Evaluation of Demand Response Resource Aggregation System Capacity Under Uncertainty

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***Abstract*—Demand response resources (DRRs) are usually aggregated in order to participate in wholesale electricity markets (e.g., capacity, energy, and ancillary service markets). In such *DRR aggregation systems*, uncertainty arising from, e.g., random failures, is unavoidable; this paper focuses on assessing the impact of such uncertain phenomena on the reliability of DRR aggregation systems. To this end, we first develop a stochastic hybrid system (SHS) model to capture DRR continuous dynamics, as well as discrete events that arise from failures and repairs. The statistics of the DRR aggregation system state variables can be obtained by using the extended generator of the SHS. Then, we can use these statistics to estimate the value of the probability that the DRR aggregation system can successfully provide a certain amount of power for a period of time. Subsequently, by varying the values of the power to be provided and period duration, we can construct a probability-capacity-duration contour. Capacity-duration curves can be then obtained by setting the probability to desired confidence levels. The proposed method is illustrated through several examples and case studies.**

I. Introduction

The motivation for this work lies in the recent trend toward ensuring more reliable, environmentally friendly, and economically operated power grids, which entails, among other things, the development of schemes in which demand resources respond to operator signals. In such demand response programs, consumers are able to adjust their consumption accordingly when the grid supply-demand mismatch is severe, or grid reliability is in jeopardy; these consumers are referred to as demand response resources (DRRs) [1]. Common examples of DRRs include residential and commercial loads with the ability to store thermal energy (e.g., residential thermostatically-controlled loads, and commercial building HVAC systems), and other interruptible loads (e.g., certain industrial consumers). Typically, in order for these DRRs to participate in a demand response program, an aggregator that coordinates their response and bids into the wholesale market is required; in this paper, we refer to a collection of DRR units coordinated by an aggregator as a *DRR aggregation system*.

The use of DRRs has been promoted by several initiatives. For example, Federal Energy Regulation Commission (FERC) Order No. 719 defines the role of the aggregator as an entity that can bid on behalf of aggregated loads in various electricity markets, e.g., capacity, energy, and ancillary service markets [2]. Independent system operators (ISOs) have established different procedures and requirements for DRRs to participate in various markets. For instance, in PJM, the aggregator of a DRR aggregation system is referred to as a curtailment service provider (CSP), and CSPs can participate in the capacity, energy, and ancillary service markets upon satisfying the requirements of these markets. DRRs that participate in the capacity and energy markets are generally required to be available for a period of time in the order of hours, while ancillary service market participants are usually required to be responsive in the order of minutes. Although the choice to participate is voluntary, the commitment is mandatory, and penalties will be applied for noncompliance. Therefore, it is critical for aggregators to consider the reliability of the participant DRRs. California ISO also allows DRRs to participate in day-ahead and real-time markets as energy resources for both reliability and economic purposes [3]. In 2014, DRRs contributed up to 10% of the peak demand in the footprint of some ISOs [4], and in 2011, DRRs played a significant role and helped avoiding power outages during peak summer load periods in the footprint of the Electric Reliability Council of Texas (ERCOT) [4]. However, there are remaining issues that are hampering the widespread deployment of DRRs.

One such issue is reliability; a participant in a DRR aggregation system may fail to respond to the command sent by the aggregator due to various reasons. For example, while the advanced metering infrastructure facilitates the implementation of demand response programs, high dependence on such infrastructure introduces the possibility of malicious cyber attacks, and device misbehavior. Moreover, the customer may opt out at any time as participation is mostly on a voluntary basis (see, e.g., the SmartAC program implemented by Pacific Gas and Electric Company [5]). As the reliance on DRRs increases, the impact of such events is no longer negligible. Thus, it is necessary to carefully perform reliability evaluation and assess the actual capacity of DRR aggregation systems. In this regard, power grid reliability may be significantly threatened if the expected capacity of DRR aggregation systems is not realized when needed, especially considering the fact that DRRs are usually utilized when the system is stressed. Also, the aggregator may face significant fines.

There is a significant number of works proposing mechanisms to enable the utilization of DRRs to provide different services to the grid. For example, in [6] and [7], different market mechanisms are explored; currently, most programs are incentive-based, where customers receive payments for their participation [8], [9], [10]. Price-based programs, where

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customers respond according to dynamic pricing, have been shown to be more efficient by the studies in [7]; however they introduce a higher level of uncertainty in customer behavior. Various pricing and market mechanisms are explored using game-theoretic frameworks (see, e.g., [11], [12], [13], [14]). The coordination of DRRs and renewable-based generation resources is also investigated in [15], [16], [17]. From an operational perspective, as mentioned above, a considerable portion of DRRs are residential and commercial loads with the ability to store thermal energy. A key difference between these DRRs and conventional generation units is their energy limits. In this regard, the authors in [18] and [19] present generalized battery models to capture the dynamics of such thermal-storage capable DRRs and their energy constraints; these constraints complicate the allocation/control algorithms. To overcome this problem, different control algorithms, mainly for providing ancillary services, e.g., frequency regulation, have been explored (see, e.g., [20], [21], [22]). Moreover, various fast, resilient, and distributed control algorithms are presented in [23], [24]. On the other hand, given that the behavior of DRRs is different from that of conventional generation units, it is also important to specifically study the reliability of DRR aggregation systems; however, although some works have recognized the potential inadequacy and uncertainty issues (see, e.g., [25] for market inadequacy issues, and [26], [27], [28] for various uncertainty concerns), to the best of our knowledge, there are no specific studies on quantifying the reliable capacity in the literature.

Reliability analysis of traditional power systems has been studied for decades (see, e.g., [29], [30], and the references therein). Many probabilistic reliability evaluation techniques are based on Markov chain models, where random failures and repairs are captured by transitions in and out of the states of the chain (see., e.g., [31]). Then, the probability distribution of generation and load can be obtained, from which other reliability metrics can be computed. However, Markov-chain models are not powerful enough to evaluate the reliability of DRR aggregation systems as they cannot capture the continuous dynamics and energy constraints of DRRs; this paper focuses on addressing this problem.

As described above, DRRs with the ability to store thermal energy possess battery-like characteristics, including constraints on the amount of energy stored; this makes the capacity of DRR aggregation systems vary with different durations of service, the relationship of which can be depicted by a *capacity-duration curve*. In this paper, we focus on DRR aggregation systems consisting of this type of DRRs, and propose a method to obtain this capacity-duration curve taking random failures in the DRR units into consideration. To this end, in order to describe the dynamics of each individual DRR unit, we adopt the virtual battery model in [19]—a set of ordinary differential equations (ODEs) with constraints on state and input. Note that if the DRR aggregation system includes other types of DRRs, basic convolution operations can be applied to evaluate the capacity of the overall DRR aggregation systems; this will be described in Section V. Note that in this paper we focus on cases where the battery-like characteristics of participants are known and properly described by a model; this is usually the case for industrial and commercial customers (see, e.g., [19]). We expect that this methodology could be also applied to residential aggregation if the corresponding battery-like models were available (see, e.g. [18], for the progress made in that direction to extract such models for residential thermostatically-controlled loads).

For the DRR aggregation system to follow a requirement signal provided by an ISO, a control algorithm is needed to allocate the commanded power among its participants. In this work, we first propose a control algorithm that make the participant units reach their energy limits at the same time. It will be shown later that this control algorithm reduces the computation complexity (e.g., the number of state variables, as well as the computation time). It will also be shown that the capacity evaluation of a DRR aggregation system does not significantly depend on the control algorithm.

In order to capture the effect of random failures on the DRR aggregation system described above, we develop a stochastic hybrid system (SHS) model. The rates at which the DRR units fail are assumed to be known, and can be either obtained from empirical data or statistically estimated using historical data. The dynamic state variables from the virtual battery model become stochastic, and by using the extended generator of the SHS, we can obtain a set of ODEs that govern the evolution of the state variable moments (see, e.g., [32], [33], [34]). Then, we can compute the probability that the DRR aggregation system can successfully provide a certain amount of power for a period of time—this probability is the reliability measure we adopt. Then, by varying the value of the power to be provided and the period duration, we can construct a probability-capacity-duration contour. By setting this probability to a desired confidence level, we can obtain a capacity-duration curve. This probability-capacity-duration contour is critical for aggregators as based on it, the aggregators can participate/bid in electricity markets with a quantifiable high confidence level, and avoid overestimating or underestimating the actual capacity of the DRRs they coordinate. Additionally, the ISO may also require the confidence level of each DRR participant and use it to more accurately evaluate overall power system reliability. Example 2 in Section III exemplifies the importance of this information.

The remainder of this paper is organized as follows. In Section II, a brief overview of SHSs is provided. Section III presents the DRR aggregation model adopted in this work. Section IV describes the proposed reliability assessment method. Discussions on the generalization of this method are presented in Section V. Section VI illustrates the assessment method via several case studies. Concluding remarks are presented in Section VII.

II. Preliminaries

In this section, we provide a brief overview of stochastic hybrid systems (SHSs) tools, including a method to evaluate the statistics of SHS state variables; based on these tools the DRR aggregation system reliability model is formulated and assessed in Sections III and IV.

A. Stochastic Hybrid System (SHS) Models

The state space of an SHS is comprised of a discrete state (also referred to as mode), $q(t) \in \mathcal{Q}$, where $\mathcal{Q}$ is a countable set, and a continuous state $x(t) \in \mathbb{R}^n$; the system state is defined by the pair $(q(t), x(t))$ (see, e.g., [32], [33], [34]). Then, an SHS model is characterized by three components:

(i) a differential equation,

$$\dot{x} = f(q, x, t); \tag{1}$$

(ii) a collection of transition rate functions $\lambda_{ab}(q, x, t)$,

$$\lambda_{ab} : \mathcal{Q} \times \mathbb{R}^n \times [0, \infty) \to \mathbb{R}^+, \forall a, b \in \mathcal{Q}; \tag{2}$$

(iii) a collection of reset maps $\phi_{ab}(q, x, t)$,

$$\phi_{ab} : \mathcal{Q} \times \mathbb{R}^n \times [0, \infty) \to \mathcal{Q} \times \mathbb{R}^n, \forall a, b \in \mathcal{Q}. \tag{3}$$

In essence, the evolution of the continuous state is described by (1) when the discrete state remains unchanged. When a transition occurs, the system state is reset according to (3), with (2) describing the likelihood of a transition from mode a to mode b. To be precise, the probability that the system transitions from mode a to mode b in an interval $(t, t+dt]$ is approximately $\lambda_{ab}(q, x, t)dt$, for $dt > 0$ and sufficiently small. In the context of this paper, we assume $f, \lambda_{ab}, \phi_{ab}$ do not explicitly depend on time.

B. State Statistics

The evolution of the state moments of an SHS can be obtained as follows. Let $\psi : \mathcal{Q} \times \mathbb{R}^n \to \mathbb{R}$ be twice continuously differentiable with respect to its second argument; following [32], we will refer to ψ as a test function. In the remainder, we use capital letters (e.g., $Q(t)$, $X(t)$) to denote random variables, and small letters (e.g., $q(t), x(t)$) to denote corresponding realizations. Dynkin's formula (see, e.g., [32]) describes the evolution of the expected value of the test function (i.e., $\mathbb{E}[\psi(Q, X)]$) as

$$\frac{d\mathbb{E}[\psi(Q, X)]}{dt} = \mathbb{E}[(L\psi)(Q, X)],$$

where the operator $\psi \mapsto L\psi$, which is called the extended generator of the system, is defined as

$$\begin{aligned}(L\psi)(q, x) :=& \frac{\partial \psi(q, x)}{\partial x} f(q, x) \\ &+ \sum_{a,b \in \mathcal{Q}} (\psi(\phi_{ab}(q, x)) - \psi(q, x))\lambda_{ab}(q, x).\end{aligned}$$

We can show that by properly choosing the test functions, the extended generator together with Dynkin's formula provides a set of ODEs, which describe the evolution of conditional moments of interest. For the case when $n = 1$, i.e., $x(t) \in \mathbb{R}$, we define the following family of test functions:

$$\psi_i^{(m)}(q, x) := \delta_i(q)x^m = \begin{cases} x^m, & q = i \\ 0, & q \neq i \end{cases}, \forall i \in \mathcal{Q}.$$

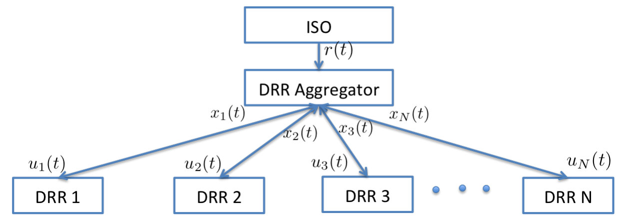


Figure 1: DRR aggregation architecture.

The expectation of these test functions, denoted by $\mu_i^{(m)}(t)$, are the product of conditional moments and occupational probabilities, i.e.,

$$\begin{aligned}\mu_i^{(m)}(t) :&= \mathbb{E}[\psi_i^{(m)}(Q(t), X(t))] \\ &= \mathbb{E}[X^m(t)|Q(t) = i]\Pr\{Q(t) = i\}.\end{aligned}$$

Useful properties about $\mu_i^{(m)}(t)$ include the following: (i) for $m = 0$, $\mu_i^{(0)}(t)$ is the occupational probability of mode i, i.e., $\mu_i^{(0)}(t) = \Pr\{Q(t) = i\}$; (ii) the conditional moments can be computed as $\mathbb{E}[X^m(t)|Q(t) = i] = \frac{\mu_i^m(t)}{\mu_i^{(0)}(t)}$; and (iii) the state moment can be calculated as $\mathbb{E}[X^m(t)] = \sum_{i \in \mathcal{Q}} \mu_i^m(t)$.

III. Modeling Framework

In this section, we present the DRR aggregation system model, including the dynamics of individual DRR units and a control scheme that we propose to coordinate the response of the DRRs. Then, based on this model, we present an SHS-based framework to capture the effect of failures and repairs.

A. Deterministic DRR Aggregation System Model

Consider an aggregator that coordinates N DRR units, as shown in Fig. 1. The aggregator receives from the ISO a signal, $r(t)$, to be followed. The aggregator also measures the state of each unit i, $x_i(t)$ (to be described later), and determines the power reduction (or increase) from the baseline power of each unit i, $u_i(t)$. The dynamic model for individual units and the control algorithm to determine the power reduction amount of each unit are described next.

1) DRR Unit Dynamics: Let $x_i(t) \in \mathbb{R}$ denote the state variable of DRR unit i, and $u_i(t)$ denote the commanded power reduction (or increase) from the baseline power consumption profile. The dynamics of individual DRR units are described by the virtual battery model proposed in [19], [18]:

$$\dot{x}_i(t) = -\xi_i x_i(t) - u_i(t),\ i = 1, \cdots, N, \tag{4}$$

$$-C_i \leq x_i(t) \leq C_i, \qquad -\underline{n}_i \leq u(t) \leq \bar{n}_i, \tag{5}$$

where the value of the state $x_i(t)$ is analogous to the state-of-charge of a battery; $\xi_i > 0$ is a constant representing the dissipation rate; and $C_i > 0$, $\underline{n}_i > 0$, $\bar{n}_i > 0$ are constants respectively representing the energy capacity, lower power limit, and upper power limit. Note that although not common, $u_i(t)$ can be negative so as to capture the case in which the unit increases its consumption as commanded. In the remainder, we will focus on the power-reduction case (i.e., $u_i(t) > 0$); the same procedure applies for the power-increase case.

2) Control: Let $r(t)$ be the amount of power requirement that the DRR aggregation system needs to follow. Assume that the unit state variables i.e., the $x_i(t)$'s are measured and available to the aggregator. The power reduction command for each unit is determined as

$$u_i(t) = \frac{C_i}{\sum_j C_j} r(t) + \frac{C_i \sum_{j=1}^N \xi_j x_j(t)}{\sum_{j=1}^N C_j} - \xi_i x_i(t). \quad (6)$$

Then, we have that $\sum_{i=1}^N u_i(t) = r(t)$, and

$$\dot{x}_i(t) = -\xi_i x_i(t) - u_i(t) = -\frac{C_i}{\sum_{j=1}^N C_j}(r(t) + \sum_{j=1}^N \xi_j x_j(t)). \quad (7)$$

By dividing C_i on both sides of (7), we can show that the variable $\frac{x_i(t)}{C_i}$ evolves according to the same dynamic equation for $i = 1, \cdots, N$:

$$\dot{z}(t) = -\frac{r(t)}{\sum_{j=1}^N C_j} - \frac{\sum_{j=1}^N \xi_j C_j}{\sum_{j=1}^N C_j} z(t),$$

where $z(t) := \frac{x_i(t)}{C_i}$. The constraints on $z(t)$ are $-1 \leq z(t) \leq 1$; thus, we refer to $z(t)$ as the normalized state variable.

The control algorithm in (6) make that participant units reach their energy limit at the same time; moreover, the "state-of-charge" variables are proportional to their energy limits at all times, i.e., $\frac{x_i(t)}{C_i} = \frac{x_j(t)}{C_j}$, $t \in \mathbb{R}^+$, $\forall i, j$, given that $\frac{x_i(0)}{C_i} = \frac{x_j(0)}{C_j}$, $t \in \mathbb{R}^+$, $\forall i, j$. This control algorithm also reduces the original N-th order model to a first-order model; we will show that this will significantly reduce the computation complexity. Moreover, we will show that the capacity assessment results do not depend on the control algorithm under some reasonable assumptions. Also note that since this work focuses on the capacity characteristics of DRR aggregation systems, we set $r(t)$ to be constant and drop the dependence on t in subsequent developments. Lastly, we believe that assuming zero initial condition, i.e., $x_i(0) = 0, \forall i$, is reasonable in most cases, as we are concerned with obtaining the capacity characteristics, which are relatively long-term metrics. For instance, when this information is used by the aggregator to participate in the energy and capacity markets, the aggregators are interested in how much energy their participant units can provide, starting from normal initial conditions (i.e., $x_i(0) = 0$). In addition, even if the state initial values of certain units are not zero (the set of these units is denoted as $\mathcal{K}$), i.e., $x_k(0) \neq 0, \forall k \in \mathcal{K}$, we can always define a new state variable, $x'_k(t) = x_k(t) - x_k(0)$, so that $x'_k(0) = 0$. By defining a virtual command variable $u'_k(t) = u_k(t) + \xi_k x_k(0)$, the dynamics of $x'_k(t)$ can be described as

$$\dot{x'}_k(t) = -\xi_k x'_k(t) - u'_k(t),$$

$$-C_k - x_k(0) \leq x'_k(t) \leq C_k - x_k(0),$$

$$-\underline{n}_k + \xi_k x_k(0) \leq u'_k(t) \leq \bar{n}_k + \xi_k x_k(0).$$

The value of C'_k is determined based on the sign of $r(t)$, i.e., if $r(t)$ is positive, $C'_k = C_k + x_k(0)$; otherwise, $C'_k = C_k - x_k(0)$. Let $r'(t) = r(t) + \sum_{k \in \mathcal{K}} \xi_k x_k(0)$, then the same control algorithm as shown in (6) still applies, i.e.,

$$\begin{aligned} u_i(t) = & \frac{C_i}{\sum_{j \notin \mathcal{K}} C_j + \sum_{k \in \mathcal{K}} C'_k} r'(t) \\ & + \frac{C_i(\sum_{j \notin \mathcal{K}} \xi_j x_j(t) + \sum_{k \in \mathcal{K}} \xi_k x'_k(t))}{\sum_{j \notin \mathcal{K}} C_j + \sum_{k \in \mathcal{K}} C'_k} - \xi_i x_i(t), \forall i \notin \mathcal{K}, \end{aligned}$$

except that

$$\begin{aligned} u_k(t) = & u'_k(t) - \xi_k x_k(0) = \frac{C_k}{\sum_{j \notin \mathcal{K}} C_j + \sum_{k \in \mathcal{K}} C'_k} r'(t) \\ & + \frac{C_k(\sum_{j \notin \mathcal{K}} \xi_j x_j(t) + \sum_{k \in \mathcal{K}} \xi_k x'_k(t))}{\sum_{j \notin \mathcal{K}} C_j + \sum_{k \in \mathcal{K}} C'_k} \\ & - \xi_i x_i(t) - \xi_k x_k(0), \forall k \in \mathcal{K}, \end{aligned}$$

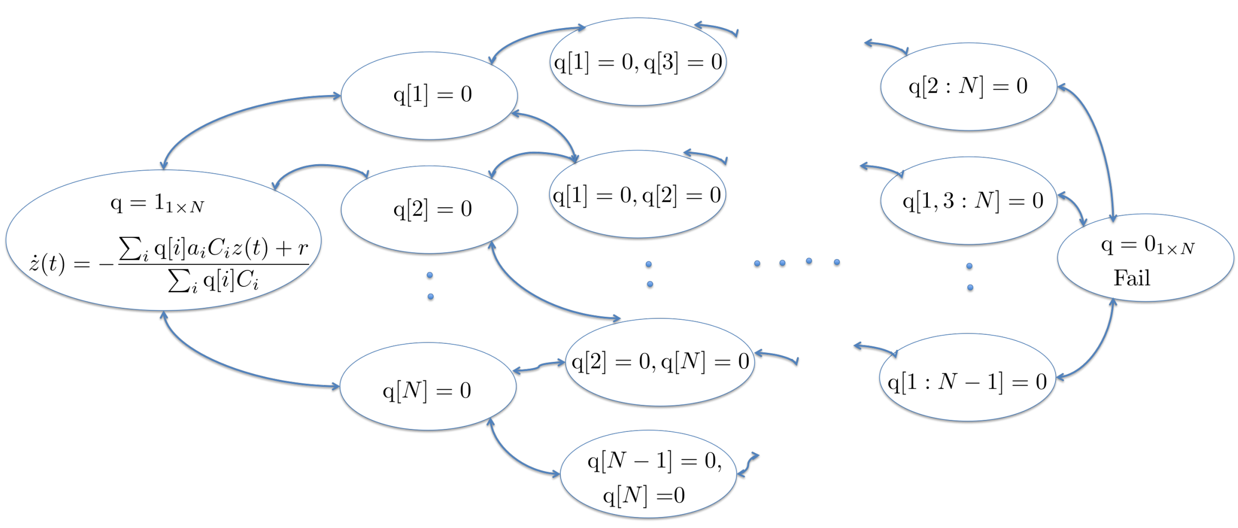


Figure 2: SHS-based model. Although only dispayed for $q = 1_{1\times N}$, the evolution of $z(t)$ is governed by (8) for all modes.

and the aforementioned properties still hold. For instance, $\sum_{i\notin\mathcal{K}} u_i(t)+\sum_{k\in\mathcal{K}} u'_k(t) = r'(t)$; therefore, $\sum_{i=1}^{N} u_i = r(t)$. Note that for $k \in \mathcal{K}$, $z(t) = \frac{x'_k(t)}{C'_k}$.

Example 1. Consider a DRR aggregation system with two DRR units, the parameters of which are listed in Table I. From (6), we have that

$$\begin{aligned} u_1(t) &= 0.4r - 0.216x_1(t) + 0.288x_2(t), \\ u_2(t) &= 0.6r + 0.216x_1(t) - 0.288x_2(t). \end{aligned}$$

Then, the evolution of the unit state variables is as follows:

$$\begin{aligned} \dot{x}_1(t) &= -0.144x_1(t) - 0.288x_2(t) - 0.4r, \\ \dot{x}_2(t) &= -0.216x_1(t) - 0.432x_2(t) - 0.6r. \end{aligned}$$

Finally, assuming $x_1(0) = x_2(0) = 0$, we have that $\frac{x_1(t)}{C_1} = \frac{x_2(t)}{C_2}$; then, by defining $z(t) := \frac{x_1(t)}{C_1}$, we obtain

$$\dot{z}(t) = -0.576z(t) - 0.02r.$$

Thus, the dynamics of this DRR aggregation system is captured by a first-order system model. ■

B. SHS-based DRR Aggregation System Model

Let $\eta_i(t)$ denote an indicator variable that takes value 1 if unit i is functional at time t, and 0 otherwise. We set $\mathcal{Q} = \{0,1\}^N$, and define $q(t) = (\eta_i(t))_{i=1,\cdots,N}$ to indicate the DRR aggregation system mode. Then, the dynamics of the DRR aggregation system can be captured by the SHS model displayed in Fig. 2, where the mode is indexed by $\mathrm{q} \in \{0,1\}^N$, and the default value of the element in q is 1 if not specified. The dynamics of the normalized variable z in each mode evolves according to:

$$\dot{z}(t) = -\frac{r}{\sum_j \eta_j(t)C_j} - \frac{\sum_j \eta_j(t)\xi_j C_j}{\sum_j \eta_j(t)C_j} z(t). \tag{8}$$

Consider that unit i fails to respond at rate α_i, and it is restored back to service at rate β_i. We neglect the case where two units fail or are restored back to service at the same time. When unit i fails, the i-th element of $q(t)$ changes from 1 to 0 and the other elements of $q(t)$ remain unchanged. Therefore, the transition rate from mode $a \in \mathcal{Q}$ to mode $b \in \mathcal{Q}$ is given by

$$\lambda_{ab} = \begin{cases} \alpha_i, \text{if } a[i] = 1,\ b[i] = 0,\ a[j] = b[j],\ \forall j \neq i, \\ \beta_i, \text{if } a \notin \mathcal{F}, a[i] = 0,\ b[i] = 1,\ a[j] = b[j],\ \forall j \neq i, \\ 0,\ \text{otherwise}, \end{cases} \tag{9}$$

where $a[i]$ denotes the i-th element of a. During a transition, $z(t)$ remains the same, i.e., the transition reset map in this case is

$$\phi_{ab}(q, z) = (b, z). \tag{10}$$

Note that the modes in which the remaining operational units cannot provide enough power as requested due to DRR unit power reduction constraints, i.e., $\sum_{i=1}^{N} \mathrm{q}[i]\bar{n}_i < r$, are referred to as "fail" modes. The set of "fail" modes is denoted by $\mathcal{F}$ and can be aggregated into one state (see, e.g., [31]). In Fig. 2,

ξ_1 [h^{-1}]	ξ_2 [h^{-1}]	C_1 [kWh]	C_2 [kWh]
0.36	0.72	20	30
α_1 [h^{-1}]	α_2 [h^{-1}]	β_1 [h^{-1}]	β_2 [h^{-1}]
0.5	0.7	6	3

Table I: Parameters of DRR units in Example 1.

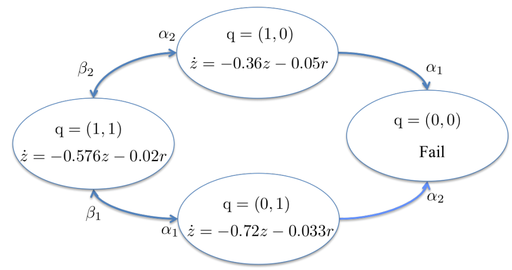


Figure 3: SHS-based model for two-unit system.

we assume that there are no constraints on the u_i's; therefore, only the mode for which all the units fail, is the "fail" mode.

Given a signal r, let $T^{(r)}$ denote the time at which $z(t)$ hits the limit -1, i.e., $z(T^{(r)}) = -1$, or the system jumps to a "fail" mode; $T^{(r)}$ is a random variable and its probability distribution will be determined in Section IV. We claim that from time $T^{(r)}$ onwards, the system can no longer provide the requested power. To substantiate this claim, let $\mathcal{R}(t)$ denote the set of units that have failed, but have been repaired before time t. Then, if the failures are permanent (i.e., $\beta = 0$), $\mathcal{R}(T^{(r)}) = \emptyset$, meaning that the DRR aggregation system cannot provide any additional energy. If failures are repairable (i.e., $\beta \neq 0$), at time $T^{(r)}$, there may be some additional energy that the units in $\mathcal{R}(T^{(r)})$ can still provide; next, we argue that this amount of energy can be neglected. First, repairable failures (e.g., command signals fail to be transmitted due to communication package drops) can usually be fixed in a fast manner (e.g., by resending command signals). Therefore, $x_j(T^{(r)})$ will be very close to $-C_j$, $\forall j \in \mathcal{R}(T^{(r)})$, meaning that the total amount of energy that the DRR units in the set $\mathcal{R}(T^{(r)})$ can provide at time $T^{(r)}$ is small. Second, the cardinality of $\mathcal{R}(T^{(r)})$ is usually relatively small as compared to N; it is then very likely that $\sum_{j\in\mathcal{R}(T^{(r)})} \bar{n}_j$ is smaller than r, meaning that the system still cannot provide enough power as requested at $T^{(r)}$.

Example 2. Consider the DRR aggregation system discussed in Example 1; unit failure and repair rates are listed in Table I. The transition diagram for the SHS-based model for this system is displayed in Fig. 3. When the aggregator participates in energy and capacity markets, it needs to know how much power the DRRs can provide with high confidence, as if the submitted offer is cleared but the DRRs fail to provide the requested power, the aggregator may face a significant fine. One possibility is to use an $N-1$ type criterion, i.e., the aggregator may submit the energy and power information based on the capability of only one unit, in this case. However, this approach is too conservative as: (i) failure rates are relatively small and the likelihood that failures occur early in the period in which the DRR aggregation system is providing service is small; and (ii) even if a failure occurs, repairs may be initiated. Again, this discussion shows the importance of developing probabilistic models in evaluating the capacity of DRR aggregation systems. ■

IV. Reliability Assessment

In this section, we apply the SHS analysis method presented in Section II to the DRR aggregation system model in Fig. 2. We first obtain the values of system state moments, based on which we can evaluate the probability that the aggregation system can successfully follow the power requirement signal r for a period of time τ, denoted by $P(r,\tau)$. By varying the values of r and τ, $P(r,\tau)$ forms a probability-capacity-duration contour. Then, by setting this probability to a desired confidence level, we can obtain the corresponding capacity-duration curves.

A. Evolution of State Statistics

Consider the DRR aggregation system SHS-based model described in Fig. 2. Applying the method presented in Section II, we can write the test function and conditional moments as follows:

$$\psi_i^{(m)}(q,z) := \delta_i(q) z^m = \begin{cases} z^m, & q = i \\ 0, & q \neq i \end{cases}, \forall i \in \mathcal{Q}.$$

$$\mu_i^{(m)}(t) := \mathbb{E}[\psi_i^{(m)}(Q(t), Z(t))].$$

Let $\mathcal{O}_i := \{j \in \mathcal{Q} : \lambda_{ij} \neq 0\}$ denote the set of modes to which transitions from mode i occur, and let $\mathcal{I}_i := \{j \in \mathcal{Q} : \lambda_{ji} \neq 0\}$ denote the set of modes from which transitions to modes i occur. Then, we can write the following set of ODEs that governs the evolution of the $\mu_i^{(m)}$'s:

$$\begin{aligned} \dot{\mu}_i^{(0)} =& -\sum_{j\in\mathcal{O}_i} \lambda_{ij}\mu_i^{(0)} + \sum_{j\in\mathcal{I}_i} \lambda_{ji}\mu_j^{(0)},\ i \notin \mathcal{F}; \\ \dot{\mu}_i^{(m)} =& -\frac{mr}{\sum_{k=1}^N i[k]C_k}\mu_i^{(m-1)} - (\frac{m\sum_{k=1}^N i[k]\xi_k C_k}{\sum_{k=1}^N i[k]C_k} + \\ & \sum_{j\in\mathcal{O}_i}\lambda_{ij})\mu_i^{(m)} + \sum_{j\in\mathcal{I}_i}\lambda_{ji}\mu_j^{(m)},\ i \notin \mathcal{F}, m \geq 1. \quad (11) \end{aligned}$$

Example 3. Consider the two-unit DRR aggregation system discussed in Example 1; then, the specific expression for (11) is given by

$$\begin{aligned} \dot{\mu}_{(1,1)}^{(0)} =& -(\alpha_1+\alpha_2)\mu_{(1,1)}^{(0)} + \beta_1\mu_{(0,1)}^{(0)} + \beta_2\mu_{(1,0)}^{(0)}, \\ \dot{\mu}_{(0,1)}^{(0)} =& -(\beta_1+\alpha_2)\mu_{(0,1)}^{(0)} + \alpha_1\mu_{(1,1)}^{(0)}, \\ \dot{\mu}_{(1,0)} =& -(\beta_2+\alpha_1)\mu_{(1,0)} + \alpha_2\mu_{(1,1)}, \\ \dot{\mu}_{(1,1)}^{(m)} =& -0.576m\mu_{(1,1)}^{(m-1)} - (0.02m+\alpha_1+\alpha_2)\mu_{(1,1)}^{(m)} \\ & + \beta_1\mu_{(0,1)}^{(m)} + \beta_2\mu_{(1,0)}^{(m)}, m > 1, \\ \dot{\mu}_{(0,1)}^{(m)} =& -0.72\mu_{(0,1)}^{(m-1)} - (0.05m+\beta_1+\alpha_2)\mu_{(0,1)}^{(m)} + \alpha_1\mu_{(1,1)}^{(m)}, \\ \dot{\mu}_{(1,0)}^{(m)} =& -0.36\mu_{(1,0)}^{(m-1)} - (0.033m+\beta_2+\alpha_1)\mu_{(0,1)}^{(m)} + \alpha_2\mu_{(1,1)}^{(m)}, \end{aligned}$$

this completes the example. ■

B. Reliability Assessment

Define the event E_1 that the DRR aggregation system can still meet the power requirement at time τ, i.e., $E_1 = \{T_r > \tau\}$. Also, define the event E_2 that the system nominal state z is still within constraints, i.e., $E_2 = \{Z(\tau) > -1 \cap Q(\tau) \notin \mathcal{F}\}$. We argued in Section III that events E_1 and E_2 are equivalent; therefore, we have that

$$\begin{aligned} P(r,\tau) =& \Pr\{T_r > \tau\} = \Pr\{Z(\tau) > -1 \cap Q(\tau) \notin \mathcal{F}\} \\ =& \sum_{i\notin\mathcal{F}} \Pr\{Z(\tau) > -1 | Q(\tau) = i\}\Pr\{Q(\tau) = i\}, \quad (12) \end{aligned}$$

where $\Pr\{Q(\tau) = i\} = \mu_i^{(0)}(\tau)$, and $\Pr\{Z(\tau) > -1 | Q(\tau) = i\}$ can be approximated by using conditional moments. For example, utilizing Cantelli's inequality [35], $\Pr\{Z(\tau) > -1 | Q(\tau) = i\}$ can be lower-bounded using the conditional first and second moments as follows:

$$\Pr\{Z(\tau) > -1 | Q(\tau) = i\} > 1 - \frac{\sigma_i^2(\tau)}{\sigma_i^2(\tau) + (-1-\mu_i(\tau))^2}, \quad (13)$$

where $\mu_i(\tau)$ and $\sigma_i^2(\tau)$ are the conditional mean and variance of $Z(\tau)$ given $Q(\tau) = i$, which can be obtained as $\mu_i(\tau) = \frac{\mu_i^{(1)}(\tau)}{\mu_i^{(0)}(\tau)}$ and $\sigma_i^2(\tau) = \frac{\mu_i^{(2)}(\tau)}{\mu_i^{(0)}(\tau)} - [\frac{\mu_i^{(1)}(\tau)}{\mu_i^{(0)}(\tau)}]^2$, respectively. Alternatively, we have that

$$\begin{aligned} \Pr\{T_r > \tau\} =& \Pr\{Z(\tau) > -1 \cap Q(\tau) \notin \mathcal{F}\} \quad (14) \\ =& \Pr\{Z(\tau) > -1 | Q(\tau) \notin \mathcal{F}\}\Pr\{Q(\tau) \notin \mathcal{F}\}, \end{aligned}$$

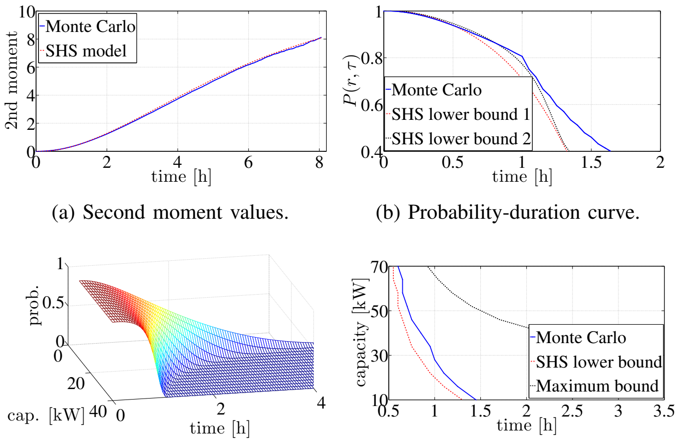
where $\Pr\{Q(\tau) \notin \mathcal{F}\} = \sum_{i\notin\mathcal{F}} \mu_i^{(0)}(\tau)$. Similarly, by utilizing Cantelli's inequality, we have that

$$\Pr\{Z(\tau) > -1 | Q(\tau) \notin \mathcal{F}\} > 1 - \frac{\sigma_{\bar{M}}^2(\tau)}{\sigma_{\bar{M}}^2(\tau) + (-1-\mu_{\bar{M}}(\tau))^2}, \quad (15)$$

where $\mu_{\bar{\mathcal{F}}}(\tau)$ and $\sigma_{\bar{\mathcal{F}}}^2(\tau)$ are the conditional mean and variance of $Z(\tau)$ given $Q(\tau) \notin \mathcal{F}$, which can be obtained as $\mu_{\bar{\mathcal{F}}}(\tau) = \frac{\sum_{i\notin s\mathcal{F}} \mu_i^{(1)}(\tau)}{\sum_{i\notin s\mathcal{F}} \mu_i^{(0)}(\tau)}$ and $\sigma_i^2 = \frac{\sum_{i\notin s\mathcal{F}} \mu_i^{(2)}(\tau)}{\sum_{i\notin s\mathcal{F}} \mu_i^{(0)}(\tau)} - [\frac{\sum_{i\notin s\mathcal{F}} \mu_i^{(1)}(\tau)}{\sum_{i\notin s\mathcal{F}} \mu_i^{(0)}(\tau)}]^2$, respectively. It will be shown in Section VI that the bounds on $\Pr\{T_r > \tau\}$ obtained using both (12) and (14) are tight.

By varying the value of r, we can obtain a probability-capacity-duration contour. Then, by setting the probability value to the desired confidence level, we can obtain the corresponding capacity-duration curve.

Example 4. Consider again the two-unit system discussed in Example 3. In Fig. 4a, we show the values of the second moments obtained through our proposed method and Monte Carlo simulations; as one can see, both methods yield very similar results. For $r = 24$ kW, the values of $P(r,\tau)$ estimated using our proposed method and Monte Carlo simulations are depicted in Fig. 4b. By varying the value of r, the probability-capacity-duration contour shown in Fig. 4c can be obtained. By setting the probability value to 0.6, we can obtain the capacity-duration curve shown in Fig. 4d. We also calculated the capacity-duration curve assuming there are no failures, which is given by $r = \frac{\xi_1 C_1 + \xi_2 C_2}{1 - e^{-(\xi_1 C_1 + \xi_2 C_2)/(C_1+C_2)\tau}}$. The difference between this curve and the curve with reliability considered represents the impact of failures on the system



(a) Second moment values.

(b) Probability-duration curve.

(c) Probability-capacity-duration contour.

(d) Capacity-duration curve.

Figure 4: Two-unit DRR aggregation system.

capacity. The proposed method approximately takes the same time to complete as that of executing one Monte Carlo run. Therefore, the computational time of the proposed method is significantly reduced compared with the Monte Carlo method that usually requires a considerable number of runs. ■

V. Generalizations

In this section, we first show that the proposed method is also applicable to DRR aggregation systems with control mechanisms that are different from the one proposed in Section III. Next, we show that the assessment method can be generalized to DRR aggregation systems with both battery-like DRR units and conventional DRR units.

A. DRR Aggregation Systems Under Alternative Controls

The proposed method developed so far is based on the control/allocation algorithm in (6); however, we conjecture that the system capacity characteristics do not vary much with other control algorithms. Moreover, we can prove that if we neglect the dissipation term, i.e., $\xi_i = 0$ as well as repair events, i.e., $\beta_i = 0$, and assume that the failure rate is the same for all participant units, i.e., $\alpha_i = \alpha_j, \forall i, j \in \{1, \cdots, N\}$, the system capacity characteristics do not depend on the allocation mechanism. The statement is intuitive for a system with a collection of battery-like DRRs, and no failures considered. The proof of this statement with $\alpha_i \neq 0$ is provided in the Appendix; in the next example, we verify it with a two-unit aggregation system example. Also note that our conjecture is based on the following observations. First, the dissipation rate is usually small, as concluded by the results in [19]. Second, the repair rates depend on the type of failure. In this regard, if the failures are due to device malfunction, it is reasonable to assume that the failures cannot be fixed during the time period of interest, e.g., in the order of hours; therefore, we can assume that $\beta_i = 0$. Third, the failure rates are usually small, so although they are not exactly the same, their difference can be bounded. Furthermore, for aggregation systems consisting of

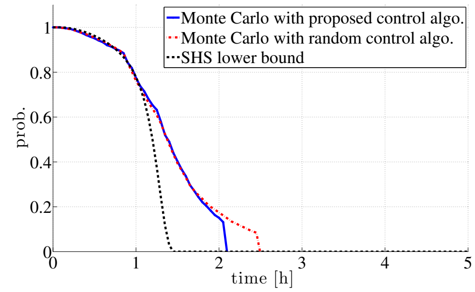


Figure 5: Comparision of two control algorithms.

homogeneous units, the failure and repair rates should be the same. On the other hand, for cases in which the assumptions above do not hold, the closeness of the capacity characteristics under different control mechanisms may vary on a case by case basis.

Example 5. Consider a two-unit DRR aggregation system. We set $\xi_i = 0$, $\beta_i = 0$, and $\alpha_i = 0.5$, $i = 1, 2$. The other parameters are the same as those in Example 1. For $r = 24$ kW, the values of $P(r, \tau)$ using the control algorithm in (6) and another control algorithm in which the aggregator randomly allocates the power requirement among the operational DRR units, are obtained via Monte Carlo simulations, respectively. The results for both control algorithms are depicted in Fig. 5, where one can see a very close matching. ■

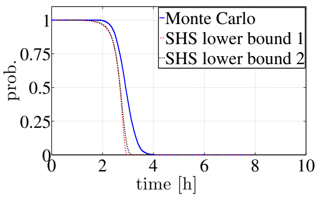
B. DRR Aggregation Systems with Different Types of DRRs

It is likely that a DRR aggregation system not only includes battery-like DRR units but also conventional DRR units with no energy limits. Following the procedure described in Section IV, we can obtain the probability-capacity-duration contour of the battery-like DRRs in the system. By setting the duration time to a fixed value, τ, we can obtain the corresponding probability-capacity curve. Denote by $R^{(\tau)}$ the maximum power that the DRR aggregation system can provide for a period of time τ. Then, the probability-capacity curve is the complementary cumulative function of $R^{(\tau)}$, denoted by $F_{R^{(\tau)}}(r) := \Pr\{R^{(\tau)} > r\}$. Denote by D the power that can be provided by the conventional DRRs, and assume we have the probability density function of D, denoted by $f_D(d) := \Pr\{D = d\}$. Let $S^{(\tau)}$ be the total power that can be provided by the total DRR aggregation system; then we have that (see, e.g., [36])

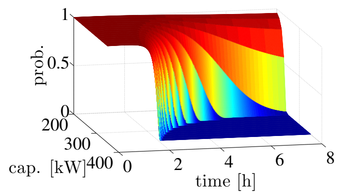
$$\begin{aligned}\Pr\{S^{(\tau)} \geq s\} &= \Pr\{R^{(\tau)} + D \geq s\}\\ &= \int_{-\infty}^{\infty} f_D(v) \cdot F_{R^{(\tau)}}(s - v) dv = f_D \circ F_{R^{(\tau)}}(s),\end{aligned}$$

where $\circ$ indicates the convolution operator. For instance, if the conventional DRR units can provide an amount of power a with probability of p and zero power with probability of $(1 - p)$, we have that $f_D(d) = p\delta(d - a) + (1 - p)\delta(d)$, and

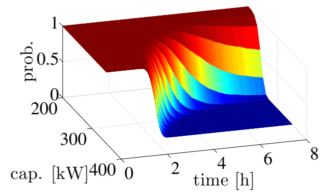
$$\Pr\{S^{(\tau)} > s\} = (1 - p)F_{R^{(\tau)}}(s) + pF_{R^{(\tau)}}(s - a).$$



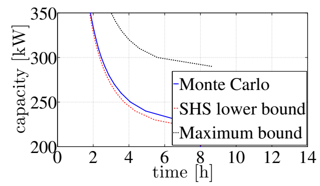
(a) Probability-duration curve.



(b) Probability-capacity-duration contour by proposed method.



(c) Probability-capacity-duration contour by Monte Carlo method.



(d) Capacity-duration curve.

Figure 6: Twenty-unit DRR aggregation system.

Then, by varying the value of τ, we can obtain the probability-capacity-duration curve for the complete DRR aggregation system.

VI. Case Studies

In this section, we analyze two DRR aggregation systems with 20 units and 1000 units in order to demonstrate the scalability of the proposed method. Conservative estimates of $P(r,\tau)$ are obtained using (12)-(15). Monte Carlo simulations are also conducted to estimate the value of $P(r,\tau)$. The results obtained through both methods are shown to be close; however, the computational time of our proposed method is much less than that of the Monte Carlo method. A way to address the explosion of the SHS state space dimension when analyzing large DRR aggregation systems is also discussed. The computation is performed on a PC with a 3.40 GHz Intel Xeon CPU processor and 8 GB memory in the MATLAB environment.

A. A DRR Aggregation System with 20 Units

Consider a DRR aggregation system that consists of 20 units. Ten of them are identical and their parameters are the same as those of unit 1 in Example 1; let $\mathcal{A}$ denote the set of these units. The parameters of the other ten units are the same as those of unit 2 in Example 1; let $\mathcal{B}$ denote the set of these units. Since there are 20 units, there should be 2^{20} discrete modes in the SHS model. However, since the units in each set are identical, we can aggregate the modes with same number of operational units in each set as one mode; then, an SHS model with 121 modes results. Assume there are no constraints on the u_i's; then only the last mode where no units are functional is a "fail" mode.

For $r = 300$ kW, the lower bound on $P(r,\tau)$, depicted in Fig. 6a, can be obtained using (12) and (14). Another way to estimate the value of $P(r,\tau)$ is to run 4000 Monte Carlo simulations and calculate the percentage of the resulting sample paths for which the state variable $z(\tau)$ is still within limits at time τ (i.e., $-1 < z(\tau) < 1$); the result is also depicted in Fig. 6a. By varying the value of r, we can obtain the probability-capacity-duration contour shown in Figs. 6b and 6c, respectively obtained by our proposed method, and by using Monte Carlo simulations. Then, by setting the probability to a desired confidence level (for example, 95%), we can obtain the capacity-duration curve shown in Fig. 6d. Note that the capacity-duration curve obtained using the bounds on $P(r,\tau)$ closely matches that obtained by Monte Carlo simulation.

We also calculated the capacity-duration curve assuming there are no failures, which is given by $r = \frac{10\xi_1 C_1 + 10\xi_2 C_2}{1-e^{-(10\xi_1 C_1+10\xi_2 C_2)/(10C_1+10C_2)\tau}}$. The difference between this curve and the curve with failures considered captures the impact on the system capacity of random failures. Also note that the occupational probability values of the states, for which large number of units fail, are almost zero; thus, we can truncate those states without affecting the results. For this case, the results obtained with a truncated SHS model having 37 modes are the same as those presented above as the occupational probabilities of the truncated modes are zero during the considered time period.

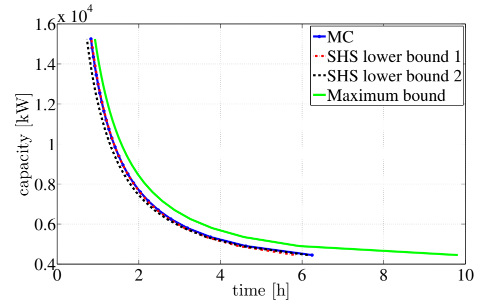


Figure 7: Capacity-duration curve for 1000-unit system.

B. A DRR Aggregation System with 1000 Identical Units

Consider a system with 1000 identical units, the parameters of which are the same as those of unit 1 in Example 1. After aggregating the modes with same number of functional units as one, we can develop an SHS model with 1001 states to describe this system. Alternatively, since the probability that a large number of units may fail is very small (which can be quantified by the occupational probability), we can truncate the 1001-mode model and obtain a 100-mode model by neglecting the modes for which more than 100 units fail. Then, following the same procedure described above, we can obtain the capacity-duration curve with 95% confidence, which is shown in Fig. 7. Note that when numerically solving the ODEs given in (11), an implicit integration method should be used instead of an explicit method to avoid numerical stability issues. The curves obtained by our proposed method closely match those obtained by Monte Carlo simulations. The computational time of the Monte Carlo simulation method is 4.13 hours, while our method takes 11.4 seconds.

VII. Concluding Remarks

In this paper, we presented a method to evaluate the capacity-duration characteristics of DRR aggregation systems consisting of battery-like DRRs with a desired confidence level. We also showed that this method is applicable to DRR aggregation systems that include other types of DRRs and control algorithms. The computation time of the proposed method is reduced significantly as compared to Monte Carlo simulations, which allows aggregators to perform analyses under different scenarios in a timely manner. The effectiveness and scalability of the proposed method was illustrated through several case studies in which we show how to address the explosion in the dimension of the SHS model state space. Future work includes investigating more special cases, including non-constant transition rate cases, and cases in which the parameters make the capacity characteristics vary significantly with different control mechanisms.

Appendix

Lemma A.1. For the DRR aggregation system described by the SHS model in (8)-(10), if $\xi_i = 0$, $\beta_i = 0$, and $\alpha_i = \alpha_j$, $\forall i, j$, the system capacity characteristics do not depend on the allocation mechanism.

Proof. For a given r, let $Q^{(r)}$ indicate the total energy provided by the DRR aggregation system; then,

$$T^{(r)} = \frac{Q^{(r)}}{r}.$$

If no failure has occurred, then the total energy is $\sum_{i=1}^{N} C_i$. Let L_j indicate the energy loss due to the j-th failure that happens in the system. For instance, no matter which unit first fails, L_1 indicates the energy loss due to this first failure that happens in the system. Then, we have

$$Q^{(r)} = \sum_{i=1}^{N} C_i - \sum_{k=1}^{M} L_k, \tag{16}$$

where M indicates the total number of failures during the time that the DRR aggregation system can meet the power request. Next, we show that L_j is independent of the control algorithm used by the aggregator; therefore, the distribution of $T^{(r)}$ is independent of the control algorithm. Take L_1 as an example; the key point is that any control mechanism guarantees that $\sum_{i=1}^{N} u_i(t) = r$. Let $p_{1,i}$ be the probability that unit i is the first unit that fails. As all the units have the same failure rate, $p_{1,i} = \frac{1}{N}$. Let T_1 be the time that the first unit fails; then, we have that

$$\begin{aligned} L_1 &= \sum_{i=1}^{N} p_{1,i}(C_i - \int_0^{T_1} u_i(t)dt)) = \frac{1}{N}(\sum_{i=1}^{N} C_i - \int_0^{T_1} \sum_{i=1}^{N} u_i(t)dt) \\ &= \frac{1}{N}(\sum_{i=1}^{N} C_i - \int_0^{T_1} rdt) = \frac{1}{N}\sum_{i=1}^{N} C_i - \frac{r}{N}T_1, \end{aligned}$$

which is independent of $u_i(t)$'s. Similarly, we have that

$$\begin{aligned} L_2 =& \frac{1}{N}\sum_{j=1}^{N}(\frac{1}{n-1}\sum_{i=1, i\neq j}^{N}(C_i - \int_0^{T_1} u_i(t)dt - \int_{T_1}^{T_2} u_i(t)dt)) \\ =& \frac{1}{N(N-1)}(\sum_{j=1}^{N}\sum_{i=1, i\neq j}^{N} C_i - \sum_{i=1, i\neq j}^{N}\int_0^{T_1}\sum_{j=1}^{N} u_i(t)dt \\ & - \sum_{j=1}^{N}\int_{T_1}^{T_2}\sum_{i=1, i\neq j}^{N} u_i(t)dt \\ =& \frac{1}{N}\sum_{j=1}^{N} C_i - \frac{r}{N}T_1 - \frac{r}{N-1}(T_2 - T_1). \end{aligned}$$

Then, by setting $T_0 = 0$, L_k is given by

$$L_k = \frac{1}{N}\sum_{i=1}^{N} C_i - \sum_{j=1}^{k}\frac{1}{N+1-j}(T_k - T_{k-1}),$$

which again is independent of the control algorithm. Therefore, the distributions of $Q^{(r)}$ and $T^{(r)}$ are independent of the control algorithm. ■

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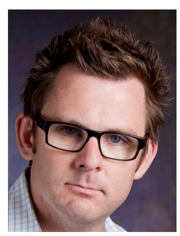

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