A Zonotope-Based Method for Capturing the Effect of Variable Generation on the Power Flow

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***Abstract*—This paper presents a set-theoretic method to capture the effects of uncertainty on the generation side of a power system; this uncertainty arises from the increasing penetration of renewable resources such as wind and solar into existing systems. Using this method, we can determine whether the system state variables are within acceptable ranges as dictated by operational requirements. We bound all possible values that the uncertain generation can take by a zonotope and propagate it through a linearized power flow model, resulting in another zonotope that captures all possible variations in the system static state variables. For verification, we test our proposed method on the IEEE 123-bus distribution system and the IEEE 145-bus, 50-machine transmission system.**

I. Introduction

The integration of renewable resources such as wind and solar introduces additional uncertainty on the generation side of a power system since these power sources are intermittent and difficult to forecast. Such uncertainty invariably affects the system operation across all time scales, from day-ahead scheduling to automatic generation control, and across all subsystems, including transmission and generation. Our work focuses on capturing the effect of this uncertainty in electricity generation using a set-theoretic approach. Given the uncertainty in renewable-based electricity generation, it is possible to determine whether the system static state variables, i.e., bus voltage magnitudes and angles, are within their acceptable ranges using the proposed method.

Deterministic power flow analysis does not capture the uncertainty associated with renewable-based electricity generation as it only provides a snapshot of the system states at a particular time for a specific generation and load profile. In order to assess the effects of uncertain generation on the power flow solution, two main approaches have been developed; they include probabilistic and set-theoretic methods. In probabilistic power flow analysis (see, e.g., [1]), uncertainty in load and generation is modeled as a random vector, which results in the power flow solution also being described by a random vector. Both numerical and analytical methods have been proposed to address the probabilistic power flow problem [2], [3], [4], [5]. Other researchers have addressed the issues of efficiency and accuracy in calculating the probability density functions of the bus voltages and line flows [6], [7], [8], [9], [10]. In set-theoretic methods, some of the system parameters and variables are assumed to be unknown, but constrained to lie within a bounded set [11]. For example, in interval analysis [12], [13], [14], [15], it is assumed that some line parameters and loads take values within a symmetric polytope. This uncertainty is propagated through the power system model, resulting in a solution that is also constrained within some symmetric polytope. A disadvantage of this method is that the polytope, which contains the set of all possible solutions, may be overly conservative and contains non-solutions as well. In our previous work, we used ellipsoids to capture the uncertainty in renewable-based electricity generation [16]; however, that method becomes progressively less efficient when applied to larger systems.

In the analysis method proposed in this paper, we capture the uncertain variations in renewable-based generation with a zonotope where its center is the nominal forecast value. As an example, the power produced by a rooftop solar installation can be assumed to lie within some interval around a nominal power output value, which may be based on the forecasted solar insolation level. Using set operations, we propogate this zonotope through a linearized model of the power system. The result is a zonotope which bounds the bus voltage magnitudes and angles. To determine whether renewable-based power generation variability has a significant impact on power system static performance, we verify that this zonotope is contained within the region of the static state space defined by system operational requirements, such as minimum and maximum bus voltage values. Zonotopes are ideal candidates for uncertainty analysis because they can be efficiently encoded, are computationally tractable, and are closed under linear transformations [17]. Additionally, our method is equally applicable to both distribution and transmission systems and scales with the size of the system.

Zonotopes have been used in several power and energy system applications to capture the impact of uncertainty on dynamic performance [18], [19], [20], [21]. Specifically, in [18], [19], zonotopes are used to address the general problem of quantifying the impact of uncertainty in initial states and inputs on power system dynamics. Similarly, [20] proposes the use of zonotopes to study the impact of high-capacity

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transmission on power system frequency dynamics. In the context of wind-energy conversion systems, zonotopes have also been used in design verification problems pertaining to voltage ride-through capabilities of the system's power electronics converter [21].

It is important to note that in relation to the works in [18], [19], [20], while we also use zonotopes for uncertainty modeling, our setting is very different in the sense that we are interested in capturing the impact of uncertainty in variable generation on the power flow. In other words, instead of dealing with a dynamic problem as in [18], [19], [20], we solve a static problem similar to the one we worked on in our earlier papers [22], [23], where we used ellipsoids instead of zonotopes; however, ellipsoids bounding the system uncertainty become more difficult to compute as its size increases. In this paper, we show that the use of zonotopes complements the ellipsoidal methods we proposed in [22], [23], as they fill in many of the shortcomings associated with ellipsoids.

The remainder of this paper is organized as follows. In Section II, we introduce the fundamental ideas behind uncertainty modeling using zonotopes and apply them to study the impact of uncertain renewable-based electricity generation in power systems. We illustrate these ideas through a simple 4-bus example. Section III presents the results of the proposed methodology applied to the IEEE 123-bus distribution system, and the IEEE 145-bus transmission system. We compare the performance of our method against the linearized and the nonlinear power flow computations and present the results in Section IV. Concluding remarks are made in Section V.

II. Preliminaries and Problem Statement

In this section, we introduce a set-theoretic method for capturing the effect of uncertainty in renewable-based electricity generation on voltage magnitudes and angles. The uncertain values of generation is upper-bounded with a zonotope. We linearize the power flow equations around a nominal operating point (based on the nominal forecast of renewable-based power generation profile), and propagate the zonotope through the linearized system to obtain another zonotope that bounds the power system static states.

A. Power System Model

Let V_i and θ_i denote the voltage magnitude and angle of bus i and P_i^g (P_i^d) and Q_i^g (Q_i^d) denote the generation (demand) of real and reactive power at bus i. Then, the power balance equations for real and reactive power at bus i can be written as,

$$\begin{aligned} P &= V_i \sum_{k=1}^{n} V_k \left[G_{ik}\cos(\theta_i-\theta_k) + B_{ik}\sin(\theta_i-\theta_k)\right], \\ Q &= V_i \sum_{k=1}^{n} V_k \left[G_{ik}\sin(\theta_i-\theta_k) - B_{ik}\cos(\theta_i-\theta_k)\right], \end{aligned} \tag{1}$$

where $P = P_i^g - P_i^d$, $Q = Q_i^g - Q_i^d$; G_{ik} and B_{ik} are the real and imaginary parts of the (i,k) entry in the network admittance matrix, respectively. For an n bus system, let m denote the number of PQ buses. Then, after removing the slack bus active and reactive power balance equations (assumed to be bus 1), and the reactive power balance equations for the PV buses, we can write the remaining equations of (1) as

$$u + w = f(x), \tag{2}$$

where the nonlinear vector function $f : \mathbb{R}^{n+m-1} \mapsto \mathbb{R}^{n+m-1}$ denotes the mapping between the system states and the power injections; $x \in \mathbb{R}^{n+m-1}$ represents unknown quantities to be solved for, which includes V_i and θ_i for PQ buses and θ_i for PV buses; $u \in \mathbb{R}^{n+m-1}$ contains active power injections in PV buses arising from conventional sources and the demands of active power in PV buses and the demand of both active and reactive power in PQ buses; $w \in \mathbb{R}^{n+m-1}$ contains renewable-based active power generation in PV and PQ buses, and the reactive power injections in PQ buses. Note that in (2), the entries of u corresponding to reactive power balance equations in PQ buses and the entries of w corresponding to buses without renewable-based generation and load are all zero.

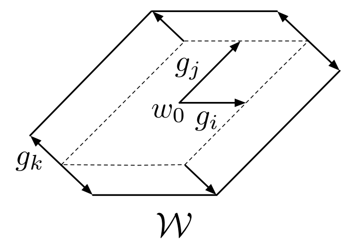


Fig. 1: Construction of a zonotope.

B. Zonotopes

Zonotopes are a special instance of polytopes which can be defined as the Minkowski sum of a finite number of line segments. Formally, a zonotope is defined as

$$\mathcal{W} = \{w : w = w_0 + \sum_{j=1}^{s} \alpha_j g_j, \ \ -1 \le \alpha_j \le 1\}. \tag{3}$$

where $w_0 \in \mathbb{R}^{n+m-1}$ is the center of the zonotope and $g_1, g_2, \dots, g_s \in \mathbb{R}^{n+m-1}$ form the set of linearly independent generators [17]. Figure 1 illustrates how a zonotope is constructed from the Minkowski sum of its generators.

Zonotopes also have the useful property of being closed under linear transformations. Thus, given a zonotope $\mathcal{W}$ and a linear transformation matrix $H \in \mathbb{R}^{(n+m-1)\times(n+m-1)}$, we can obtain another zonotope $\mathcal{X}$ after applying the linear transformation as follows:

$$\begin{aligned} \mathcal{X} &= H\mathcal{W} \\ &= \{x : x = Hw_0 + H\sum_{j=1}^{s} \alpha_j g_j, \ \ -1 \le \alpha_j \le 1\}. \end{aligned} \tag{4}$$

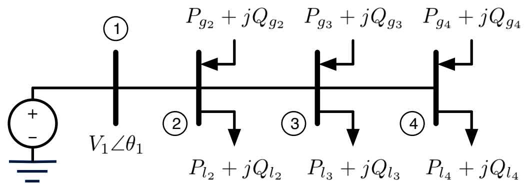


Fig. 2: Four-bus system with renewable power injection.

C. Problem Statement: Unknown-but-bounded Model

In general, load forecasts are more accurate than renewable-generation forecast [24]. Therefore, we assume that uncertainty in the power injections only appear in w (although we can easily extend our formulation to include uncertainty in the load). The vector w can be expressed as $w = w_0 + \Delta w$ where w_0 denotes the vector of nominal values that w takes from the generation forecast. Assuming the values that w can take are unknown-but-bounded, we can capture the uncertainty in w with a zonotope $\mathcal{W}$ according to (3). The magnitudes of the generators correspond to the amount of uncertainty in the values of w. In addition, the angles between each pair of generator vectors provide a measure of correlation between the variables of w.

Example 1 (Four-bus system): Consider the four-bus system shown in Fig. 2 with renewable-based resources connected to buses 2, 3, and 4. Suppose the nominal forecast for renewable generation is $w_0 = [P_{g2}^0 \; P_{g3}^0 \; P_{g4}^0]^T = [0.4 \; 0.3 \; 0.5]^T$ p.u. with an uncertainty of $\pm 50\%$ of the nominal forecast. Then, following the notation of (3), the generator vectors are $g_1 = [0.2 \;\, 0 \;\, 0]^T$, $g_2 = [0 \;\, 0.15 \;\, 0]^T$, and $g_3 = [0 \;\, 0 \;\, 0.25]^T$. For this particular example, the zonotope $\mathcal{W}$ capturing the uncertainty in w is a rectangular prism with sides having lengths of 0.4, 0.3, and 0.5, centered at $[0.4 \;\, 0.3 \;\, 0.5]^T$. ■

We are interested in propagating this uncertainty set $\mathcal{W}$ through the power system model of (1) to obtain a set $\mathcal{X}$ that bounds all possible values that the power system states x can take. To do this, we first linearize the nonlinear mapping $f(\cdot)$ about its nominal solution x_0 corresponding to $w = w_0$ and $u = u_0$. Let $\Delta w := w - w_0$ where $w \in \mathcal{W}$. Then for sufficiently small w,

$$\Delta w \approx J \Delta x, \tag{5}$$

where $J = \frac{\partial f}{\partial x}\Big|_{x_0}$ is the power flow Jacobian evaluated at x_0. Assuming that the Jacobian is invertible, we can express the sensitivity of the state variable x to w as

$$\Delta x \approx M \Delta w, \tag{6}$$

where $M = J^{-1}$.

Example 2 (Four-bus system): Continuing with the four-bus system of Example 1, we compute the zonotope $\mathcal{X}$ that bounds the possible values of the state vector x resulting from uncertainty in the active power injections in w using (4). The center of zonotope $\mathcal{X}$, x_0, is computed from w_0 using the nonlinear power flow equations of (1). Table I shows the nominal power flow solutions of this system. The zonotope $\mathcal{W}$ capturing the uncertainty in w is projected onto the P_{g3}-P_{g4} axis and displayed in Fig. 3(a) and the resulting projection of zonotope $\mathcal{X}$ bounding the system states V_2 and V_3 is shown in Fig. 3(b). In addition, we sampled the input space $\mathcal{W}$ and calculated the corresponding exact solutions to the power flow equations of (1). In Fig. 3(b), we see that one of the solution points lie on the outside of zonotope $\mathcal{X}$, which is attributed to error from the power flow model's linearization. ■

III. Case Studies

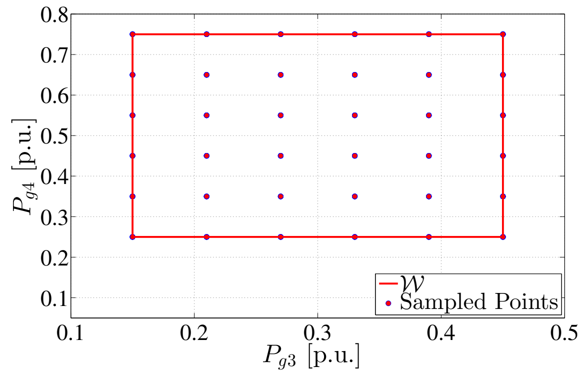
In this section, we validate the framework developed in Section II by comparing the results obtained using the proposed analysis method against the exact solutions computed from the power flow equations (1). The case studies are performed on the IEEE 123-bus distribution system [25], and the IEEE 145-bus, 50-machine transmission system [26], [27]. These systems are modified to include power injection resulting from renewable resources at a subset of buses. For each case study we linearize the system before propagating the uncertainty in renewable-based generation through the model. Then we examine the impact of uncertainty on the bus voltage magnitudes.

A. 123-bus Distribution System

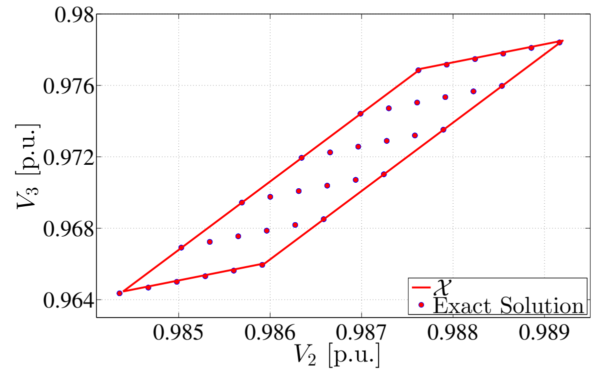
The one-line diagram and complete description for this system can be found in [25]. Suppose that for this system, renewable-based electricity generation resources are installed at buses 80, 95, 96, 103, 108, 110, 115, 121, 122, and 123, with nominal real power injection of 1 p.u. and an uncertainty of $\pm 50\%$ (± 0.5 p.u.) around the nominal value. In order to determine whether the system states will remain within bounds dictated by performance requirements, we first bounded the power injection space with a zonotope $\mathcal{W}$. Then we computed the corresponding state-bounding zonotope $\mathcal{X}$ using (4). The resulting zonotope is projected onto the subspace defined by the V_{110}-V_{123} axis and shown in Fig. 4(a). Additionally, we sampled the input power injection space and obtained the corresponding solutions of the linearized power flow as well as the exact solutions of the nonlinear power flow and depicted them with squares and circles, respectively. The resulting projection of the zonotope $\mathcal{X}$ contained all the linearized power flow solutions and all but the extrema of the nonlinear power flow solution. Thus, we can conclude that the linearization is fairly accurate. In fact, for this case study, we computed the percent

TABLE I: Four-bus system nominal power flow solution.

w_0	$P_{g_2}^0$	$Q_{g_2}^0$	$P_{g_3}^0$	$Q_{g_3}^0$	$P_{g_4}^0$	$Q_{g_4}^0$
	0.4	0	0.3	0	0.5	0
u_0	$P_{l_2}^0$	$Q_{l_2}^0$	$P_{l_3}^0$	$Q_{l_3}^0$	$P_{l_4}^0$	$Q_{l_4}^0$
	0.8	0.25	0.5	0.1	0.9	0.5
x_0	V_2^0	θ_2^0	V_3^0	θ_3^0	V_4^0	θ_4^0
	0.987	$-0.124°$	0.972	$-0.273°$	0.965	$-0.302°$



(a) Projection of bounding zonotope $\mathcal{W}$ and input uncertainty set.



(b) Projection of state-bounding zonotope $\mathcal{X}$ and exact solution points.

Fig. 3: Projection of Four-bus test system's input and state bounding zonotopes.

error between the voltage magnitudes obtained through the linearized power flow and the nonlinear power flow for each sample point and found the maximum to be only 2.5%.

From Fig. 4(a), we can also conclude that for the uncertainty levels selected, a portion of the input space maps to a region in the solution state space that violates the voltage constraints of 1.05 p.u., which are depicted with dashed lines. This conclusion is also verified by the linearized and exact nonlinear power flow solutions. Therefore, we cannot conclude that the system states will remain within the performance requirements if this system were subjected to this level of uncertainty in power injection arising from renewable resources.

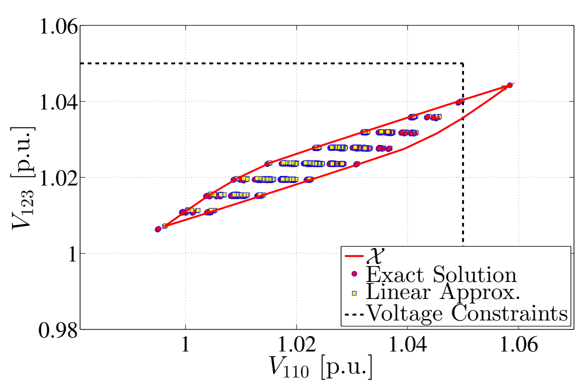
Now suppose the nominal real power injection is 0.8 p.u. (instead of 1 p.u.) at the same buses and that the uncertainty of the power injections at the affected buses remains at $\pm 50\%$ (± 0.4 p.u.) around the nominal value. The result is shown in Fig. 4(b) along with the linearized and exact nonlinear power flow solutions. We conclude that for the the power injection and uncertainty levels chosen, no voltage magnitude violations for buses 110 and 123 are detected.

B. 145-bus Transmission System

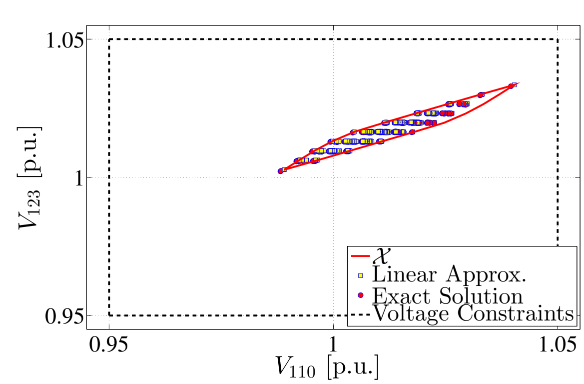
A full description of the IEEE 145-bus transmission system can be found in [26]. Suppose renewable-based electricity resources are installed at the buses listed in Table II, which also specifies their respective nominal power outputs and uncertainty in power injection. We captured this uncertainty with a zonotope $\mathcal{W}$, and used it to compute the state-bounding zonotope $\mathcal{X}$ that contains all possible values of x. The projection of $\mathcal{X}$ onto the V_{100}-V_{133} axes is shown in Fig. 5 along with the exact power flow solutions. As in the 123-bus case, all of the solution points lie within $\mathcal{X}$ except for one lower extreme point, which can be attributed to the error resulting from linearization. Again, we computed the maximum error of the voltage magnitudes between the linearized power flow and nonlinear power flow to be only 1.13%. Therefore, with this particular level of uncertainty in the power injection, the linearization provided an accurate estimate for the nonlinear power flow solutions.

IV. Performance Evaluation

In this section, the computation time of our method is evaluated against those of solving the linearized and nonlinear power flows in MATLAB. Our code is executed on a computer



(a) Renewable-based power injections with variability of $\pm 50\%$ around each nominal value of 1 p.u.



(b) Renewable-based power injections with variability of $\pm 50\%$ around each nominal value of 0.8 p.u.

Fig. 4: 123-bus system: nonlinear and linearized power flow solutions, and state-bounding zonotope projections.

TABLE II: 145-bus system: data for renewable-based power injection variations.

Bus	70	85	96	110	112	120	125	128	130	133
Nominal Value [p.u.]	1	1.5	1	1	1	1.5	1	1.5	1	1
Variation [p.u.]	0.3	0.4	0.2	0.3	0.6	0.7	0.4	0.4	0.5	0.4

equipped with Intel Core 2 Quad 8400 processor running at 2.66 GHz. For our method, we provide the time required to compute $\mathcal{X}$ from the input uncertainty space $\mathcal{W}$ along with the time required to compute the linearized and nonlinear power flow solutions by sampling the extremas of $\mathcal{W}$.

The computation times required for each of the test cases in Section III and the 4-bus example are shown in Table III. As the number of buses with renewable-based generation increased, the time required to compute the nonlinear power flow solutions corresponding to the extremas of $\mathcal{W}$ grew much more quickly than that required for the linearized power flow and the zonotope method. In fact, obtaining the exact power flow solutions required more than 12 hours to solve when there were more than 12 buses with renewable-based generation, while the computation time for the zontope method remained nearly constant. Lastly, if interior points of the input space were also sampled in addition to the extremas of $\mathcal{W}$, then significantly longer times were required for obtaining the corresponding nonlinear and linearized power flow solution points, rendering the computations intractable. On the other hand, our proposed method does not exhibit any significant increase in computation time when used on a larger system.

V. Concluding Remarks

This paper proposes a set-theoretic method to assess the impact of unknown-but-bounded power injections resulting from renewable resources. The unknown-but-bounded uncertainty in the power injections are captured using a zonotope which is subsequently propagated through the linearized power-flow model of the system to obtain another zonotope that bounds the worst-case deviation of the system state variables. From this method, we can determine whether the system state variables will remain within the ranges specified by operational requirements when subjected to uncertainty in renewable generation.

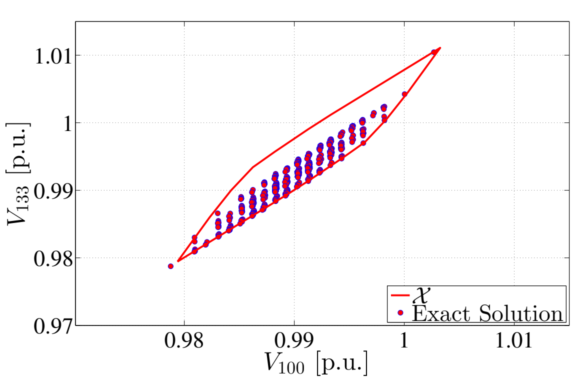


Fig. 5: 145-bus power system.

TABLE III: Comparison of overall computation times [s] for for 4-, 123-, and 145-bus systems.

	4-bus	123-bus	145-bus
Zonotope	0.00037	0.03812	0.15394
Linear Approx.	0.00002	0.00360	0.02149
Nonlinear PF	0.01111	9.46197	103.151

The validity of this method is verified on two test systems. From the test cases, we showed that the results using our method matches closely to those obtained from repeatedly solving the nonlinear power flow for different power injections associated with various levels of uncertainty. We have also shown that our method is highly scalable with the dimensionality and the size of the system. Future work may include an analysis of the limits of the small-signal approximation to the power flow model in order to capture solution points that currently lie outside of the set obtained from the linearized model.

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