

# Offer Strategies for Wholesale Energy and Regulation Markets

Dariusz Fooladivanda

Hanchen Xu

Alejandro D. Domínguez-García

Subhonmesh Bose

**Abstract**—Following FERC Order 755 in October 2011, wholesale markets in the US adopted a two-part compensation structure in their regulation markets, paying for regulation capacity made available as well as the quantity of regulation actually provided. In this letter, we first present a mathematical model for PJM’s market clearing process for their energy and regulation markets. We then compute the optimal offer of a single energy resource offering its services into such an energy and regulation market. Finally, we empirically illustrate our offer strategy through case studies.

**Index Terms**—Regulation markets.

## I. INTRODUCTION

Regulation signals encode the mismatch of demand and supply of power over a geographical area. Efficient tracking of regulation signals helps to maintain the demand supply balance, thereby keeping the system frequency around its nominal value. Independent System operators (ISOs) procure energy and regulation services through their wholesale electricity markets. While dispatchable generators have typically provided such services, many have advocated utilizing distributed energy resources (DER), or an aggregation thereof [1]. See [2] and [3] for DER’s offers into energy-only markets, and see [4] for a different offer model for microgrids.

In this letter, we consider how a single resource, henceforth denoted as  $\mathbf{R}$ , can offer such services in today’s performance-based energy and regulation markets to maximize its revenues. The resource can be a single generator or a DER with a single point of interconnection to the electric power network<sup>1</sup>. Our market model closely follows the one managed by the Independent System Operator for Pennsylvania-Jersey-Maryland (PJM). Our contributions are twofold. First, we present a mathematical model for PJM’s market clearing process. Second, we propose a stochastic control formulation for  $\mathbf{R}$  to optimally offer in that market.

*Notation:* Let  $\mathbb{R}$  be the set of real numbers. Denote by  $\mathbf{x} := (x[1], \dots, x[K]) \in \mathbb{R}^K$ , a signal over  $K$  time-slots. Associate with  $\mathbf{x}$ , the signal  $\mathbf{x}[k_1 : k_2] := (x[k_1], \dots, x[k_2])$  for each  $1 \leq k_1 \leq k_2 \leq K$ .

## II. PJM’S ENERGY AND REGULATION MARKETS

We begin by presenting a mathematical model for PJM’s market clearing process for their energy and regulation markets, as outlined in [5]. For simplicity, the transmission network is ignored throughout. PJM currently offers two types of regulation signals—the slowly-varying *regA* signal, and the fast-ramping *regD*. The market model presented here only

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The authors are affiliated with the Department of Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign, Urbana, IL, USA. Emails: {dfooladi, hxu45, aledan, bores}@illinois.edu

<sup>1</sup>The extension to the case with an aggregation of DERs interconnected via a distribution network is relegated to future work.

considers *regD* signals, but can be easily extended to include both signal types.

Denote by  $\mathcal{U}$ , the collection of market participants. Assume that the market proceeds in three distinct stages. In the first stage (call it  $k = -1$ ), each participant  $i \in \mathcal{U}$  provides the following information to the ISO:

- $[p_i, \bar{p}_i]$ : generation capacity limits (MW);
- $J_i : [p_i, \bar{p}_i] \rightarrow \mathbb{R}_+$ : cost function (\$/MWh) for energy as inferred from a supply offer;
- $\bar{r}_i$ : offered regulation capacity (MW);
- $\alpha_i$ : offer price for regulation capacity (\$/MWh); and
- $\beta_i$ : offer price for regulation performance (\$/ $\Delta$ MWh).

For each participant  $i$ , the ISO maintains the following scores:

- $S_i^H$ : historical performance score; and
- $\mu_i^H$ : historical mileage ratio ( $\Delta$ MW/MW);

where  $S_i^H \in [0, 1]$  and  $\mu_i^H \geq 0$ . The ISO utilizes these historical scores in its market clearing process that we describe next.

The ISO forecasts a total demand  $D$  and a required regulation capacity  $R$  to maintain the balance between demand and supply over the contract period. Given the forecasts  $D$ ,  $R$ , a collection of offers  $(J_i, \bar{r}_i, p_i, \bar{p}_i, \alpha_i, \beta_i)$ , and historical scores  $S_i^H$ ,  $\mu_i^H$  for each participant  $i$ , the ISO solves the following problem to clear the market at the second stage (call it  $k = 0$ ).

$$\text{minimize}_{p_i, r_i, i \in \mathcal{U}} \sum_{i \in \mathcal{U}} [J_i(p_i) + \frac{1}{S_i^H} (\alpha_i + \beta_i \mu_i^H) r_i], \quad (1a)$$

$$\text{subject to} \quad \sum_{i \in \mathcal{U}} p_i = D, \quad \sum_{i \in \mathcal{U}} S_i^H r_i \geq R, \quad (1b)$$

$$p_i \leq p_i - r_i, \quad p_i + r_i \leq \bar{p}_i, \quad r_i \leq \bar{r}_i, \quad i \in \mathcal{U}. \quad (1c)$$

The problem in (1) seeks to minimize the energy and regulation procurement costs. Participant  $i$ ’s contribution to the cost has two parts—the energy cost, denoted by  $J_i(p_i)$ , and the cost of regulation capacity and performance cost, denoted by  $(\alpha_i + \beta_i \mu_i^H) r_i / S_i^H$ . The latter is discounted by the historical scores  $S_i^H$  and  $\mu_i^H$ . The constraints in (1b) ensure that the procured energy covers the forecasted load  $D$ , and the regulation capacity (weighted by historical scores) covers the projected regulation needs  $R$ . The technological limits of participant  $i$  are modeled in (1c).

Let  $z^*$  denote an optimal value of any variable  $z$  in (1). Further, let  $\mathcal{U}_r$  denote the participants that are cleared to provide regulation, i.e., for which  $r_i^* \neq 0$ . The ISO calculates the compensations for each participant from an optimal solution of (1) as follows. First, it computes the so-called *regulation market clearing price* (RMCP), given by

$$\text{RMCP} := \max_{i \in \mathcal{U}_r} \left\{ \frac{\alpha_i + \beta_i \mu_i^H}{S_i^H} + \frac{\lambda_e^* - J_i'(p_i^*)}{S_i^H} \right\}, \quad (2)$$

where  $J'_i(p_i^*)$  represents the derivative of  $J_i$  at  $p_i^*$ . And,  $\lambda_e^*$  denotes the energy price that equals the optimal Lagrange multiplier associated with the demand-supply balance constraint in (1b). RMCP in (2) is comprised of two terms. The first term defines a price for regulation capacity and performance. The second one is a payment for the opportunity cost of offering regulation capacity instead of offering the same in the energy market. The ISO further computes

$$\lambda_p^* := \max_{i \in \mathcal{U}_r} \frac{\beta_i \mu_i^H}{S_i^H}, \quad \lambda_c^* := \text{RMCP} - \lambda_p^*, \quad (3)$$

where  $\lambda_p^*$  and  $\lambda_c^*$  define the regulation performance and capacity prices, respectively.

Identify the contract window for regulation service as the third stage of the market, given by  $K$  time slots, labeled  $k = 1, 2, \dots, K$ . At the end of the contract period, each participant  $i \in \mathcal{U}_r$  receives a two-part payment for providing regulation  $-\lambda_c^* r_i^* S_i$  for offering regulation capacity, and  $\lambda_p^* r_i^* \mu_i S_i$  for the regulation performance provided. Here,  $S_i$  and  $\mu_i$  are the performance score and mileage ratio of participant  $i$ , respectively, calculated based on how well the participant responded to the instructed regulation signals. In addition to the payment for regulation, each participant  $i$  also receives an energy payment  $\lambda_e^* p_i^*$ . Henceforth, denote by  $\lambda^* := (\lambda_p^*, \lambda_c^*, \lambda_e^*)$ , the collection of market clearing prices.

The values of  $S_i$  and  $\mu_i$  are calculated as follows. Denote by  $\boldsymbol{\rho}_i := (\rho_i[1], \dots, \rho_i[K])$ , the instructed regulation signal, and by  $\boldsymbol{\delta}_i := (\delta_i[1], \dots, \delta_i[K])$ , the deviations of the power outputs from  $p_i^*$  during the contract period. Then,  $S_i$  captures how well  $\boldsymbol{\delta}_i$  tracks  $\boldsymbol{\rho}_i$  over that period, and  $\mu_i$  captures how much movement (ramps) that participant  $i$  provides. The nature of  $S_i$  differs across markets; an example is given by

$$S_i(\boldsymbol{\rho}_i, \boldsymbol{\delta}_i) := 1 - \frac{\|\boldsymbol{\delta}_i - \boldsymbol{\rho}_i\|_1}{\|\boldsymbol{\rho}_i\|_1}, \quad (4)$$

where  $\|\cdot\|_1$  represents the  $\ell_1$ -norm of a signal. Further, the mileage ratio  $\mu_i$  is given by a scaled total variation of the regulation provided, i.e.,

$$\mu_i(\boldsymbol{\delta}_i) := \frac{1}{r_i^*} \text{TV}(\boldsymbol{\delta}_i) = \frac{1}{r_i^*} \sum_{k=1}^{K-1} |\delta_i[k+1] - \delta_i[k]|. \quad (5)$$

The ISO maintains a rolling average of  $S_i$  and  $\mu_i$  over last 100 contract periods in  $S_i^H$  and  $\mu_i^H$ , respectively.

### III. OFFERING INTO THE REGULATION MARKET

We now compute the optimal offer of a single resource  $\mathbf{R}$  in the energy and regulation market, described above. Let  $\mathbf{R}$  produce  $p[k]$  within its generation limits  $[\underline{p}[k], \bar{p}[k]]$  at time  $k = 1, \dots, K$ . Further, let  $C(p[k])$  denote the cost of producing  $p[k]$ , where  $C: [p, \bar{p}] \rightarrow \mathbb{R}$ , is convex, smooth, and increasing. Here, we have used the notation  $\underline{p} = \min_k \underline{p}[k]$  and  $\bar{p} = \max_k \bar{p}[k]$ . With this notation, we now formulate the stochastic control problem that  $\mathbf{R}$  solves to maximize its profits from the market.

#### A. Control Policies for Providing Regulation

Suppose  $\mathbf{R}$  has been cleared to provide a nominal power output  $p^*$  and a regulation capacity  $r^*$  at market prices given by  $\lambda^*$ . The instructed regulation  $\boldsymbol{\rho}$  is a discrete-time random process over the contract period. Assume that the distribution of its normalized variant,  $\boldsymbol{\rho}^N := \boldsymbol{\rho}/r^*$ , is known. A risk-neutral  $\mathbf{R}$  will seek to maximize its *expected* profits over the contract

period, where the expectation is computed according to the distribution of  $\boldsymbol{\rho}^N$ . To identify its profits, write the provided regulation  $\delta[k]$  as

$$\delta[k] = p[k] - p^*. \quad (6)$$

Then,  $\mathbf{R}$ 's profit for a given sample path of  $\boldsymbol{\rho}^N$  is given by

$$\pi(p^*, r^*, \lambda^*, \boldsymbol{\delta}) := r^* S(\boldsymbol{\rho}, \boldsymbol{\delta}) (\lambda_c^* + \lambda_p^* \mu(\boldsymbol{\delta})) + \lambda_e^* p^* - \sum_{k=1}^K C(p[k]).$$

We seek a causal control policy to provide regulation. A causal policy is a collection of maps  $\boldsymbol{\Delta} := (\Delta^1, \dots, \Delta^K)$ , such that  $\delta[k] = \Delta^k(I[k])$ , and

$$\mathcal{I}[k] = \{p^*, r^*, \lambda^*, \boldsymbol{\rho}[1:k], \boldsymbol{\delta}[1:k-1]\}, \quad (7)$$

describes the information set at time  $k = 1, \dots, K$ . Policy  $\boldsymbol{\Delta}$  is admissible, if the power outputs induced by  $\boldsymbol{\Delta}$  satisfies

$$S(\boldsymbol{\rho}, \boldsymbol{\delta}) \geq S_{\min}, \quad (8a)$$

$$|p[k+1] - p[k]| \leq \chi_j, \quad (8b)$$

$$\underline{p}[k] \leq p[k] \leq \bar{p}[k], \quad (8c)$$

for each  $k = 1, \dots, K$ , almost surely.<sup>2</sup> Regulation markets often bar resources to participate when the performance score falls below a threshold. The constraint in (8a) encodes that requirement. According to [6], PJM sets it at  $S_{\min} = 0.7$ . The inequalities in (8b)-(8c) delineate the ramping capabilities and the capacity constraints of  $\mathbf{R}$ . Denoting the set of all admissible policies by  $\mathcal{D}(p^*, r^*, \lambda^*)$ , we arrive at the following revenue maximization problem.

$$\begin{aligned} & \text{maximize} && \mathbb{E}[\pi(p^*, r^*, \lambda^*, \boldsymbol{\delta}(\boldsymbol{\Delta}))], \\ & \text{subject to} && \boldsymbol{\Delta} \in \mathcal{D}(p^*, r^*, \lambda^*). \end{aligned} \quad (9)$$

Rather than searching over all admissible policies, that is generally challenging, consider a *proportional* controller of the form

$$\delta[k] := \gamma \rho[k], \quad (10)$$

where  $\gamma \geq 0$ . The above controller yields

$$S(\boldsymbol{\rho}, \boldsymbol{\delta}) = 1 - |1 - \gamma|, \quad \mu(\boldsymbol{\delta}) = \gamma \text{TV}(\boldsymbol{\rho}^N). \quad (11)$$

For a given set of  $r^*$ ,  $p^*$ , and  $\lambda^*$ , the profit-maximizing  $\gamma^*$  is a solution to the following optimization problem.

$$\begin{aligned} & \text{maximize}_{\gamma} && \mathbb{E}[r^* (1 - |1 - \gamma|) (\lambda_c^* + \lambda_p^* \gamma \text{TV}(\boldsymbol{\rho}^N)) \\ & && + \lambda_e^* p^* - \sum_{k=1}^K C(p^* + \gamma_j r^* \rho^N[k])], \quad (12a) \\ & \text{subject to} && 1 - |1 - \gamma| \geq S_{\min}, \quad \gamma r^* \leq \nu, \quad (12b) \end{aligned}$$

where  $\nu := \frac{1}{2} \min\{\bar{p} - \underline{p}, \chi\}$ .

Notice that  $S$  measures how well  $\boldsymbol{\delta}$  tracks  $\boldsymbol{\rho}$ , while  $\mu$  measures how  $\boldsymbol{\delta}$  fluctuates. Depending upon the cost parameters and the market prices, it may be profitable to earn from providing mileage at the expense of tracking the regulation signal. Said otherwise,  $\gamma^*$  from (12a) may not always be unity.

To explore the value of  $\gamma^*$  numerically, we considered  $\mathbf{R}$  with  $\underline{p}[k] = 0$  and  $\bar{p}[k] = 200$  MW, whose cost of production was varied among generators in the IEEE test cases in MATPOWER [7]. We further used the regulation signals and market clearing prices in PJM for December 2016 [8].

<sup>2</sup>In the above equations,  $\delta[K+1]$  can be arbitrary.

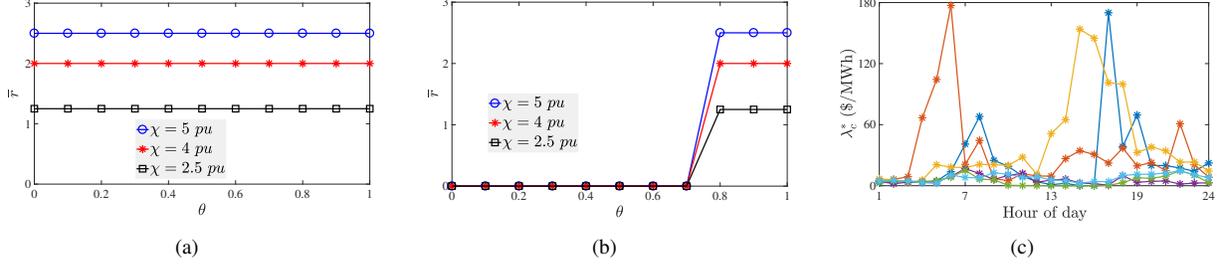


Fig. 1: The optimal  $\bar{r}$  is plotted in (a) and (b) as a function of the pessimism parameter  $\theta$  for three different values of ramp limits  $\chi$  for PJM’s day-ahead regulation market on May 1, 2017. Figure (a) plots the offers for 7 – 8 a.m., and (b) plots that for 8 – 9 a.m. Figure (c) shows temporal variations in market prices for regulation capacity for selected days in May 2017.

For different combinations of  $p^*$  and  $r^*$  that satisfied (1c), we computed  $\gamma^*$  from (12a). In all our experiments, we obtained  $\gamma^* = 1$ , implying that a resource typically does not have any incentive to deviate from tracking the instructed regulation.

### B. Computing an Optimal Offer Strategy

Recall that (9) defines the revenue maximization problem that  $\mathbf{R}$  solves upon receiving the market outcome  $(p^*, r^*, \lambda^*)$  at stage  $k = 0$ . Let  $V^*(p^*, r^*, \lambda^*)$  denote the optimal objective value of (9). Then,  $\mathbf{R}$  seeks an offer to maximize  $V^*$  at the market outcome. This outcome not only depends on  $\mathbf{R}$ ’s offer alone, but also on that of other regulation providers. One can model such interactions in a game-theoretic framework. For simplicity, we take an alternate route –  $\mathbf{R}$  computes its offer based on *conjectures* on how the market will respond to its offer, that we delineate next.

$\mathbf{R}$  assumes that its offer does not affect  $\lambda^*$  or  $\rho^N$ , and the statistics of the latter is independent of  $\lambda^*$ . It truthfully reports its cost and production capacity, and forecasts  $\lambda^*$  based on historical values (call them  $\lambda^H$ ). It further conjectures the regulation capacity and energy cleared in the market to be given by the maps  $r^*(\bar{r})$  and  $p^*(\bar{r})$ , respectively. The conjectured  $r^*(\bar{r})$  can take values between zero and  $\nu$ , and  $p^*(\bar{r})$  takes values between  $\underline{p} + r^*(\bar{r})$  and  $\bar{p} - r^*(\bar{r})$ . One can estimate such maps by repeatedly playing in these markets.

Computing the optimal offer in our framework is given by maximize  $V^*(p^*(\bar{r}), r^*(\bar{r}), \lambda^H)$ , subject to  $\bar{r} \in [0, \nu]$ . (13)

With the optimal affine controllers described in (10), and utilizing the resulting  $V^*$  from (12a), the above problem reduces to solving

$$\text{maximize}_{\bar{r}} \mathbb{E} [r^*(\bar{r}) (\lambda_c^H + \lambda_p^H \text{TV}(\rho^N)) + \lambda_e^H p^*(\bar{r})] \quad (14a)$$

$$- \sum_{k=1}^K C(p^*(\bar{r}) + r^*(\bar{r})\rho^N[k]) \quad (14b)$$

$$\text{subject to } 0 \leq \bar{r} \leq \nu. \quad (14c)$$

In the next section, we compute the optimal offer with simple maps  $p^*$  and  $r^*$ .

## IV. NUMERICAL EXPERIMENTS

Consider  $\mathbf{R}$  with generation capacity limits  $\bar{p} = 5$  MW and  $p = 0$ , offering into PJM’s day-ahead regulation market for the first day of May 2017 over two one-hour periods – 7 to 8 a.m. and 8 to 9 a.m. Let  $C(p[k]) = 20p[k]$  \$/MWh. Let  $\mathbf{R}$  conjecture the following maps for the market outcomes.

$$r^*(\bar{r}) = \frac{4}{5}\bar{r}, \quad \text{and} \quad p^*(\bar{r}) = \theta \left( \underline{p} + \frac{4}{5}\bar{r} \right) + (1-\theta) \left( \bar{p} - \frac{4}{5}\bar{r} \right).$$

The parameter  $\theta \in [0, 1]$  reflects  $\mathbf{R}$ ’s level of pessimism on the amount of energy to be cleared in the market. The larger the  $\theta$ , the lesser that amount of energy it expects to be cleared. Consider price forecasts given by the values of  $\lambda_e^*$ ,  $\lambda_c^*$ , and  $\lambda_p^*$  at the same hour the day before. These prices for PJM’s day ahead market on May 1, 2017 [8] are given by

$$7 \text{ to } 8 \text{ a.m.: } \lambda_e^H = 27.03, \quad \lambda_c^H = 68.00, \quad \lambda_p^H = 3.50,$$

$$8 \text{ to } 9 \text{ a.m.: } \lambda_e^H = 28.67, \quad \lambda_c^H = 24.90, \quad \lambda_p^H = 5.04,$$

measured in \$/MWh.

We used Gurobi to compute the optimal regulation capacity offers for the aforementioned hours over different values of the pessimism parameter  $\theta$ . The results are shown in figures 1(a)-1(b). The difference between the two plots highlights the fact that the offer depends heavily on the energy and regulation prices. When the regulation prices are higher,  $\mathbf{R}$  offers regulation regardless of its pessimism. When the regulation prices are comparable to that of energy, it only offers regulation services when it does not expect enough amount of energy to be cleared in the market. Market clearing prices for regulation capacity being quite variable (cf. Figure 1(c)), price forecasting will likely play a significant role in the participants’ offer.

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