\circ$  p.u. In Fig. 1,  $X_{n,m}$  is the imaginary part of the impedance of the line connecting buses  $n$  and  $m$ . The parameter values are listed in Table 1 and, unless otherwise stated, all quantities are in per unit. For ease of notation, we suppress the dependence on time instant  $k$ . Then, the nonlinear real power balance equations are*

$$\begin{aligned} P_1 &= \frac{V_1 V_2}{X_{1,2}} \sin(\theta_1 - \theta_2) + \frac{V_1 V_3}{X_{1,3}} \sin(\theta_1 - \theta_3), \\ P_2 &= \frac{V_2 V_1}{X_{1,2}} \sin(\theta_2 - \theta_1) + \frac{V_2 V_3}{X_{2,3}} \sin(\theta_2 - \theta_3), \\ P_3 &= \frac{V_3 V_1}{X_{1,3}} \sin(\theta_3 - \theta_1) + \frac{V_3 V_2}{X_{2,3}} \sin(\theta_3 - \theta_2). \end{aligned} \quad (4)$$

Since bus 1 is the reference bus with  $\theta_1 = 0$ , we remove the first equation from (4); and under the DC assumptions, the model in (4) becomes

$$\begin{aligned} P_2 &= \frac{1}{X_{1,2}} \theta_2 + \frac{1}{X_{2,3}} (\theta_2 - \theta_3), \\ P_3 &= \frac{1}{X_{1,3}} \theta_3 + \frac{1}{X_{2,3}} (\theta_3 - \theta_2). \end{aligned} \quad (5)$$

We differentiate (5) with respect to  $\theta = [\theta_2, \theta_3]^T$  to get a small-signal linear model of the form in (3), i.e.,  $\Delta P[k] = H\Delta\theta[k]$ , where

$$H = \begin{bmatrix} \frac{1}{X_{1,2}} + \frac{1}{X_{2,3}} & -\frac{1}{X_{2,3}} \\ -\frac{1}{X_{2,3}} & \frac{1}{X_{1,3}} + \frac{1}{X_{2,3}} \end{bmatrix}. \quad (6)$$

■

Suppose, at  $k = \gamma$ , a line outage occurs for the line connecting buses  $n$  and  $m$ , denoted by  $(n, m) \in \mathcal{E}$ . Then, for  $k \geq \gamma$ , the matrix  $H$  in (3) changes to a new matrix  $\tilde{H}_{n,m}$ . Without loss of generality, we can write the post-change matrix  $\tilde{H}_{n,m}$  as the sum of the pre-change matrix  $H$  and some perturbation matrix  $\Delta H_{n,m}$ , i.e.,  $\tilde{H}_{n,m} = H + \Delta H_{n,m}$ . Then we get the following post-change equation:

$$\Delta P[k] \approx \tilde{H}_{n,m} \Delta \theta[k] = (H + \Delta H_{n,m}) \Delta \theta[k]. \quad (7)$$

Since  $H$  has the same structure as the graph Laplacian of the network, we conclude that the only non-zero terms in the matrix  $\Delta H_{n,m}$  are  $\Delta H_{n,m}[n, n] = -\Delta H_{n,m}[m, n] = \Delta H_{n,m}[m, m] = -\Delta H_{n,m}[n, m] = -1/X_{n,m}$ , where  $X_{n,m}$  is the imaginary part of the impedance of the outaged line. Thus, the matrix  $\Delta H_{n,m}$  is a rank-one matrix and can be written as

$$\Delta H_{n,m} = -\frac{1}{X_{n,m}} h_{n,m} h_{n,m}^T, \quad (8)$$

where  $h_{n,m}$  is a vector with the  $n^{\text{th}}$  entry equal to 1,  $m^{\text{th}}$  entry equal to -1, and all other entries equal to 0.

## 2.2. Statistical Model

We now obtain a statistical model for the measurements  $\{\Delta \theta[k]\}_{k \geq 1}$ . Small variations in measurements of real power injection,  $\Delta P[k]$ , can be attributed to (i) random fluctuations in electricity consumption by end users, and (ii) random noise in communication channels between PMUs and the control center. Hence, we model  $\Delta P[k]$ 's as independent and identically distributed (i.i.d.) with a jointly Gaussian probability density function (p.d.f.), i.e.,  $\Delta P[k] \sim \mathcal{N}(0, \Sigma)$ . Further, we assume the measured  $\Delta P_n[k]$  is independent from all other  $\Delta P_m[k]$ ,  $m \neq n$ . Thus, the matrix  $\Sigma$  contains nonzero entries only on its diagonal. Since the statistics of  $\Delta P[k]$  are known, we consider  $\Delta P[k]$  as the input to the system described in (3) with  $\Delta \theta[k]$  as the observation and rewrite (3) as

$$\Delta \theta[k] \approx M \Delta P[k], \quad k < \gamma, \quad (9)$$

where  $M = H^{-1}$  and  $\gamma$  is the line outage instance. Thus,

$$\Delta \theta[k] \sim \mathcal{N}(0, M \Sigma M^T) \quad \text{for } k < \gamma. \quad (10)$$

For  $k > \gamma$ , the post-change system described in (7) can be rewritten as

$$\Delta \theta[k] \approx \tilde{M}_{n,m} \Delta P[k], \quad k > \gamma, \quad (11)$$

where,  $\tilde{M}_{n,m} = M + \Delta M_{n,m}$ . Thus, when the fault is at line  $(n, m)$ ,

$$\Delta \theta[k] \sim \mathcal{N}(0, \tilde{M}_{n,m} \Sigma \tilde{M}_{n,m}^T) \quad \text{for } k > \gamma, \quad (12)$$

In this paper we ignore the effect of the sample at  $k = \gamma$  and only aim to detect the change that is persistent. Using the matrix inversion lemma, we obtain

$$\Delta M_{n,m} = \beta_{n,m} g_{n,m} g_{n,m}^T,$$

where  $\beta_{n,m} = 1/(X_{n,m} - h_{n,m}^T H^{-1} h_{n,m})$  and  $g_{n,m} = H^{-1} h_{n,m}$ .

**Example 2 (3-Bus System)** In this example, we begin with the 3-bus system from Example 1 and consider the outage of the line (2, 3). In this case,  $\tilde{H}_{2,3} = H + \Delta H_{2,3}$ , where

$$\Delta H_{2,3} = -\frac{1}{X_{2,3}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = -\frac{1}{X_{2,3}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix},$$

a rank-one matrix. Then  $\tilde{M}_{2,3} = \tilde{H}_{2,3}^{-1} = (H + \Delta H_{2,3})^{-1}$ . Using the matrix inversion lemma, we compute  $\tilde{M}_{2,3} = H^{-1} + \beta_{2,3} g_{2,3} g_{2,3}^T$ , where

$$\beta_{2,3} = \frac{1}{X_{2,3} - h_{2,3}^T H^{-1} h_{2,3}}, \quad g_{2,3} = H^{-1} \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

with  $H$  given in (6). ■

## 3. QUICKEST CHANGE DETECTION ALGORITHM

The analysis and arguments provided in the previous section allow us to reduce the problem of line outage detection to the problem of detecting a change in the probability distribution of the sequence of random vectors  $\{\Delta \theta[k]\}_{k \geq 1}$ . We would like to detect this change in distribution as quickly as possible while avoiding false alarms. This is a well studied problem in statistics and is called the problem of quickest change detection. Next, we provide a precise mathematical description of this problem and the QCD algorithm that we will use to detect a line outage. We refer the readers to [6] for a detailed survey on QCD theory and algorithms.

### 3.1. Quickest Change Detection Problem Formulation

We assume that the sequence  $\{\Delta \theta[k]\}_{k \geq 1}$  of random vectors is being observed by a central controller or a decision maker. At some random time  $\gamma$ ,  $\gamma \geq 1$ , there is a line outage, and the p.d.f. of the sequence  $\{\Delta \theta[k]\}$  changes from  $f_0$  to  $f_1$ . The objective is to find a stopping time  $\tau$  for the sequence  $\{\Delta \theta[k]\}$  to detect this  $f_0$  to  $f_1$  transition as quickly as possible. In the absence of a change we would like to keep  $E[\tau]$  as large as possible (avoid false alarms). Once the change occurs, we would like to have  $E[\tau]$  as small as possible. A popular formulation in the literature that captures the above trade-off is due to Pollak [8]:

$$\begin{aligned} & \min_{\tau} \sup_{\gamma \geq 1} E_{\gamma}[\tau - \gamma | \tau \geq \gamma] \\ & \text{subject to } E_{\infty}[\tau] \geq \beta, \end{aligned} \quad (13)$$

where  $E_{\gamma}$  denotes the expectation with respect to probability measure when change occurs at point  $\gamma$ ,  $E_{\infty}$  denotes the corresponding expectation when the change never occurs, and  $\beta > 0$  is the given constraint on the mean time to false alarm.

### 3.2. Algorithm for Solving the QCD Problem

When both the pre- and post-change p.d.f.s  $f_0$  and  $f_1$  are known, a popular algorithm in the literature that enjoys some

optimality properties with respect to Pollak's formulation is the Cumulative Sum (CuSum) algorithm [9]. The algorithm is described next. First, compute a sequence of statistics recursively so that for  $k \geq 0$ ,

$$W_{k+1} = \left( W_k + \log \frac{f_1(\Delta\theta[k+1])}{f_0(\Delta\theta[k+1])} \right)^+, \quad W_0 = 0, \quad (14)$$

where  $(x)^+ = x$  if  $x \geq 0$ , otherwise  $(x)^+ = 0$ . Next, declare a change the first time this statistic is above a pre-designed threshold  $A$ :

$$\tau_C = \inf\{k \geq 1 : W_k > A\}. \quad (15)$$

Prior to change,  $E[\log(f_1(\Delta\theta)/f_0(\Delta\theta))] < 0$ , and the statistics  $W_k$  remain close to zero. The threshold  $A$  can be chosen to control the mean time to false alarm. After the change,  $E[\log(f_1(\Delta\theta)/f_0(\Delta\theta))] > 0$ , and the statistic  $W_k \uparrow \infty$ . Thus, if  $E[\log(f_1(\Delta\theta)/f_0(\Delta\theta))]$  is large, the change is detected quickly after it occurs.

In the line outage problem we consider in this paper the line outage can occur in more than one way. Mathematically, for the application in hand, the post-change p.d.f.  $f_1$  is unknown. However, since the single line outage can occur in, at most,  $|\mathcal{E}|$  ways, the post-change distribution is known to belong to a finite set, i.e.,

$$f_1 \in \{f_1^{n,m}, (n,m) \in \mathcal{E}\}.$$

Specifically,

$$\Delta\theta[k] \sim f_0 = \mathcal{N}(0, M\Sigma M^T) \quad \text{for } k < \gamma, \quad (16)$$

and

$$\Delta\theta[k] \sim f_1^{n,m} = \mathcal{N}(0, M_{n,m}\Sigma M_{n,m}^T) \quad \text{for } k \geq \gamma. \quad (17)$$

See Section 2.2 for expressions for  $M$  and  $M_{n,m}$ .

A standard way to solve this problem is to apply the *generalized likelihood ratio test* approach. That is to compute  $|\mathcal{E}|$  CuSum statistics in parallel, one for each post-change scenario, and stop the first time a change is detected in any one of the CuSums. Mathematically, compute, for each  $(n,m) \in \mathcal{E}$ ,

$$W_{k+1}^{n,m} = \left( W_k^{n,m} + \log \frac{f_1^{n,m}(\Delta\theta[k+1])}{f_0(\Delta\theta[k+1])} \right)^+, \quad (18)$$

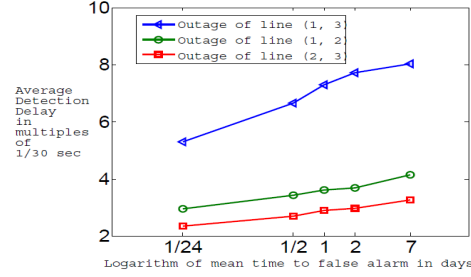
with  $W_0^{n,m} = 0$ , and stop at

$$\tau_{\max} = \inf \left\{ k \geq 1 : \max_{(n,m) \in \mathcal{E}} W_k^{n,m} > A \right\}. \quad (19)$$

See [10] for a detailed performance analysis of this algorithm. We use this algorithm or the stopping time  $\tau_{\max}$  to detect a line outage in the power system.

This same algorithm can also be used for line outage identification as well. Let  $\hat{L}$  denote the estimate for the line experiencing a fault. Then we set

$$\hat{L} = \arg \max_{(n,m) \in \mathcal{E}} W_{\tau_{\max}}^{n,m}. \quad (20)$$



**Fig. 2.** Average detection delay vs logarithm of mean time to false alarm for the 3-bus system.

It can be shown that

$$\sup_{\gamma \geq 1} E_{\gamma}[\tau_{\max} - \gamma | \tau_{\max} \geq \gamma] = E_1[\tau_{\max} - 1], \quad (21)$$

that is the supremum in (13) is achieved at  $\gamma = 1$  [6]. This is useful from the view point of simulating the test.

#### 4. NUMERICAL RESULTS AND DISCUSSION

We now apply the algorithm  $\tau_{\max}$  to detect a line outage in our running example of the three bus system. Due to the generality the QCD algorithm in (19), we can use it to detect line outages in more complex power systems as well. In our case study,  $\Sigma \in \mathbb{R}^{2 \times 2}$ , and  $\Sigma[1,1] = \Sigma[2,2] = 0.5$  while  $\Sigma[1,2] = \Sigma[2,1] = 0$ .

We simulate the performance of  $\tau_{\max}$  when applied to  $\{\Delta\theta[k]\}$  generated from the full nonlinear system. We choose  $\gamma = 1$  as the time at which the line outage occurs as this is the worst possible value of  $\gamma$ ; see (21). In Fig. 2, we plot  $E_1[\tau_{\max} - 1]$  against  $\log E_{\infty}[\tau_{\max}]$ , for three different post-change scenarios. The threshold  $A$  for  $\tau_{\max}$  is chosen to satisfy mean time to false alarms of 1 hour, half a day, 1 day, 2 days, and 1 week. At 30 samples/sec (typical of PMU measurement sampling rate [11]), this corresponds to  $\log E_{\infty}[\tau_{\max}]$  values of 11.58, 14.07, 14.76, 15.4, and 16.7, respectively. To choose the threshold  $A$  for such large values of mean time to false alarm, we observed that  $\log E_{\infty}[\tau_{\max}]$  is approximately linear in the threshold  $A$ . We make use of this fact to simulate the performance of  $\tau_{\max}$  for small values of  $A$  and then use linear regression to obtain an estimate of  $A$  that can achieve the given values of  $\log E_{\infty}[\tau_{\max}]$ .

As shown in Fig. 2, even for such high yet reasonable values of the mean time to false alarm, the detection delay is quite small. In fact, at the rate of 30 samples/sec, the delays correspond to real time detection delay of less than one third of a second. Although one may not get such a performance for large systems, one can certainly expect to get a good trade-off. Furthermore, due to the optimality property associated with such a QCD procedure, this is the best one can hope to achieve in principle. We also used (20) to isolate the faulty line for the 3-bus example. We obtained a perfect fault isolation.

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