Quickest Line Outage Detection and Identification: Measurement Placement and System Partitioning

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***Abstract*—This paper addresses the problem of detecting and identifying transmission line outages in near real-time. To this end, we utilize the statistical properties of electricity generation and demand, and apply a quickest change detection algorithm to measurements of voltage phase angles collected using phasor measurement units (PMUs). We propose a procedure for optimally selecting the buses for PMU placement with the goal of minimizing the worst case line outage detection delay. In addition, we propose a method to optimally partition the power system into several areas such that the algorithm for line outage detection can be applied in parallel to each area for faster detection. We illustrate the proposed ideas via case studies involving the IEEE 30-bus test system.**

I. Introduction

Timely detection of line outages in a power system is crucial for maintaining operational reliability as it plays a role in ensuring the correct operation of many decision-making tools. In this regard, many of the current methods for online power system monitoring rely on a system model that is obtained offline, which can be inaccurate due to bad historical or telemetry data. Such inaccuracies have been a contributing factor in many recent blackouts. For example, in the 2011 San Diego blackout, operators were unable to determine overloaded lines because the network model was not up to date [1]. This lack of situational awareness limited the ability of the operators to identify and prevent the next critical contingency, and led to a cascading failure. Similarly, during the 2003 Northeast blackout, operators failed to initiate the correct remedial schemes because they had an inaccurate model of the power system and could not identify the loss of key transmission elements [2]. These blackouts highlight the importance of developing online measurement-based techniques to detect and identify system topological changes that arise from line outages.

Our work builds on the results reported in [3]–[5], where the authors developed methods for line outage detection and identification, based on the theory of quickest change detection (QCD). In these methods, the incremental changes in real power injections at load buses are modeled as independent random variables. Then, the probability distribution of such incremental changes is mapped to that of the incremental changes in voltage phase angles via a linear transformation obtained from the power flow equations. When a line outage occurs, the probability distribution of the incremental changes in the voltage phase angles changes abruptly. The objective is to detect a change in this probability distribution after the occurrence of a line outage as quickly as possible while maintaining a desired false alarm rate.

In this paper, we consider the problem of identifying a subset of buses for placing the PMUs to be utilized by the QCD algorithms proposed in [3]–[5]. We propose an optimal PMU placement strategy in the sense of minimizing the worst-case line outage detection delay, for a fixed false alarm rate. We formulate this as an integer programming problem and present a greedy algorithm for solving it, and show that for our test cases, optimally deploying PMUs at approximately one third of the system buses is adequate for timely line outage detection. Next, we achieve a faster detection time by proposing a method to partition the power system into smaller subsystems so that the proposed QCD-based detection algorithms can be applied to all the areas concurrently. The optimality criteria considered for the partitioning algorithm include balancing the number of lines within each area, minimizing the number of tie-lines between areas, and minimizing the worst-case detection delay. We then propose a method based on the Kernighan-Lin algorithm [6] for solving this partitioning problem. Lastly, we improve the algorithm proposed in [3] by using the so-called governor power flow model (see, e.g., [7]), which more realistically represents the actual behavior of the power system.

Early approaches for topological change detection include algorithms based on state estimation [8], [9], and rule-based algorithms that mimic system operator decisions [10]. More recent proposed methods exploit the fast sampling of voltage magnitudes and phases provided by PMUs [11]–[13]. In terms of the PMU placement problem, most of the research has been focused on achieving network observability with minimum number of PMUs; on the other hand, the objective of our research is to find the optimal PMU placement for quickly detecting network topological changes. Heuristic techniques for determining optimal placement include simulated annealing, nondominated sorting genetic algorithms, and particle swarm methods [14]–[17].

The remainder of this paper is organized as follows. Section II describes the model of the power system adopted in this work, and introduces the statistics describing the voltage phase angles. Section III outlines the proposed QCD-based line outage identification algorithm. In Section IV, we present an algorithm for optimal PMU placement and a system partitioning scheme so that the QCD-based algorithm can be applied concurrently to each area of the partitioned system. Section V illustrates the proposed ideas via numerical case studies on the IEEE 30-bus test system. Finally, Section VI provides the concluding remarks and directions for future work.

II. SYSTEM MODEL

We represent the power system network by a graph consisting of N nodes and L edges, corresponding to buses and transmission lines, respectively. The set of buses is denoted by $\mathcal{V} = \{1, \ldots, N\}$, and the set of transmission lines is denoted by $\mathcal{E}$, where for $m, n \in \mathcal{V}$, $(m, n) \in \mathcal{E}$ if there exists a transmission line between buses m and n.

At time t, let $V_i(t)$ and $\theta_i(t)$ respectively denote the voltage magnitude and phase angle at bus i, and let $P_i(t)$ and $Q_i(t)$ respectively denote the net active and reactive power injections at bus i. Then, the quasi-steady-state behavior of the system can be described by the power flow equations, which for bus i can be compactly written as:

$$\begin{aligned} P_i(t) &= p_i(\theta_1(t), \ldots, \theta_N(t), V_1(t), \ldots, V_N(t)), \qquad (1)\\ Q_i(t) &= q_i(\theta_1(t), \ldots, \theta_N(t), V_1(t), \ldots, V_N(t)). \end{aligned}$$

A. Pre-outage Incremental Power Flow Model

Let $P_i[k] := P_i(k\Delta t)$ and $Q_i[k] := Q_i(k\Delta t)$, $\Delta t > 0$, $k = 0, 1, 2, \ldots$, denote the k^{th} measurement sample of active and reactive power injections into bus i. Similarly, let $V_i[k]$ and $\theta_i[k]$, $k = 0, 1, 2, \ldots$, denote bus i's k^{th} voltage magnitude and angle measurement sample. Furthermore, define variations in voltage magnitudes and phase angles between consecutive sampling times $k\Delta t$ and $(k+1)\Delta t$ as $\Delta V_i[k] := V_i[k+1] - V_i[k]$ and $\Delta\theta_i[k] := \theta_i[k+1] - \theta_i[k]$, respectively. Similarly, variations in the active and reactive power injections at bus i between two consecutive sampling times are defined as $\Delta P_i[k] = P_i[k+1] - P_i[k]$ and $\Delta Q_i[k] = Q_i[k+1] - Q_i[k]$.

We linearize (1) about $(\theta_i[k], V_i[k], P_i[k], Q_i[k])$, $i = 1, \ldots, N$ and use the DC power flow assumptions to decouple the active and reactive power flow equations. Then, the variations in the voltage phase angles can be mapped to the variations in the active power injection as:

$$\Delta P[k] \approx H_0 \Delta\theta[k], \qquad (2)$$

where $\Delta P[k]$, $\Delta\theta[k] \in \mathbb{R}^{(N-1)}$ and $H_0 \in \mathbb{R}^{(N-1)\times(N-1)}$. Note that the $N-1$ dimension of the vectors is the result of omitting the reference bus equation, which we designate to be bus 1.

In an actual power system, random fluctuations in the load drive the generator response. Therefore, in this paper, we use the so-called governor power flow model (see, e.g., [7]) to capture the system behavior, which is more realistic than the conventional power flow model where the slack bus picks up any changes in the load power demand. In the governor power flow model, at time instant k, the relation between changes in the load demand vector, $\Delta P^d[k] \in \mathbb{R}^{N_d}$, and changes in the power generation vector, $\Delta P^g[k] \in \mathbb{R}^{N_g}$, is described by

$$\Delta P^g[k] = B\Delta P^d[k], \qquad (3)$$

where B is a matrix of participation factors and $N_d + N_g = N - 1$. Let $M_0 := H_0^{-1}$. We can then substitute (3) into (2) to obtain a pre-outage relation between the changes in the voltage angles to the active power demand at the load buses as follows:

$$\begin{aligned} \Delta\theta[k] &\approx M_0 \Delta P[k] \\ &=: [M_0^1 \;\; M_0^2] \begin{bmatrix} \Delta P^g[k] \\ \Delta P^d[k] \end{bmatrix} \\ &= [M_0^1 \;\; M_0^2] \begin{bmatrix} B\Delta P^d[k] \\ \Delta P^d[k] \end{bmatrix} \\ &= (M_0^1 B + M_0^2)\Delta P^d[k] \\ &= \tilde{M}_0 \Delta P^d[k]. \qquad (4) \end{aligned}$$

B. Post-outage Incremental Power Flow Model

Now suppose a persistent outage occurs for the line (m, n) at time $t = t_f$, where $\gamma\Delta t \le t_f < (\gamma+1)\Delta t$. In addition, assume that the loss of line (m, n) does not cause islands to form in the post-event system (i.e., the underlying graph representing the internal power system remains connected).

In order to relate the post-outage $\Delta\theta[k]$ to $\Delta P^d[k]$ as in (4), we first express the change in matrix H_0 resulting from the outage as the sum of the pre-change matrix and a perturbation matrix, $\Delta H_{(m,n)}$, i.e., $H_{(m,n)} = H_0 + \Delta H_{(m,n)}$. Then, letting $M_{(m,n)} := H_{(m,n)}^{-1} = [M_{(m,n)}^1 \;\; M_{(m,n)}^2]$, and substituting into (4) for M_0 and simplifying, we obtain the post-outage relation between the changes in the voltage angles to the active power demand as:

$$\Delta\theta[k] \approx \tilde{M}_{(m,n)} \Delta P^d[k], \;\; k \ge \gamma, \qquad (5)$$

where $\tilde{M}_{(m,n)} = M_{(m,n)}^1 B + M_{(m,n)}^2$.

C. Instantaneous Change During Outage

At the time of outage, $t = t_f$, there is an instantaneous change in the mean of the voltage phase angle measurements that affects only one incremental sample, namely, $\Delta\theta[\gamma] = \theta[\gamma+1] - \theta[\gamma]$. The measurement $\theta[\gamma]$ is taken immediately prior to the outage, whereas $\theta[\gamma+1]$ is the measurement taken immediately after the outage. We model the effect of an outage in line (m, n) via a power injection of $P_{(m,n)}[\gamma]$ at bus m and $-P_{(m,n)}[\gamma]$ at bus n, where $P_{(m,n)}[\gamma]$ is the pre-outage line flow across line (m, n). Following a similar approach as [3], the relation between the incremental voltage phase angle at the instant of outage, $\Delta\theta[\gamma]$, and the variations in the real power flow can be expressed as:

$$\Delta\theta[\gamma] = M_0 \Delta P[\gamma] - P_{(m,n)}[\gamma+1] M_0 r_{(m,n)}, \qquad (6)$$

where $r_{(m,n)} \in \mathbb{R}^{N-1}$ is a vector with the $(m-1)^{\text{th}}$ entry equal to 1, the $(n-1)^{\text{th}}$ entry equal to -1, and all other entries equal to 0. Furthermore, by using the governor power flow model of (3) and substituting into (6), and simplifying, we obtain:

$$\Delta\theta[\gamma] = \tilde{M}_0 \Delta P^d[\gamma] - P_{(m,n)}[\gamma+1] M_0 r_{(m,n)}. \qquad (7)$$

D. Measurement Model

We allow for the situation where the angles are measured at only a subset of the load buses and denote this reduced measurement set by $\hat{\theta}[k]$. Suppose that there are N_d load buses and we select $p \leq N_d$ locations to deploy the PMUs. As a result, there are $\binom{N_d}{p}$ possible locations to place the PMUs. Let

$$\tilde{M} = \begin{cases} \tilde{M}_0, & \text{if } k \leq \gamma, \\ \tilde{M}_{(m,n)}, & \text{if } k > \gamma. \end{cases} \tag{8}$$

Then, the absence of a PMU at bus i corresponds to removing the i^{th} row of $\tilde{M}$. Thus, let $\hat{M} \in \mathbb{R}^{p \times N_d}$ be the matrix obtained by removing $N-p-1$ rows from $\tilde{M}$. Therefore, we can relate $\hat{M}$ to $\tilde{M}$ in (8) as follows:

$$\hat{M} = C\tilde{M}, \tag{9}$$

where $C \in \mathbb{R}^{p \times (N-1)}$ is a matrix of 1's and 0's that appropriately selects the rows of $\tilde{M}$ based on buses with PMUs. Accordingly, the increments in the phase angle can be expressed as follows:

$$\Delta\hat{\theta}[k] = \hat{M}\Delta P^d[k]. \tag{10}$$

The small variations in the active power injections at the load buses, $\Delta P^d[k]$, can be attributed to random fluctuations in electricity consumption. In this regard, we assume that the vectors, $\Delta P^d[k]$, are independent and identically distributed (i.i.d.) in time before, during, and after the outage. In addition, we assume that the entries of $\Delta P^d[k]$ are independent random variables with a joint Gaussian probability density function (p.d.f.), i.e., $\Delta P^d[k] \sim \mathcal{N}(0, \Lambda)$. Since the statistics of $\Delta P^d[k]$ in (10) are known, $\Delta P^d[k]$ is the independent variable and $\Delta\hat{\theta}[k]$ is the observation that depends on $\Delta P^d[k]$. Consequently, we have that:

$$\Delta\hat{\theta}[k] \sim \begin{cases} f_0 := \mathcal{N}(0, \hat{M}_0\Lambda\hat{M}_0^T), & \text{if } k < \gamma, \\ f^{\mu}_{(m,n)} := \mathcal{N}(-P_{(m,n)}[\gamma+1]CM_0 r_{(m,n)}, \\ \qquad\qquad \hat{M}_0\Lambda\hat{M}_0^T), & \text{if } k = \gamma, \\ f^{\sigma}_{(m,n)} := \mathcal{N}(0, \hat{M}_{(m,n)}\Lambda\hat{M}^T_{(m,n)}), & \text{if } k > \gamma. \end{cases} \tag{11}$$

It is important to note that for $\mathcal{N}\left(0, \hat{M}\Lambda\hat{M}^T\right)$ to have a nondegenerate p.d.f., the covariance matrix, $\hat{M}\Lambda\hat{M}^T$, must be full rank. We enforce this by ensuring that the number of PMUs allocated, p, is less than or equal to the number of load buses, N_d.

III. Quickest Line Outage Detection and Identification

In the setting described in Section II, the goal is to detect the change in the probability distribution of the sequence $\{\Delta\hat{\theta}[k]\}_{k\geq 1}$ (that results from the line outage) as quickly as possible while maintaining a certain level of detection accuracy (e.g., the false alarm rate). This problem is referred to as quickest change detection (QCD). Next, we provide a precise mathematical description of the QCD problem and an algorithm that we will use to detect a line outage.

The entries of voltage phase angle measurements, $\Delta\hat{\theta}[k]$, are i.i.d. in time before, during, and after the line outage. For the base case where no line outage is present, we have that $\Delta\hat{\theta}[k] \sim f_0$. At some random time, t_f, an outage occurs on line (m,n) and the p.d.f. of the sequence $\{\Delta\hat{\theta}[k]\}$ changes from f_0 to $f^{\mu}_{(m,n)}$ and then to $f^{\sigma}_{(m,n)}$. The objective is to detect this transition in the p.d.f. of $\{\Delta\hat{\theta}[k]\}$ as quickly as possible, which is equivalent to optimizing the stopping time τ on the sequence of observations. In the absence of a change, the expectation of τ, $\mathbb{E}[\tau]$, should be maximized so as to avoid false alarms. On the other hand, once a line outage occurs, we desire $\mathbb{E}[\tau]$ to be as small as possible.

A. Generalized CuSum Algorithm for Line Outage Detection

Suppose that the p.d.f.'s f_0, $f^{\mu}_{(m,n)}$, and $f^{\sigma}_{(m,n)}$ are known. From the sequence of phase angle measurements, define the CuSum statistic corresponding to line (m,n) as:

$$W_{(m,n)}[k+1] = \max\left\{0, \log\frac{f^{\mu}_{(m,n)}(\Delta\hat{\theta}[k+1])}{f_0(\Delta\hat{\theta}[k+1])}, \right. \\ \left. W_{(m,n)}[k] + \log\frac{f^{\sigma}_{(m,n)}(\Delta\hat{\theta}[k+1])}{f_0(\Delta\hat{\theta}[k+1])}\right\}, \tag{12}$$

where $W_{(m,n)}[0] = 0$ for all $(m,n) \in \mathcal{E}$. Denote τ_{C} to be the time at which the Generalized CuSum algorithm declares the occurrence of a line outage [5]; then,

$$\tau_{\text{C}} = \inf_{(m,n)\in\mathcal{E}}\left\{\inf\{k \geq 1 : W_{(m,n)}[k] > A_{(m,n)}\}\right\}. \tag{13}$$

where $A_{(m,n)}$ is a threshold selected for the corresponding $W_{(m,n)}[k]$ statistic. In addition, this algorithm also identifies the line that is outaged at τ_{C} to be

$$(\hat{m}, \hat{n}) = \arg\max_{(m,n)\in\mathcal{E}} W_{(m,n)}[\tau_{\text{C}}]. \tag{14}$$

B. Intuition Behind the Operation of the CuSum Algorithm

The algorithm we presented in (12) for line outage detection is based on the Kullback-Leibler (KL) divergence, which for any two probability densities f and g, is defined as follows:

$$D(f \parallel g) := \int f(x)\log\frac{f(x)}{g(x)}dx \geq 0, \tag{15}$$

with equality if and only if $f = g$ almost surely. In the context of the line outage detection problem, for an outage of line (m,n), the KL divergence is

$$D(f_{(m,n)} \parallel f_0) = \\ \mathbb{E}\left[\log\left(\frac{f_{(m,n)}(\Delta\hat{\theta}[k])}{f_0(\Delta\hat{\theta}[k])}\right)\middle| (m,n) \text{ outage}\right], \tag{16}$$

which provides a bound on the delay for detecting an outage in line (m,n); a larger KL divergence results in lower detection delay and vice versa. Prior to any changes, the mean of the log likelihood ratio is negative due to (15). Therefore, $W_{(m,n)}[k]$ would remain close to or at 0 prior to a line outage. On the other hand, after an outage occurs, the mean of the log likelihood ratio is positive. As a result, $W_{(m,n)}[k]$ increases unboundedly after the outage in line (m,n), and the CuSum algorithm in (12) declares the occurrence of an outage in line (m,n) the first time that $W_{(m,n)}[k]$ reaches $A_{(m,n)}$.

IV. Optimal PMU Placement and System Partitioning

In this section, we formulate the optimal PMU placement strategy for the given power system network so that the worst case detection delay is minimized. Additionally, we show that the QCD-based algorithm can be applied to each area of a partitioned power system concurrently for faster detection, and provide several criteria to obtain such partitions.

A. PMU Placement

The KL divergence presented in (15) has a closed form solution if the two distributions f and g are Gaussian. Since the pre-outage distribution, $f_0 \sim \mathcal{N}(0, \hat{M}_0 \Lambda \hat{M}_0^T)$, and the post-outage distribution, $f_{(m,n)} \sim \mathcal{N}(0, \hat{M}_{(m,n)} \Lambda \hat{M}_{(m,n)}^T)$, are both Gaussian, we can express $D(f_{(m,n)} \parallel f_0)$ as

$$\frac{1}{2}\big[\mathrm{Tr}(\Gamma_0^{-1}\Gamma_{(m,n)}) - p + \log(\det(\Gamma_0 \Gamma_{(m,n)}^{-1}))\big], \qquad (17)$$

where $p \leq N_d$ is the number of PMUs allocated for the system, $\Gamma_0 = \hat{M}_0 \Lambda \hat{M}_0^T$, and $\Gamma_{(m,n)} = \hat{M}_{(m,n)} \Lambda \hat{M}_{(m,n)}^T$.

From (17), it is evident that the KL divergences depend on p. In addition, for a fixed p, the locations of the PMUs also affect the KL divergences. In order to minimize the worst case detection delay for all line outages, the following optimization can be solved for the optimal placement of the p PMUs:

$$\max_{C} \min_{\Gamma_{(m,n)}} \frac{1}{2}\big[\mathrm{Tr}(\Gamma_0^{-1}\Gamma_{(m,n)}) - p + \log(\det(\Gamma_0 \Gamma_{(m,n)}^{-1}))\big], \quad (18)$$

where C is defined in (9). Since we would like to minimize the detection delay for the worst possible line outage, the inner minimization is performed over all possible line outages in the system.

The integer programming problem in (18) is NP hard; therefore, in order to speed up the combinatorial search, we propose a greedy algorithm, the pseudocode of which is provided in Algorithm 1; this algorithm provides a lower bound to the globally optimal solution. The algorithm chooses the locations of the PMUs sequentially. At each step, the additional location of the PMU is selected such that the location maximizes the current minimum KL divergence for all possible line outages. The algorithm stops when the number of PMUs selected reaches p. We show in the case studies that this method is computationally tractable with good performance.

B. Power System Network Partitioning

For scalability, the graph describing the topology of a power system could be partitioned into subgraphs and the QCD-based algorithm described in Section III could be applied to each partition concurrently. There are many ways we can partition the overall system according to some optimality criteria; we consider three possible criteria here:

C1. Equal number of edges within each partition (balanced size for each partition).

C2. A partition such that the number of detectable single-line outages for the overall system is maximized.

C3. Minimum KL divergence for all the partitions is maximized (to minimize detection delay).

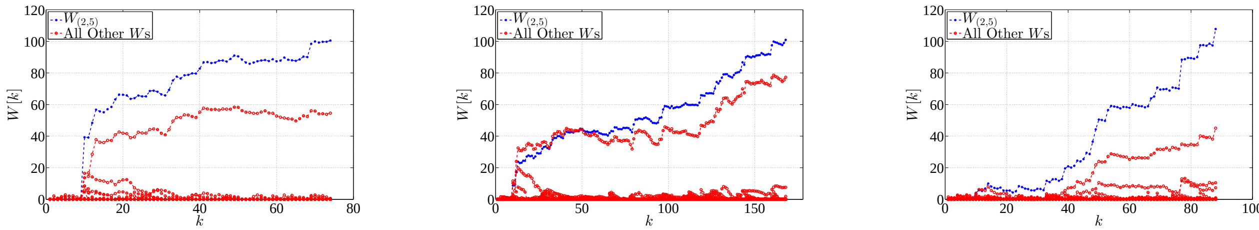
1) Criterion **C1**: Suppose each processor executing the line-outage detection algorithm could perform computations on K streams of data in parallel and there are L total lines in the overall system. Then the ideal number of partitions for the system such that all processors are fully-utilized is $\lceil \frac{L}{K} \rceil$. A graph partitioning algorithm that achieves this goal is proposed in [18]. This algorithm is based on computing the spectral factorization of the partition matrix. A software for implementing this scheme is the METIS package [19]. This software partitions a graph into k partitions based on two possible objective functions, minimum edgecut or minimum communication volume (based on weights assigned to border vertices). Hence, one form of quasi-optimality for partitioning is to balance the number of edges *within* each partition; once the number of edges within each partition is specified, then the problem becomes finding the set of partitions such that the minimum KL divergence in each partition is maximized.

2) Criterion **C2**: If the removal of an edge in the partition further divides the graph into subgraphs (this corresponds to islanding in the power system partition), then such a line outage is undetectable by the QCD method. Thus, a good partitioning scheme maximizes the number of detectable single-line outages for the overall system.

3) Criterion **C3**: The optimal partition should minimize the false alarm and false isolation rates. The false isolation rate decays exponentially with the threshold $A_{(m,n)}$ while the average detection delay is inversely proportional to the KL divergence [20]. The problem of finding the optimal

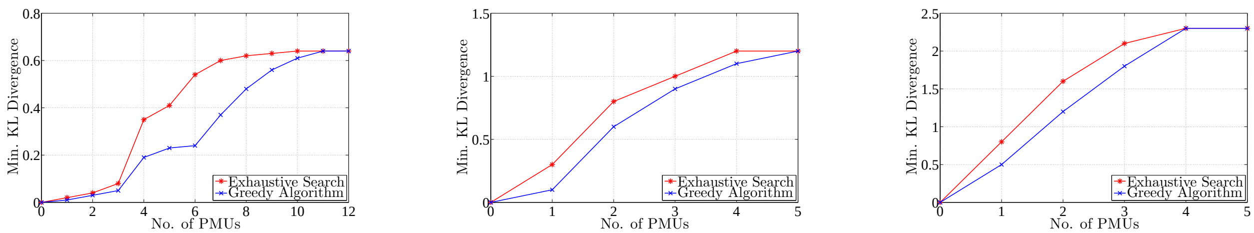
Algorithm 1: Greedy Algorithm for PMU placement

```
Data: N, p
C = 0, k = 0;
for k = 1 to p do
    g = 0;
    for n = 1 to N do
        C(k,:) = e_n^T;
        KL = min_{Γ_i} 1/2 [Tr(Γ_0^{-1} Γ_i) − p + log(det(Γ_0 Γ_i^{-1}))];
        if g < KL then
            g = KL;
            l = n;
        end
    end
    C(k,:) = e_l^T;
end
return C
```



(a) Full measurement set. (b) Reduced measurement set. (c) Optimal reduced measurement set.

Fig. 1: 14-bus system: Sample paths of $W_{(m,n)}[k]$ for outage of $(2, 5)$.



(a) Full System: Buses 1-14. (b) Area 1: Buses 1-5, 7-8. (c) Area 2: Buses 6, 9-14.

Fig. 2: Minimum KL divergence of 14-bus system.

partitioning of the system would then be formulated as

$$\max_{\text{all partitions}} \min_{\Gamma_{(m,n)}} \frac{1}{2}\Big[\mathrm{Tr}(\Gamma_0^{-1}\Gamma_{(m,n)}) - p + \log(\det(\Gamma_0\Gamma_{(m,n)}^{-1}))\Big]. \quad (19)$$

Constraints can be added to the optimization problem as necessary. These constraints include the maximum number of edges in each partition, the maximum number of vertices in each partition, or the number of partitions for the overall system.

V. Case Studies

In this section, we illustrate the ideas proposed in this paper on the IEEE 14-bus and the 30-bus test systems.

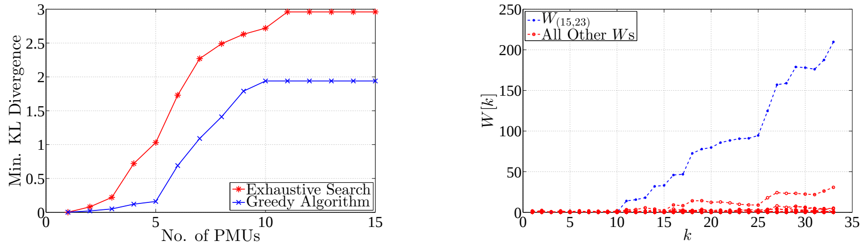
A. 14-bus System

An outage in line $(2, 5)$ is simulated at $k = 10$. We apply the Generalized CuSum algorithm in (12) by computing each CuSum statistic $W_{(m,n)}[k]$ with a threshold of $A = 100$. Figure 1 shows the typical progressions of $W_{(m,n)}[k]$ for an outage in line $(2, 5)$. Figure 1(a) assumes a full measurement set of voltage phase angles, while Fig. 1(b) is simulated with a reduced measurement set, where PMUs are deployed randomly at only nine of the buses. In both cases, the line outage is correctly identified when the $W_{(2,5)}$ statistic crosses the threshold of $A = 100$ first. With a full measurement set, the correct line outage is identified 65 samples after the outage occurs while a much longer detection delay of 170 samples is needed for the reduced measurement set.

Now suppose that we select nine buses via the procedure in Algorithm 1. The typical progressions of $W_{(m,n)}[k]$ for this case is shown in Fig. 1(c). By optimally placing the PMUs, we have reduced the detection delay to 79 samples, which is significantly better than randomly choosing the nine PMU locations.

We then adopt Criterion **C1** and use the METIS software package to partition the network of the IEEE 14-bus system. For a partition size of 2, this program separates the 14-bus system into areas with approximately equal number of nodes while minimizing the number of tie-lines between the two areas. The result from the graph partitioning algorithm is as follows. For Area 1, Buses 1, 2, 3, 4, 5, 7, 8; Area 2, Buses 6, 9, 10, 11, 12, 13, 14. Note that while such a partition choice is favorable from the perspective of requiring the fewest number of direct tie-line flow measurements for the tie lines, it does not minimize the total number of unobservable line outages for the overall system. This is evidenced by the fact that an outage on lines $(10, 11)$ or $(13, 14)$ causes islands to form in Area 2.

Next, we compute the minimum KL divergences for the entire 14-bus system, Area 1, and Area 2 of the partitioned 14-bus system to compare how they are affected by the number of PMUs deployed. Figure 2 shows how the minimum KL divergence increases as more PMUs are added, using both an exhaustive search that is globally optimal and the greedy Algorithm 1. The results for the entire 14-bus system is shown in Fig. 2(a) while those for Area 1 and Area 2 of the partitioned 14-bus system are shown in Fig. 2(b) and Fig. 2(c), respectively. From the plots, we conclude that the greedy algorithm provides a lower bound to the exhaustive search because the greedy algorithm is only traversing one of the many branches of the branch and bound method. Although



(a) Minimum KL divergence of Area 2.

(b) Sample paths for line outage of $(15, 23)$ of Area 2.

Fig. 3: 30-bus test system

not globally optimal, the greedy algorithm is attractive in the sense that is it tractable for larger systems.

B. 30-bus System

For the IEEE 30-bus system, we partition the system into two areas using the METIS software and compute the minimum KL divergence in Area 2 (buses 10, $12-30$) to see how it is affected by the number of PMUs deployed throughout the partition. We apply both the greedy algorithm (the pseudocode of which is provided in Algorithm 1) and an exhaustive search and show the results in Fig. 3(a). Using MATLAB running on an Intel Core i7 Processor, the greedy algorithm required less than 2 minutes to run, while 3 days were required for the optimal PMU placement via exhaustive search. For the exhaustive search method, the number of computations required for each partition is $\binom{N_i}{p}$ while the greedy algorithm requires only $N_i p$ computations, where N_i is the number of buses in partition i.

Next we simulate a line outage on line $(15, 23)$ of Area 2, which has a KL divergence of 14.3. The threshold is set at $A = 200$ with the variance of active power injections assumed to be 0.3 at all of the load buses. Typical progressions of $W_{(m,n)}[k]$ are shown in Fig. 3(b). For this particular example, $W_{(15,23)}$ crosses the threshold of $A = 200$ for the first time 23 samples after the outage occurs, resulting in a detection delay of 0.76 seconds given a PMU sampling rate of 60 samples/s.

VI. Concluding Remarks

In this paper, we employed a QCD-based algorithm to detect and identify line outages and proposed a method to optimally deploy a fixed number of PMUs across the system buses to minimize the worst case line outage detection delay, which is formulated as an integer programming problem. For solving this problem, we presented a greedy algorithm and showed that it performs well compared to an exhaustive search for the globally optimal solution. In addition, we developed strategies to partition the power system into multiple areas so that the proposed line outage detection could be applied concurrently to each area for better performance.

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