Measurement-Based Real-Time Economic Dispatch

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***Abstract*—In this paper, we propose a measurement-based approach to the real-time economic dispatch (ED). The real-time ED is a widely used market scheduling problem seeking to economically balance electricity system supply and demand and provide locational marginal prices (LMPs) while respecting system reliability requirements. The ED is a convex optimization problem with a linear or quadratic objective, typically the minimization of generator costs or the maximization of social surplus. The constraints capture power balance and network flow capacity limits and are formulated using a linearized power flow model. Our approach utilizes power system sensitivities estimated from phasor measurement unit (PMU) measurements to reformulate the model-based power flow and network flow constraints. The resulting measurement-based real-time ED overcomes the vulnerabilities of the model-based real-time ED. The dispatch instructions and LMPs calculated with our measurement-based real-time ED accurately, and adaptively, reflect real-time system conditions. We illustrate the strengths of the proposed approach via several case studies.**

I. Introduction

The majority of electricity consumers in the United States are served by electricity procured in Independent System Operator (ISO)- or Regional Transmission Organization (RTO)-run markets [1]. ISO/RTOs oversee large-scale market scheduling procedures that consists of a sequence of forward markets based on security-constrained unit commitment- and economic dispatch-based algorithms (see, e.g., [2], [3]). The goal of these processes is to schedule resources on various time-scales such that the system operator can maintain the supply-demand balance around-the-clock and satisfy operational and physical constraints imposed by the electricity network. Moreover, the markets outcome include the prices for energy, the locational marginal prices (LMPs), and the prices for ancillary services [4]. The LMPs are an important outcome of the market clearing process and serve two primary functions: i) they provide prices at which to settle energy supplied/consumed by resources in the system; and ii) they provide the system operator with an indication of the existence of localized scarcity due to network constraints, which also indicates the buses at which injections/withdrawals would have the highest impact on relieving such congestion.

The so-called real-time economic dispatch (ED) in the real-time electricity markets is the final stage in the scheduling process at which LMPs are determined. The ED is a widely used market scheduling problem seeking to economically balance electricity system supply and demand and provide LMPs while respecting system reliability requirements. The real-time ED has four primary components: i) the objective, typically the maximization of social surplus, which consists of the producer and consumer offer and bid functions, respectively, which are commonly quadratic or piecewise-linear functions; ii) power flow and network flow constraints obtained from a model of the system; iii) equipment constraints, e.g., generator power output limits; and iv) additional reliability-driven constraints, e.g., reserve requirement and security constraints [8]. In those ISOs/RTOs with a real-time ED, the process typically takes place on a five-minute basis and determines LMPs and dispatch targets for resources for the next five minutes (see, e.g., [2], [3]).

In order to formulate power flow and network flow constraints, the real-time ED requires an up-to-date model of the ISO/RTO's electricity system and that of neighboring systems, which is typically derived from the output of a state-estimator. The state estimator-based model is vulnerable to errors due to numerous phenomena, e.g., undetected changes in the internal system topology and erroneous model parameters [5], as well as inaccurate representations of neighboring systems [6]. As such, the accuracy of the dispatch targets and LMPs determined in the model-based real-time ED is coupled with the accuracy of the system model and is subject to the same vulnerabilities. Inaccurate dispatch targets and LMPs have economic implications, such as sending incorrect local price signals and over/under payment to resources, as well as system reliability implications, such as unintended equipment overloads and outages [7].

In this paper, we propose a measurement-based approach to the real-time ED. Typically, the real-time ED is formulated using model-based linear flow sensitivities, known as the injection shift factors (ISFs), to represent network flows [8]. Our approach instead utilizes the measurement-based method of estimating the ISFs described in [9]. As we demonstrate in this paper, by moving away from a model-based representation of the network, our approach virtually eliminates the impacts of phenomena such as undetected changes in system topology and erroneous model parameters on the dispatch targets and LMPs determined in the real-time ED. In doing so, we ensure that the real-time ED outcomes reflect actual real-time system conditions.

Our approach relies on the availability of phasor measurement units (PMUs), the preponderance of which in power systems is facilitating the proliferation of a new generation of operational tools that harness the very high frequency and time synchronicity of their measurements (see, e.g., [10], [11]). The use of PMU data in real-time operations has been promoted as a means by which to circumvent the shortcomings of the existing telemetry system, upon which state estimation is based, and reduce the frequency of the occurrence and the magnitude of the impact of preventable outages [7]. The real-time ED process can likewise be enhanced by the deployment of PMU measurements.

II. Preliminaries

In this section, we first give an overview of the fundamental components of the market-based electricity scheduling process known as the real-time ED. Next, we review the calculation of the measurement-based line sensitivities, which provide the basis for our measurement-based approach to the real-time ED.

A. Model-Based Real-Time ED

The real-time ED is based on the classical optimal power flow (OPF) [8]. The main components of the OPF are the objective, typically the maximization of the social surplus or the minimization of generator costs [12], the nonlinear power flow and network flow constraints, and the equipment operational constraints. For a number of computational and practical reasons, the real-time ED is commonly formulated using a simplified OPF formulation, the DC-OPF [13].

The DC-OPF relies on the so-called "DC" assumptions: (i) the system is lossless, (ii) the voltage at each bus is approximately equal to one p.u., (iii) the difference in the voltage angles between at each pair of connected buses is small [8]. These assumptions result in a linear approximation of the nonlinear power flow and network flow constraints. There are two primary approaches to the representation of the network in the DC-OPF, the conventional "B-θ" approach and the ISF-based approach [13]. In this work, we focus on the ISF-based approach as it explicitly motivates our proposed measurement-based real-time ED.

1) General Notation: We consider a system that consists of N buses indexed by $\mathcal{N} = \{1, \ldots, N\}$, and L lines indexed by $\mathcal{L} = \{\ell_1, \ldots, \ell_L\}$, where each ℓ_l is an ordered pair (n, m), $n, m \in \mathcal{N}$, representing a transmission line between buses n and m, with the convention that positive flow on such a line is in the direction *from* n *to* m. Moreover, let there be G generators indexed by $\mathcal{G} = \{1, \ldots, G\}$, and D demands indexed by $\mathcal{D} = \{1, \ldots, D\}$. Let $\mathcal{G}_n \subseteq \mathcal{G}$ be the subset of generators at bus $n \in \mathcal{N}$, and let $\mathcal{D}_m \subseteq \mathcal{D}$ be the subset of loads at bus $m \in \mathcal{N}$.

Let $P_i^g[k]$ be the output of generator $i \in \mathcal{G}$ at time k and let $P_j^d[k]$ be the demand of load $j \in \mathcal{D}$ at time k with the convention that $P_i^g[k] > 0$ is a generator real power injection into the system, and $P_j^d[k] > 0$ is a load real power withdrawal from the system. Then, define the vectors of generation and demand as $P^g[k] = [P_1^g[k], \ldots, P_G^g[k]]^T$ and $P^d[k] = \left[P_1^d[k], \ldots, P_D^d[k]\right]^T$, respectively. With these quantities, we define the net injection at a bus $n \in \mathcal{N}$ at time k as

$$P_n[k] = \sum_{i \in \mathcal{G}_n} P_i^g[k] - \sum_{j \in \mathcal{D}_n} P_j^d[k],$$

with the convention that $P_n[k] > 0$ is a real power injection *into* the system. Then, define the vector of net injections at all buses as $P[k] = [P_1[k], \ldots, P_N[k]]^T$.

2) Power Balance Constraint: In the ISF-based network representation, the voltage angles are not explicitly represented, rather the bus power balance and power flowing on each line are written in terms of the system-wide power balance, the linear flow sensitivities, and the bus injections. The system-wide power balance for a time k can be written in the form

$$\mathbb{1}_G P^g[k] - \mathbb{1}_D P^d[k] - \mathbb{1}_L P^\ell[k] = 0, \tag{1}$$

where $\mathbb{1}_G$, $\mathbb{1}_D$, and $\mathbb{1}_L$ are all-ones row vectors of dimensions G, D, and L, respectively, and $P^\ell[k]$ is the L-dimensional column vector of line real power losses at time k, which we will assume takes the form of a loss sensitivity-factor-based loss model, such as that given in [14].

3) Network Flow Constraints: To represent the transmission network, we denote the $L \times N$ system incidence matrix by $A = [a_1, \ldots, a_i, \ldots, a_N]$, where a_i is an L-dimensional column vector the jth entry of which is equal to 1 if bus i is the *from* bus of line j, -1 if bus i is the *to* bus of line j, and zero otherwise. Further, let b denote the L-dimensional column vector of branch susceptances, and define the diagonal $L \times L$ branch susceptance matrix as $B_b = \text{diag}\{b\}$, where $\text{diag}\{\cdot\}$ denotes a diagonal matrix such that $B_b[i, i] = b_i$, $\forall i$, and the $N \times N$ nodal susceptance matrix as $B = A^T B_b A$.

The $L \times N$ linear flow sensitivity matrix, or ISF matrix, denoted by Ψ, provides the basis of the ISF-based DC-OPF network flow representation. An entry of Ψ, denoted by $\Psi[l, i]$, provides the sensitivity of the flow on line $\ell_l \in \mathcal{L}$ to an injection at bus i that is withdrawn at the slack bus. Under the DC assumptions, Ψ can be calculated directly from the network connectivity and parameters as follows,

$$\Psi = B_b A B^{-1}. \tag{2}$$

It is important to note that Ψ is invariant to changes in bus injections/withdrawals and changes in the system topology.

With the model-based ISFs from (2), we define the vector of line flows at a time k in terms of the bus injections as

$$P^f[k] = \Psi P[k], \tag{3}$$

which are bounded above and below by the line upper and lower limits, which are denoted by $\bar{P}^f$ and $\underline{P}^f$, respectively.

4) Objective Function: Let $\mathscr{O}_i(\cdot)$ be the offer function of generator i and $\mathscr{B}_j^d(\cdot)$ be the bid function of demand j, which are functions of the $P_i^g[k]$ and $P_j^d[k]$, respectively. These functions represent the preferences of the generators (sellers) and loads (buyers) and their willingness to accept or pay, respectively, for electricity transacted in the real-time ED. The objective of the real-time ED is the maximization of the social surplus [12], which is defined for a time k as:

$$\mathscr{S}(P^g[k], P^d[k]) = \sum_{j\in\mathcal{D}} \mathscr{B}_j(P^d_j[k]) - \sum_{i\in\mathcal{G}} \mathscr{O}_i(P^g_i[k]). \quad (4)$$

5) Model-Based Real-Time ED Problem Formulation: Combining the objective in (4) with the constraints that result from the power balance and network flow expressions in (1) and (3), respectively, we formulate the model-based real-time ED for a time k as follows:

$$\max_{P^g[k], P^d, [k]} \mathscr{S}(P^g[k], P^d[k]) \quad (5a)$$

$$s.t.$$

$$\mathbb{1}_G P^g[k] - \mathbb{1}_D P^d[k] - \mathbb{1}_L P^\ell[k] = 0 \leftrightarrow \lambda[k] \quad (5b)$$

$$\underline{P}^g \le P^g[k] \le \bar{P}^g \quad (5c)$$

$$\underline{P}^d \le P^d[k] \le \bar{P}^d \quad (5d)$$

$$\underline{P}^f \le \Psi P[k] \le \bar{P}^f \qquad \leftrightarrow \underline{\mu}^f[k], \bar{\mu}^f[k], \quad (5e)$$

where $\lambda_r[k]$ and $\underline{\mu}^f[k], \bar{\mu}^f[k]$ are the dual variables of their respective constraints, also referred to as "shadow prices" due to their well-known economic interpretation [15]. The shadow price of the system-wide power balance constraint, $\lambda_r[k]$, is often referred to as the system reference or energy price.

The main outcomes of the real-time ED are i) the optimal generator and load dispatch instructions, which are a direct result of the solution to (5), and ii) the LMPs, which are not a direct result of the solution to (5), but may be calculated from the ISFs, the loss sensitivity vector, and the shadow prices [16].

Let ζ be the N-dimensional column vector of the system-wide loss sensitivity to nodal injections, assuming injections are balanced by the slack bus. Then, the LMPs are as follows,

$$\lambda[k] = \lambda_r[k]\mathbb{1}_N^T + \Psi^T(\bar{\mu}^f[k] - \underline{\mu}^f[k]) + \zeta\lambda_r[k],$$

where $\mathbb{1}_N$ is an all-ones N-dimensional row vector. Note that for clarity of presentation in (5), we have left out of the real-time ED the security, reserve requirement, and ramping constraints that would be present in a practical real-time ED [4]. The exclusion of these constraints, however, has no bearing on the formulation of the measurement-based real-time ED and the constraints may easily be included in our measurement-based approach.

B. Measurement-Based ISFs

In this section, we review the measurement-based ISF estimation approach developed in [9], which we will employ later in the formulation of the measurement-based real-time ED. Consider the same power system defined in Section II-A. Suppose the net real power injected into the system at bus i at time t, $P_i(t)$, varies by a small amount $\Delta P_i(t)$ from time t to time $t + \Delta t$, where $\Delta t > 0$ and small. Further, let $\Delta P^i_{n\text{-}m}(t)$ be the change in real power flow on line $\ell_l = (n, m)$ due to ΔP_i. Define the measurement-based ISF for line ℓ_l with respect to an injection at bus i as

$$\Gamma^i_{n\text{-}m} := \frac{\Delta P^i_{n\text{-}m}}{\Delta P_i}. \quad (6)$$

While $\Delta P^i_{n\text{-}m}(t)$ is not directly available through PMU measurements, we can, however, measure $\Delta P_{n\text{-}m}(t)$, the total change in flow on line ℓ_l due to bus injections at time t. We observe that the variation in the flow on line ℓ_l is due to variations in the injections at each bus i:

$$\Delta P_{n\text{-}m} = \Delta P^1_{n\text{-}m}(t) + \cdots + \Delta P^N_{n\text{-}m}(t). \quad (7)$$

Employing (6) in (7) we obtain

$$\Delta P_{n\text{-}m} \approx \Delta P_1(t)\Gamma^1_{n\text{-}m} + \cdots + \Delta P_N(t)\Gamma^N_{n\text{-}m}.$$

Now suppose we have $M + 1$ sets of synchronized measurements. Let

$$\Delta P_i[j] = \Delta P_i[(j+1)\Delta t] - \Delta P_i[j\Delta t],$$

$$\Delta P_{n\text{-}m}[j] = \Delta P_{n\text{-}m}[(j+1)\Delta t] - \Delta P_{n\text{-}m}[j\Delta t]$$

for $j = 1, \ldots, M$ and define

$$\Delta P_i = [\Delta P_i[1] \ \cdots \ \Delta P_i[j] \ \cdots \ \Delta P_i[M]]^T,$$

$$\Delta P_{n\text{-}m} = [\Delta P_{n\text{-}m}[1] \ \cdots \ \Delta P_{n\text{-}m}[j] \ \cdots \ \Delta P_{n\text{-}m}[M]]^T.$$

Let $\Gamma_{n\text{-}m} = [\Gamma^1_{n\text{-}m}, \ldots, \Gamma^i_{n\text{-}m}, \ldots, \Gamma^N_{n\text{-}m}]$. Then, clearly,

$$\Delta P_{n\text{-}m} = [\Delta P_1 \ \cdots \ \Delta P_i \ \cdots \ \Delta P_N]\,\Gamma^T_{n\text{-}m} \quad (8)$$

Let ΔP denote the $M \times N$ matrix $[\Delta P_1 \ \cdots \ \Delta P_i \ \cdots \ \Delta P_N]$. Then, the system in (8) becomes

$$\Delta P_{n\text{-}m} = \Delta P \Gamma^T_{n\text{-}m}. \quad (9)$$

If $M \ge N$, then (9) is an overdetermined system. Moreover, assuming the ISFs are approximately constant over the $M+1$ measurements, we can obtain an estimate of $\Gamma_{n\text{-}m}$ from least-squares error estimation (see, e.g., [9]) as

$$\hat{\Gamma}^T_{n\text{-}m} = (\Delta P^T \Delta P)^{-1} \Delta P^T \Delta P_{n\text{-}m}. \quad (10)$$

In the next section, we discuss the application of the measurement-based ISFs in the context of the real-time ED.

III. Measurement-Based Real-Time ED

It is clear from (5) that the dispatch targets and LMPs calculated from the results of the model-based real-time ED depend the model-based ISFs and how accurately these ISFs reflect the conditions in the system at the time the real-time ED is formulated. However, due to potential inaccuracies in telemetry and state estimation that propagate to the underlying system model, the model-based ISFs may not always reflect the real-time system conditions resulting in reliability and economic issues. We address this shortcoming of the model-based ED via the measurement-based ISFs described in Section II-B.

Let $\hat{\Gamma}$ be the $L \times N$ matrix of the measurement-based ISF estimates, each row of which is obtained from (10). As described in Section II-A, model-based ISFs form the basis of the network description in the real-time ED. To remove the dependence of (5) on a power system model, we deploy $\hat{\Gamma}$ to re-formulate the network constraints (5e) as

$$\underline{P}^f \le \hat{\Gamma} P[k] \le \bar{P}^f. \tag{11}$$

With the reformulated network constraints in (11), the real-time ED no longer relies on a system model. Instead, the system operator continuously updates the estimate of $\hat{\Gamma}$ via (10) as new PMU measurements become available and uses the most up-to-date estimate to formulate the real-time ED. This measurement-based real-time ED is adaptive to changing system conditions, such as detected or undetected topology changes, variations in bus injections, and even changes in line and other system parameters due to loading or extreme temperature conditions. The measurement-based ISFs can also be used to formulate other sensitivities, such as the power transfer distribution factors and line outage distribution factors [13], which could then be used to formulate security constraints in the real-time ED.

A key strength of our measurement-based ED approach is its consistency with the current real-time ED framework; the structure of the real-time ED formulation is left largely unchanged, but more appropriate data is used as the basis for that structure. The result is an enhanced and adaptive real-time ED.

IV. Case Studies

In this section, we illustrate two strengths of the measurement-based real-time ED by using the Western Electricity Coordinating Council (WECC) 3-generator, 9-bus test system. The real power flow limit on line $(5, 6)$ has been reduced to 20 MW so as to introduce transmission congestion. In each study, we use simulated PMU measurements of the random fluctuations in bus injections to estimate $\hat{\Gamma}$, which are generated according to

$$P_i[j] = P_i^0[j] + \sigma\nu,$$

where $P_i^0[j]$ is the nominal bus injection and $\sigma\nu$ is a pseudorandom value drawn from a normal distribution with zero mean and standard deviation σ. Furthermore, we assume each load has an infinite willingness to pay, i.e., demand is inelastic, and each generator i submits a quadratic offer function of the form $a_i(P_i^g)^2 + b_i P_i^g + c_i$ and report the parameters of the functions used in the studies in Table I.

TABLE I: Generator Offer Function Parameters

generator	a_i (\$/MWh^2)	b_i (\$/MWh)	c_i (\$)
1	0.1100	5.0	150
2	0.0850	1.2	600
3	0.1225	1.0	335

A. Undetected Line Outage

In this study, we assume there is an undetected outage of line $(6, 7)$ and compare the LMPs that would be realized in the real-time ED with a model-based approach vs our

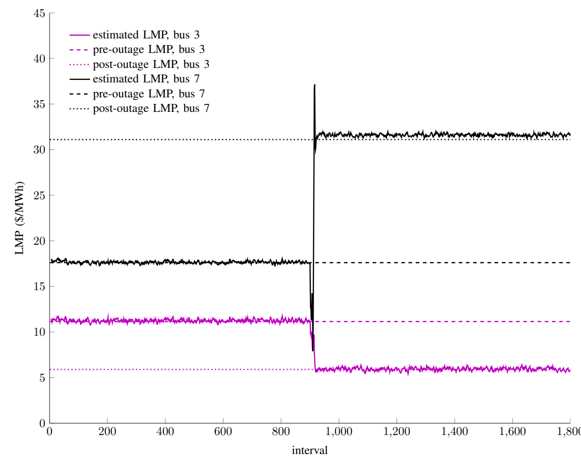

Fig. 1: Pre- and post-outage LMPs and measurement-based LMP evolution at buses 3 and 7 with line $(6, 7)$ outaged.

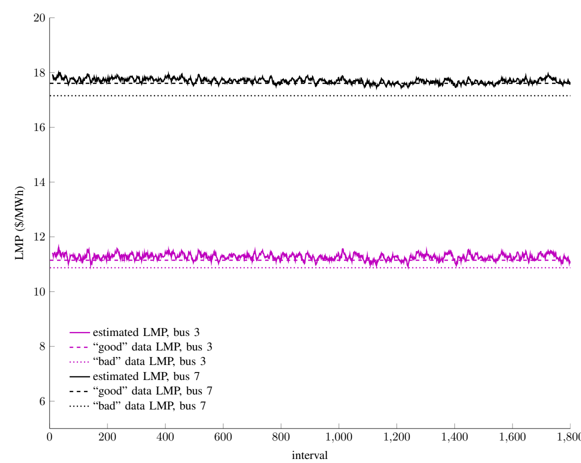

Fig. 2: "good" and ''bad" data LMPs and measurement-based LMP evolution at buses 3 and 7.

measurement-based approach. For bus 3 (a generator bus) and bus 7 (a load bus), Fig. 1 shows: i) the pre-outage model-based LMPs, which are the LMPs that would be realized in the presence of the undetected outage; ii) the post-outage model-based LMPs, which are the LMPs that would be realized if the outage was detected; and iii) the measurement-based estimates of the LMPs. Unsurprisingly, there is a large discrepancy between the pre- and post-outage model-based LMPs, more than \$10/MWh at bus 7, which might raise market fairness issues of such a situation were to arise. The measurement-based LMPs, however, closely track the 'true' LMPs regardless of whether or not the outage is detected. This example illustrates how a measurement-based real-time ED formulation can enhance the ability of the real-time ED to provide LMPs that capture real-time system conditions.

B. Inaccurate System Model Data

We now demonstrate the robustness of our measurement-based real-time ED approach against erroneous model data. To simulate the impact of "bad" system model data on the LMPs calculated with the real-time ED results, we increase the impedance of line $(8, 9)$ by 20%, leaving all other system data unchanged. Figure 2 shows for buses 3 and 7: i) the model-based LMPs with the "good" line $(8, 9)$ impedance data; ii) the model-based LMPs with the "bad" line $(8, 9)$ impedance data; and iii) the measurement-based estimates of the LMPs. In this case, the bad line data causes the model-based LMPs at buses 3 and 7 to appear less than their true values, sending an incorrect price signal to resources at these buses. However, our measurement-based approach is able to capture the true LMP values due to its independence from the system model data. Thus, the measurement-based LMPs send price signals to the resources at buses 3 and 7 that accurately reflect the true values of the system parameters, which dictate actual system performance.

V. Concluding Remarks

In this paper, we developed a measurement-based approach to the real-time ED. Our approach harnesses sensitivities estimated from PMU measurements to reformulate the model-based power flow and network flow constraints of the existing model-based real-time ED. As shown in our case studies, the measurement-based real-time ED is robust to undetected system disturbances and inaccurate model data and results in LMPs that more accurately reflect real-time system conditions.

Our future work will focus on the evaluation of the comparative performance of the measurement-based vs model-based real-time ED in interconnected systems with little or no information exchange. Furthermore, we will expand the formulation of the measurement-based real-time ED presented in this work to include security-based constraints and demonstrate its effectiveness on large-scale systems.

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