Microgrid Distributed Frequency Control Over Time-Varying Communication Networks

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Abstract— **In this paper, we study AC microgrid dynamics under a completely decentralized primary control, and a secondary frequency control the implementation of which is distributed over a communication network with communication links that are time-varying and can be (i) bidirectional, or (ii) unidirectional. For a certain class of controllers, the closed-loop system dynamics solve a certain multi-agent optimization problem by performing two steps: (i) gradient-descent, and (ii) distributed averaging. The proposed framework allows to explore many of the existing distributed algorithms developed for solving general multi-agent optimization problems over time-varying communication networks. In particular, we use the *subgradient-push algorithm* to design a distributed frequency controller, and we present the convergence analysis for the closed-loop system. We also dwell on this framework and propose a distributed frequency controller that does not require agents (power generators) to know their out-degree, which is a necessary assumption for the convergence of the subgradient-push algorithm.**

I. Introduction

Compared to the bulk power grid, microgrids are much smaller in terms of their physical footprint, and are equipped with generators that have limited power delivery capabilities, serving only a few local loads, but with the ability to provide a faster response to changes in load to that of centralized power plants. Due to the nature of the power generation technologies used, these generators are often interfaced with inverters, thereby turning the microgrids into low-inertia systems. Lack of rotational inertia in the system causes difficulties in stabilizing it and maintaining the system frequency at the nominal value. Additional difficulties in achieving a stable operation of microgrids emerge from the likely event that the generation levels associated with renewable-based resources being low during hours of peak demand, which requires carefully designed energy management strategies, especially, when the maximum load is much higher than the average load [1].

With regard to the control architecture being utilized, a distributed implementation is, in general, more preferable since it offers more resiliency to failures in the processing units where the controllers reside, and entails less stringent requirements on the sensing and communication ranges [1]. Moreover, if the microgrid topology changes, parameter tuning for the controller with the centralized architecture can be challenging; in this regard, a distributed implementation allows to design controls that are more adaptable and easier to scale.

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Our work is inspired by recent results in the area of distributed control and optimization, where novel algorithms that are robust against communication delays and packet drops have been proposed (see, e.g., [2]–[9]). In this paper, we apply some of these results to designing frequency controllers over time-varying communication networks with (i) bidirectional, and (ii) unidirectional communication links.

For bidirectional communications, we consider the case when information exchange between neighboring nodes is not necessarily synchronized due to delays and packet drops, and propose a distributed subgradient method with alternating averaging, which we utilize to develop a frequency controller for microgrids.

For the case of unidirectional communication links, we propose a frequency controller based on the subgradient-push method proposed in [7], and present convergence results. We note that, for a fixed communication network topology, and without the subgradient step, this method reduces to the basic version of the so-called *ratio-consensus algorithm* (see, e.g., [5]), which is a special case of the *push-sum protocol* proposed in [2], [4]. [All these algorithms rely on each node maintaining two linear iterations the ratio of which converges to the ratio of the sum of the initial conditions for the numerator iteration and the sum of the initial conditions for the denominator iteration.] One limitation of the subgradient-push method is that for the averaging step, each agent needs to know its out-degree; robust versions of the ratio-consensus algorithms proposed in [6] and [8] are known to overcome this limitation. For future work, it is of interest to further investigate whether these robustified ratio-consensus algorithms merged with the subgradient step enjoy the same convergence properties as the subgradient-push method.

Some of the recent works related to the design of distributed frequency controllers focused on the stability analysis of the closed-loop system under communication delays (see, e.g., [10]–[16]). A common approach is to use robust control techniques to quantify delay effects on stability by constructing Lyapunov-Krasovskii functionals (see, [10], [12], [14], [16]); typically, some delay-dependent conditions are derived to guarantee robust stability. Several control schemes have been proposed for mitigating delay effects; one of them is to design a Smith predictor-based controller [13]. Based on the measured communication delay, the authors in [11] propose a gain-scheduling approach for adjusting gain values in the frequency controller. A distributed observer-based control approach has been proposed in [15] to cope with switching communication network topologies. Finally, the authors in [17] propose a distributed generation control approach that relies on the robust ratio-consensus algorithm proposed in [6], [8], which mitigates the effects of information loss due to the presence of unreliable communication links.

II. Preliminaries

In this section, we introduce the microgrid and communication network models adopted in this work, and formulate the problem of interest.

A. Physical Layer

Consider a group of three-phase inverter-interfaced generators and loads interconnected via a lossless network, the topology of which is described by an undirected graph, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with $\mathcal{V} = \mathcal{V}^{(I)} \cup \mathcal{V}^{(L)}$, where $\mathcal{V}^{(I)} = \{1, \dots, m\}$ denotes the set of nodes (buses) with an inverter, and $\mathcal{V}^{(L)} = \{m+1, \dots, n\}$ denotes the set of nodes with a load; and where $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, with $\{i, j\} \in \mathcal{E}$ if nodes i and j are electrically connected.

The active power injected into the network via node $i \in \mathcal{V}$ is given by

$$P_i(\theta(t), V(t)) = \sum_{j \in \mathcal{V}} V_i(t) V_j(t) B_{ij} \sin(\theta_i(t) - \theta_j(t)), \quad (1)$$

where $V_i(t)$ is the instantaneous voltage magnitude, $\theta_i(t)$ is the phase angle in a coordinate frame rotating with nominal frequency ω^*, $\theta(t) = [\theta_1(t), \dots, \theta_n(t)]^\top$, $V(t) = [V_1(t), \dots, V_n(t)]^\top$, and $B_{ij} = -b_{ij}$, with $b_{ij} < 0$ denoting the susceptance of the line connecting nodes i and j. Let $P_j^{(I)}(t)$ denote the active power generated by the inverter at node $j \in \mathcal{V}^{(I)}$, and let $P_{l-m}^{(L)}(t) > 0$ denote the active power consumed by the load at node $l \in \mathcal{V}^{(L)}$; then,

$$P_i^{(I)}(t) = P_i(\theta(t), V(t)), \; i \in \mathcal{V}^{(I)}, \quad (2a)$$
$$-P_{l-m}^{(L)}(t) = P_l(\theta(t), V(t)), \; l \in \mathcal{V}^{(L)}. \quad (2b)$$

Assuming fixed voltage magnitudes and constant load demand, we differentiate and linearize the power flow equations around a stable equilibrium point $\theta(t) = \theta^*$ and apply Kron reduction to eliminate the equations corresponding to loads, obtaining

$$\dot{\theta}^{(I)}(t) = u(t), \quad (3a)$$
$$\dot{P}^{(I)}(t) = S(\theta^*) u(t), \quad (3b)$$

where $u(t)$ is the frequency controller to be designed, $P^{(I)}(t) = [P_1(t), \dots, P_m(t)]^\top$, $\theta^{(I)}(t) = [\theta_1(t), \dots, \theta_m(t)]^\top$, and $S(\theta^*)$ is a positive-semidefinite matrix (assuming $|\theta_i^* - \theta_j^*| < \frac{\pi}{2}, \forall \{i, j\} \in \mathcal{E}$). [The model (3) has also been studied in [18].] In subsequent developments, we drop the argument θ^* in $S(\theta^*)$ and just write S.

B. Cyber Layer

We establish some notation and state our assumptions on the communication network utilized by the processing units located at the inverter-interfaced generators. During any time interval (t_k, t_{k+1}), successful data transmissions among processors can be captured by graph $\mathcal{G}^{(I)}[k] = (\mathcal{V}^{(I)}, \mathcal{E}^{(I)}[k])$, where $\mathcal{V}^{(I)}$ is the set of inverter buses, and $\mathcal{E}^{(I)}[k]$ is the set of successfully established communication links. In this work, we consider two different cases for $\mathcal{G}^{(I)}[k]$: undirected and directed.

When $\mathcal{G}^{(I)}[k]$ is undirected, $\{i, j\} \in \mathcal{E}^{(I)}[k]$ if nodes i and j simultaneously exchange information with each other during time interval (t_k, t_{k+1}). Let $(\mathcal{V}^{(I)}, \mathcal{E}_\infty^{(I)})$ denote an undirected graph, where $\mathcal{E}_\infty^{(I)}$ is the edge set with $\{i, j\} \in \mathcal{E}_\infty^{(I)}$ if communication link between i and j is established infinitely often, i.e., $\{i, j\} \in \bigcup_{l \geq k} \mathcal{E}^{(I)}[l]$, for any k. Let $\mathcal{N}_i := \{j \in \mathcal{V} : \{i, j\} \in \mathcal{E}_\infty^{(I)}\}$.

When $\mathcal{G}^{(I)}[k]$ is directed, $(i, j) \in \mathcal{E}^{(I)}[k]$ if node j receives information from node i during time interval (t_k, t_{k+1}) but not vice versa. Also, $\mathcal{E}_\infty^{(I)}$ is the edge set with $(i, j) \in \mathcal{E}_\infty^{(I)}$ if $(i, j) \in \bigcup_{l \geq k} \mathcal{E}^{(I)}[l]$, for any k. Let $\mathcal{N}_i^+[k]$ and $\mathcal{N}_i^-[k]$ denote the sets of out-neighbors and in-neighbors of node i, respectively, during time interval (t_k, t_{k+1}), i.e., $\mathcal{N}_i^+[k] := \{j \in \mathcal{V} : (i, j) \in \mathcal{E}^{(I)}[k]\}$ and $\mathcal{N}_i^-[k] := \{j \in \mathcal{V} : (j, i) \in \mathcal{E}^{(I)}[k]\}$, and define $d_i[k] := |\mathcal{N}_i^+[k]|$.

Regarding the communication model, we also make the following two assumptions.

Assumption 1. *When $\mathcal{G}^{(I)}[k]$ is undirected, for every $\{i, j\} \in \mathcal{E}_\infty^{(I)}$, communication link between i and j is established at least once every B iterations, i.e., $\{i, j\} \in \bigcup_{l=k+1}^{k+B} \mathcal{E}^{(I)}[l]$, $\forall k$. When $\mathcal{G}^{(I)}[k]$ is directed, $(i, j) \in \bigcup_{l=k+1}^{k+B} \mathcal{E}^{(I)}[l]$, $\forall k$, for every $(i, j) \in \mathcal{E}_\infty^{(I)}$.*

Assumption 2. *If $\mathcal{G}^{(I)}[k]$ is undirected (directed), $(\mathcal{V}^{(I)}, \mathcal{E}_\infty^{(I)})$ is connected (strongly connected).*

C. Problem Statement

We recall from [18], [19] that the role of primary and secondary frequency controllers is to ensure

O1. *proportional active power sharing:* $P_i^{(I)}(t)/\overline{P}_i - P_j^{(I)}(t)/\overline{P}_j \to 0, \forall i, j$,

O2. *frequency regulation:* $\dot{\theta}^{(I)}(t) \to 0$ as $t \to \infty$,

where $\overline{P}_i$ is the maximum power capacity of generator i.

Our objective is to design a distributed frequency controller robust against communication delays and packet drops. For this purpose, we study the microgrid dynamics under a purely decentralized primary control, and a secondary frequency control the implementation of which is distributed over a time-varying communication network with (i) bidirectional, or (ii) unidirectional communication links. For a certain class of frequency controllers, the closed-loop microgrid dynamics solve a certain multi-agent optimization problem by executing two steps: (i) gradient-descent, and (ii) averaging with neighbors (distributed averaging step). We exploit this fact to design primary and distributed secondary frequency controllers that are able to operate over time-varying communication networks.

We begin our analysis by looking at the *DAPI controller* proposed in [19]:

$$u(t) = -W(P^{(I)}(t) - P^r(t)), \quad (4a)$$
$$W\dot{P}^r(t) = -\beta(P^r(t) - P^{(I)}(t)) - L^{(I)} W P^r(t), \quad (4b)$$

where W is a diagonal matrix with $W_{ii} = 1/D_i$, $\forall i \in \mathcal{V}^{(I)}$, $D_i > 0$ is the (inverse) droop coefficient, $P^r(t) = [P_1^r(t), \ldots, P_m^r(t)]^\top$ with $P_i^r(t)$ being the generator i set-point, $L^{(I)}$ is the cyber layer graph Laplacian with $L_{ij}^{(I)} = L_{ji}^{(I)} < 0$ if $\{i,j\} \in \mathcal{E}_\infty^{(I)}$, and $L_{ii}^{(I)} = -\sum_{j\in\mathcal{N}_i} L_{ij}^{(I)}$. The DAPI controller has been shown to achieve Objectives O1 and O2. By using (3) together with (4), the closed-loop system under the DAPI controller takes the following form:

$$u(t) = -W(P^{(I)}(t) - P^r(t)), \tag{5a}$$

$$W\dot{P}^r(t) = -\beta(P^r(t) - P^{(I)}(t)) - L^{(I)}WP^r(t), \tag{5b}$$

$$\dot{P}^{(I)}(t) = Su(t). \tag{5c}$$

Later in the paper, we will analyze a slight modification of (5), formulated in discrete time as follows:

$$u[k] = -\alpha[k](WP^{(I)}[k] - \xi[k])/s, \tag{6a}$$

$$\xi[k+1] = A\xi[k] - \alpha[k]\beta(W^{-1}\xi[k] - P^{(I)}[k]), \tag{6b}$$

$$P^{(I)}[k+1] = P^{(I)}[k] + sSu[k], \tag{6c}$$

where $s > 0$ is a stepsize, $\xi[k] := WP^r[k]$, I is an identity matrix, $A := I - sL^{(I)}$ is doubly stochastic, and $\alpha[k] > 0$ is a time-varying gain.

In its current form, (6b) implicitly assumes that, at each time instant, information exchange between every pair of neighboring nodes is bidirectional and successful, and $\mathcal{G}^{(I)}[k]$ is fixed. In this work, we consider a more realistic scenario when information exchange between each pair of neighbors is not guaranteed to happen. To this end, we examine two cases: (i) $\mathcal{G}^{(I)}[k]$ is time-varying and undirected; (ii) $\mathcal{G}^{(I)}[k]$ is time-varying and directed.

III. Primary and Secondary Frequency Regulation as a Multi-Agent Optimization Problem

In this section, we relate the problem of implementing frequency controllers via time-varying communication networks to solving a multi-agent optimization problem.

We define $\tilde{P}^{(I)}[k] := P^{(I)}[k] - P^{(I)}[0]$, and rewrite the system (6) as follows:

$$\tilde{P}^{(I)}[k+1] = \tilde{P}^{(I)}[k] - \alpha[k]S\big(W\tilde{P}^{(I)}[k] + WP^{(I)}[0] - \xi[k]\big), \tag{7a}$$

$$\xi[k+1] = A[k]\xi[k] - \alpha[k]\beta\big(W^{-1}\xi[k] - \tilde{P}^{(I)}[k] - P^{(I)}[0]\big). \tag{7b}$$

Later, we show that the system (7) can be thought of as a scaled distributed subgradient method for solving the following unconstrained multi-agent optimization problem:

$$\mathbf{P} : \min_{\tilde{P}^{(I)},\zeta} \frac{\beta}{2}\big(P^{(I)}[0] + \tilde{P}^{(I)} - W^{-1}\mathbf{1}\zeta\big)^\top W \times \big(P^{(I)}[0] + \tilde{P}^{(I)} - W^{-1}\mathbf{1}\zeta\big), \tag{8a}$$

$$\text{subject to } \tilde{P}^{(I)} \in \mathbb{R}^m,\ \zeta \in \mathbb{R}, \tag{8b}$$

with $\tilde{P}^{(I)}[k]$ being the estimate of $\tilde{P}^{(I)*}$, and $\xi_i[k]$ being the node i estimate of ζ^* at iteration k, where $(\tilde{P}^{(I)*}, \zeta^*)$ denotes a minimum of $\mathbf{P}$. It can be seen that $\tilde{P}^{(I)*} = P^{(I)*} - P^{(I)}[0]$, $P^{(I)*} := W^{-1}\mathbf{1}\zeta^*$, and $\zeta^* = \sum_{j=1}^m P_j^{(I)}[0]/\sum_{j=1}^m D_j$. We also note that if $\tilde{P}^{(I)}[k]$ converges to $\tilde{P}^{(I)*}$, Objectives O1 and O2 are achieved.

Problem $\mathbf{P}$ is a special case of the following unconstrained multi-agent optimization problem extensively studied in the literature (see, e.g., [3], [7]):

$$\mathbf{P}_0 : \min_{x\in\mathbb{R}^n} \phi(x) := \sum_{i=1}^n \phi_i(x), \tag{9}$$

with $x = [x_1, x_2, \ldots, x_n]^\top$, where each node i represents an agent with a local cost $\phi_i(x)$, and state x_i, and all agents collectively aim to minimize $\phi(x)$ through information exchange with neighboring nodes. Distributed algorithms for solving $\mathbf{P}_0$ (see, e.g., [3], [7]) exploit one common idea of having each node locally estimate a solution of $\mathbf{P}_0$ and, then, perform estimate averaging with its neighbors. More precisely, they solve the following modified version of $\mathbf{P}_0$:

$$\mathbf{P}_0' : \min_{x^{(i)},\forall i} \sum_{i=1}^n \phi_i(x^{(i)}), \tag{10a}$$

$$\text{subject to } x^{(i)} = x^{(j)},\ \forall i,j, \tag{10b}$$

where $x^{(i)}$ is an agent i estimate of x^*, denoting a minimum of $\mathbf{P}_0$. Once $\mathbf{P}_0'$ is solved, $x^{(i)}$ converges to x^*, $\forall i$. The algorithms for solving $\mathbf{P}_0'$ include a consensus-like step to drive all estimates to a common (consensus) point, and a subgradient step to steer this consensus point to x^*. Next, we look more closely at the problem of implementing frequency controllers via time-varying undirected and directed $\mathcal{G}^{(I)}[k]$.

A. $\mathcal{G}^{(I)}[k]$ is Time-Varying and Undirected

When $\mathcal{G}^{(I)}[k]$ is undirected, $\mathbf{P}_0'$ can be solved using the following algorithm [3]:

$$x^{(i)}[k+1] = \sum_{j=1}^n a_{ij}[k]x^{(j)}[k] - \alpha[k]\delta_i[k], \tag{11}$$

where $x^{(i)}[k] \in \mathbb{R}^n$ is the estimate of x^* at instant k, maintained at node i, $\delta_i[k]$ is a subgradient of the cost function $\phi_i(x)$ evaluated at $x = x^{(i)}[k]$, $\alpha[k] > 0$ is a stepsize, and the weights $a_{ij}[k] = a_{ji}[k] > 0$ if i and j exchanged information during time interval (t_k, t_{k+1}), and $a_{ij}[k] = a_{ji}[k] = 0$, otherwise. Here, all weights add up to 1, which is ensured choosing $a_{ii}[k] = 1 - \sum_{j\neq i} a_{ij}[k]$.

The algorithm in (11) implements the subgradient method and estimate averaging between neighboring nodes. These two features can be seen in (7), where the averaging term in (7b) ensures convergence of P_i^r/D_i's to a common (consensus) value, and the gradient terms steer $P_i^{(I)}/D_i$ to this consensus value, thereby minimizing the cost in $\mathbf{P}$.

For the considered $\mathcal{G}^{(I)}[k]$, the mixing matrix A in (6b) needs to be time-varying to handle unsuccessful information exchange between inverters; namely, we set $A_{ij}[k] = A_{ji}[k] \geq \eta$ for some constant $\eta > 0$ if inverters i and j simultaneously exchange information during time interval (t_k, t_{k+1}), and $A_{ij}[k] = A_{ji}[k] = 0$, otherwise. Also, we choose $A_{ii}[k] = 1 - \sum_{j\neq i} A_{ij}[k] \geq \eta$, $\forall i$. Then, we rewrite the system (6) as follows:

$$P^{(I)}[k+1] = P^{(I)}[k] - \alpha[k]S(WP^{(I)}[k] - \xi[k]), \quad (12a)$$

$$\xi[k+1] = A[k]\xi[k] - \alpha[k]\beta(W^{-1}\xi[k] - P^{(I)}[k]). \quad (12b)$$

In the next result, we prove the convergence of (12) to $(P^{(I)*}, \mathbf{1}\zeta^*)$, where $(P^{(I)*} - P^{(I)}[0], \zeta^*)$ is a minimum of $\mathbf{P}$. [We omit the proof due to space limitations.]

Proposition 1. *Let $\mathcal{G}^{(I)}[k]$ be undirected and assume that Assumptions 1 – 2 hold. Then, the system* (12) *trajectories converge to $(P^{(I)*}, \mathbf{1}\zeta^*)$ if $\alpha[k]$ satisfies $\sum_{k=1}^{\infty} \alpha^2[k] < \infty$ and $\sum_{k=1}^{\infty} \alpha[k] = \infty$.*

We note that if the stepsize $\alpha[k]$ is constant, then, $P^{(I)}[k]$ converges to $P^{(I)*}$ within some error, which can be made small by choosing $\alpha[k]$ small enough.

B. $\mathcal{G}^{(I)}[k]$ is Time-Varying and Directed

When $\mathcal{G}^{(I)}[k]$ is directed, we apply the subgradient-push algorithm (see, e.g., [7]) to the frequency regulation problem, and modify (12b) as follows:

$$w_i[k+1] = \sum_{j\in\mathcal{N}_i^-[k]\cup i} \frac{x_j[k]}{d_j[k]+1}, \quad (13a)$$

$$y_i[k+1] = \sum_{j\in\mathcal{N}_i^-[k]\cup i} \frac{y_j[k]}{d_j[k]+1}, \quad (13b)$$

$$\xi_i[k] = \frac{w_i[k+1]}{y_i[k+1]},\ i \in \mathcal{V}^{(I)}, \quad (13c)$$

$$x_i[k+1] = w_i[k+1] - \alpha[k]\beta(D_i\xi_i[k] - P_i^{(I)}[k]), \quad (13d)$$

where w_i, y_i, x_i are scalar variables, and $y_i[0] = 1$ for all i. Each node i transmits two variables, $\frac{x_i[k]}{d_i[k]+1}$ and $\frac{y_i[k]}{d_i[k]+1}$, to its out-neighbors. Note that the subgradient-push algorithm requires each node i to know its out-degree $d_i[k]$. In the next section, we propose a different algorithm that does not require this assumption to hold. In the next result, without loss of any generality, we assume that $|D_i\xi_i[k] - P_i^{(I)}[k]|$ is bounded, $\forall i$, since, otherwise, we can impose saturation limits on the control signal.

Proposition 2. *Let $\mathcal{G}^{(I)}[k]$ be directed and assume that Assumptions 1 – 2 hold. Then, the trajectories $(P^{(I)}[k], \xi[k])$ of the system* (12a) *and* (13) *converge to $(P^{(I)*}, \mathbf{1}\zeta^*)$ if $\alpha[k]$ is monotonically non-increasing, and satisfies $\sum_{k=1}^{\infty} \alpha^2[k] < \infty$ and $\sum_{k=1}^{\infty} \alpha[k] = \infty$.*

C. Multi-Agent Optimization Interpretations for Frequency Control

Since the sparsity structure of the matrix S does not necessarily coincide with that of the Laplacian matrix associated with $\mathcal{G}^{(I)}[k]$, it is not obvious how to interpret (12a) using the multi-agent optimization perspective. We mentioned earlier that the distributed averaging step moves agents' estimates towards the same point, and, then, this point is steered by the subgradient term towards the optimal set. In other words, estimates follow a certain average process, which converges to some optimal point. In the following, we show that the dynamics for power injections in (12a) can viewed as some average process converging to an optimal point. For this purpose, we perform the following linear transformation:

$$\tilde{P}^{(I)}[k] = cS^{\frac{1}{2}}\tilde{p}[k] \quad (14)$$

where $c := \sqrt{m}/\sqrt{\beta}$, and $\tilde{p}[k]$ evolves according to

$$\tilde{p}[k+1] = \tilde{p}[k] - \alpha[k]\frac{1}{c}S^{\frac{1}{2}}\Big(cWS^{\frac{1}{2}}\tilde{p}[k] + WP^{(I)}[0] - \xi[k]\Big). \quad (15)$$

Note that if the transformation in (14) is applied to (15), (12a) can be recovered. By substituting $\tilde{p}$ for $\tilde{P}^{(I)}$ in (8), we rewrite $\mathbf{P}$ in terms of $\tilde{p}$ and ζ as follows:

$$\mathbf{P}_1 : \min_{\tilde{p},\zeta}\ f(\tilde{p},\zeta) := \sum_{i=1}^{m} f_i(\tilde{p},\zeta), \quad (16a)$$

$$\text{subject to } \tilde{p} \in \mathbb{R}^m,\ \zeta \in \mathbb{R}, \quad (16b)$$

where $f_i(\tilde{p},\zeta) := \frac{\beta}{2D_i}\big(P_i^{(I)}[0] + c(S^{\frac{1}{2}})_i\tilde{p} - D_i\zeta\big)^2$ with $(S^{\frac{1}{2}})_i$ being the i-th row of $S^{\frac{1}{2}}$, and

$$f(\tilde{p},\zeta) := \frac{\beta}{2}\big(P^{(I)}[0] + cS^{\frac{1}{2}}\tilde{p} - W^{-1}\mathbf{1}\zeta\big)^\top W \times \big(P^{(I)}[0] + cS^{\frac{1}{2}}\tilde{p} - W^{-1}\mathbf{1}\zeta\big). \quad (17)$$

The result in Lemma 1 below establishes a linear relationship between a minimum of $\mathbf{P}_1$ and $(\tilde{P}^{(I)*}, \zeta^*)$, a minimum of $\mathbf{P}$, where we recall that $\tilde{P}^{(I)*} = P^{(I)*} - P^{(I)}[0]$ and $\zeta^* = \sum_{j=1}^{m} P_j^{(I)}[0] / \sum_{j=1}^{m} D_j$.

Lemma 1. *Let $(\tilde{p}^*, \zeta^{(1)})$ denote a minimum of $\mathbf{P}_1$. Then, the following relations hold: $\zeta^{(1)} = \zeta^*$, and $\tilde{P}^{(I)*} = cS^{\frac{1}{2}}\tilde{p}^*$.*

Clearly, $\xi_i[k]$ generated by (12b) or (13) is the estimate of ζ^* maintained by node i, and $\tilde{p}[k]$ can be viewed as the estimate of $\tilde{p}^*$ averaged among inverter nodes since (15) can be rewritten as follows:

$$\tilde{p}[k+1] = \tilde{p}[k] - \frac{\alpha[k]}{m}\sum_{i=1}^{m} \nabla_{\tilde{p}} f_i(\tilde{p}[k], \xi_i[k]), \quad (18)$$

where $\nabla_{\tilde{p}} f_i(\tilde{p},\xi_i)$ is the gradient of the node i local cost function, $f_i(\tilde{p},\xi_i)$, with respect to $\tilde{p}$. Define $\tilde{\xi}[k] := \frac{1}{m}\sum_{j=1}^{m} \xi_j[k]$. By using (12b) or (13), we have that

$$\tilde{\xi}[k+1] = \tilde{\xi}[k] - \frac{\alpha[k]}{m}\sum_{i=1}^{m} \nabla_{\xi_i} f_i(\tilde{p}[k], \xi_i[k]). \quad (19)$$

If local estimates $\xi_i[k]$ are close to the average $\tilde{\xi}[k]$, (18) – (19) can be seen as the gradient descent method for solving $\mathbf{P}_1$. Then, the goal of the distributed averaging step is to ensure that the local estimates stay close to the average.

IV. Distributed Frequency Control With Asynchronous Information Exchange

In this section, we assume $\mathcal{G}^{(I)}[k]$ is undirected, and relax the assumption that information exchange between neighboring nodes happens simultaneously. Thereby, we allow the possibility that if node i receives information from node j during time interval (t_k, t_{k+1}), node j might not receive information from node i during the same time interval.

A. Distributed Subgradient Method with Alternating Averaging

Here, we propose a method, in which, instead of having one estimate of ζ^*, node i maintains $|\mathcal{N}_i|$ local estimates with each estimate corresponding to a communication link. Via distributed averaging between i and j, the estimates corresponding to link $\{i,j\}$ are driven to a common value. Additional gradient terms are included into the gradient step to perform *local averaging*, i.e., to drive all local estimates at each node to the same value. Thus, all distributed averaging steps across the network are decoupled from each other since updates of any two estimates corresponding to the same link involve only two nodes, the ends of the link, as opposed to performing averaging across all nodes of the network. Another idea used in the proposed algorithm is to alternate averaging between neighboring nodes by using a simple message passing protocol. In other words, node i performs averaging of its estimate with the estimate of node j, and before it performs another averaging with node j, it waits for node j to perform averaging.

We arbitrarily assign an orientation to each edge in $\mathcal{E}^{(I)}_\infty$, and denote by (i,j) the edge oriented from i to j, i.e., (i,j) is incident to j. Let $\vec{\mathcal{E}}^{(I)}_\infty$ denote the set of oriented edges that result from this arbitrarily chosen orientation. Suppose $(i,j) \in \vec{\mathcal{E}}^{(I)}_\infty$. Let z_{ij} denote the estimate of ζ^* at node i corresponding to communication link $\{i,j\} \in \mathcal{E}^{(I)}[k]$; next, we explain how z_{ij} is updated. Let $\{k_l\}_{l=0}^\infty$ denote a strictly increasing scalar sequence, which describes the averaging instances between nodes i and j. Suppose node j performs averaging using node i estimate at $\{k_{2t}\}_{t=0}^\infty$, and node i performs averaging using node j estimate at $\{k_{2t+1}-1\}_{t=0}^\infty$, where $k_{2t+1}-1 > k_{2t}$, which means that nodes i and j perform averaging in alternating way. We include additional terms into gradient-like step responsible for performing local averaging as follows:

$$\begin{aligned} g_{ij}^{(i)}[k] := &\sum_{l\in\mathcal{N}_i} b_{jl}[k](z_{ij}[k]-z_{il}[k]) \\ &+ \frac{\beta}{|\mathcal{N}_i|}(D_i z_{ij}[k]-P_i^{(I)}[k]), \end{aligned}$$

in which the first gradient term is responsible for performing local averaging, and the second one for eliminating the frequency error. Then, the update iteration for z_{ji} at $k=k_{2t}$ is given by

$$z_{ji}[k+1] = (1-a_{ji})z_{ji}[k] + a_{ji}z_{ij}[k] - \alpha[k]g_{ji}^{(j)}[k], \tag{20}$$

where $0 < a_{ji} < 1$ and $0 < b_{li} < 1$ are constant and symmetric weights, i.e., $a_{ji} = a_{ij}$ and $b_{li} = b_{il}$. Here, the update (20) includes three steps: (i) averaging with neighbor i estimate, z_{ij}, (ii) the gradient term driving all node j estimates, z_{jl}'s, to a common value, and (iii) the gradient term for estimating ζ^*. The update for z_{ji} at $k \notin \{k_{2t}\}_{t=0}^\infty$ is given by

$$z_{ji}[k+1] = z_{ji}[k] - \alpha[k]g_{ji}^{(j)}[k]. \tag{21}$$

Node i estimate, z_{ij}, is updated as follows. At $k \notin \{k_{2t+1}-1\}_{t=0}^\infty$,

$$z_{ij}[k+1] = z_{ij}[k] - \alpha[k]g_{ij}^{(i)}[k], \tag{22}$$

and, otherwise, for any $t \geq 0$,

$$\begin{aligned} z_{ij}[k_{2t+1}] = &(1-a_{ij})z_{ij}[k_{2t}] + a_{ij}z_{ji}[k_{2t}] \\ &+ (z_{ij}[k_{2t+1}-1] - z_{ij}[k_{2t}]) \\ &+ \alpha[k_{2t+1}-1]g_{ij}^{(i)}[k_{2t+1}-1], \end{aligned} \tag{23}$$

which, by using (22), can also be rewritten as

$$\begin{aligned} z_{ij}[k_{2t+1}] = &(1-a_{ij})z_{ij}[k_{2t}] + a_{ij}z_{ji}[k_{2t}] \\ &- \sum_{s=0}^{\Delta k_{2t}-1} \alpha[k_{2t}+s]g_{ij}^{(i)}[k_{2t}+s], \end{aligned} \tag{24}$$

where $\Delta k_i = k_{i+1} - k_i$.

The frequency controller in (4a) will then take the following form:

$$\dot{\theta}_i = -\frac{P_i^{(I)}}{D_i} + \frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i} z_{ij}, \tag{25}$$

resulting in the following closed-loop system:

$$P^{(I)}[k+1] = P^{(I)}[k] - \alpha[k]S(WP^{(I)}[k] - \xi[k]), \tag{26a}$$

$$\xi_i[k] = \frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i} z_{ij}[k],\ i \in \mathcal{V}^{(I)}. \tag{26b}$$

Now, we present one of the main convergence results.

Proposition 3. *Let $\mathcal{G}^{(I)}[k]$ be undirected and assume that Assumptions 1 – 2 hold. Consider the scenario when the simultaneous information exchange between neighboring nodes is not guaranteed. Then, $P^{(I)}[k]$ generated by the closed-loop system* (20) – (22)*,* (26) *converges to $P^{(I)*}$ if $\alpha[k]$ satisfies $\sum_{k=1}^\infty \alpha^2[k] < \infty$, and $\sum_{k=1}^\infty \alpha[k] = \infty$.*

In the following, we introduce the message passing protocol so as to ensure that the iterations in (20) – (23) are properly executed.

B. Message Passing Protocol

Here, we propose the message passing protocol for implementing alternating averaging between neighboring nodes. For every link $\{i,j\} \in \mathcal{E}^{(I)}_\infty$, node i maintains $\nu_{ij}^{(i)}[k]$ and $\nu_{ji}^{(i)}[k]$, and additional variables $\mu_{ij}^{(i)}[k]$ and $\mu_{ji}^{(j)}[k]$ if $(j,i) \in \vec{\mathcal{E}}^{(I)}_\infty$, at iteration k, and updates them as follows:

$$\nu_{ji}^{(i)}[k] = \begin{cases} \nu_{ji}^{(j)}[k] & \text{if } (j,i) \in \mathcal{E}^{(I)}[k], \\ \nu_{ji}^{(i)}[k-1] & \text{otherwise,} \end{cases} \tag{27}$$

$$\nu_{ij}^{(i)}[k] = \begin{cases} \nu_{ij}^{(i)}[k-1] & \text{if } \nu_{ji}^{(i)}[k] = \nu_{ji}^{(i)}[k-1], \\ 1-\nu_{ij}^{(i)}[k-1] & \text{otherwise,} \end{cases} \tag{28}$$

and, if $(j,i) \in \vec{\mathcal{E}}_\infty^{(I)}$,

$$\mu_{ij}^{(i)}[k] = \begin{cases} \mu_{ij}^{(i)}[k-1] & \text{if } \nu_{ji}^{(i)}[k] = \nu_{ji}^{(i)}[k-1], \\ z_{ij}[k] & \text{otherwise,} \end{cases} \tag{29}$$

$$\mu_{ji}^{(j)}[k] = \begin{cases} \mu_{ji}^{(j)}[k-1] & \text{if } \nu_{ji}^{(i)}[k] = \nu_{ji}^{(i)}[k-1], \\ z_{ji}[k] & \text{otherwise.} \end{cases} \tag{30}$$

If $(j,i) \in \vec{\mathcal{E}}_\infty^{(I)}$, then, $\nu_{ij}^{(i)}[k]$, $\mu_{ij}^{(i)}[k]$ and $\mu_{ji}^{(j)}[k]$ comprise the information that node i needs to send to node j during time interval (t_k, t_{k+1}). Otherwise, if $(i,j) \in \vec{\mathcal{E}}_\infty^{(I)}$, then, $\nu_{ij}^{(i)}[k]$ and $z_{ij}[k]$ comprise the information sent by node i to node j. Assuming the scheme execution starts at $k=0$ and $(i,j) \in \vec{\mathcal{E}}_\infty^{(I)}$, the initial values of $\nu_{ij}^{(i)}[k]$, $\nu_{ji}^{(i)}[k]$, $\nu_{ji}^{(j)}[k]$, and $\nu_{ij}^{(j)}[k]$ need to be set to $\nu_{ij}^{(i)}[0]=1$, $\nu_{ji}^{(i)}[0]=0$, $\nu_{ji}^{(j)}[0]=0$, and $\nu_{ij}^{(j)}[0]=0$. Also, node j sets $\mu_{ij}^{(i)}[0]=0$, and $\mu_{ji}^{(j)}[0]=0$. The reason for setting $\nu_{ij}^{(i)}[0]=1$, and other variables to zero is that for each link $(i,j) \in \vec{\mathcal{E}}_\infty^{(I)}$ we choose i to be the node that initiates the execution of the message passing protocol at $k=0$.

The scheme based on the bidirectional acknowledgement in (27) – (30) synchronizes estimate averaging between each pair of neighboring nodes. In particular, $\nu_{ij}^{(i)}[k]$ assumes a role of acknowledgement flag sent by node i to node j to indicate that information from node j has been received and used to perform estimate averaging in (24). Node i initiates information exchange if $(i,j) \in \vec{\mathcal{E}}_\infty^{(I)}$ by sending the most recent value of z_{ij}. If acknowledgment flag sent by node i is new, node j averages its estimate z_{ji} with the received z_{ij}, updates $\mu_{ji}^{(j)}$, $\mu_{ij}^{(i)}$ and its acknowledgement flag, $\nu_{ji}^{(j)}$, and sends them to node i. Once a new acknowledgement flag arrives from node j, node i uses $\mu_{ij}^{(i)}$ and $\mu_{ji}^{(j)}$ to perform averaging in (24) to update z_{ij}.

By using the message passing protocol in (27) – (30), we rewrite the updates for $z_{ij}[k]$ and $z_{ji}[k]$ in (20) – (23) as shown next. Assuming $(i,j) \in \vec{\mathcal{E}}_\infty^{(I)}$, the updates for z_{ji} are given by

$$z_{ji}[k+1] = (1-a_{ji})z_{ji}[k] + a_{ji}z_{ij}[k] - \alpha[k]g_{ji}^{(j)}[k], \tag{31}$$

if $(i,j) \in \mathcal{E}^{(I)}[k]$ and $\nu_{ij}^{(j)}[k] \neq \nu_{ij}^{(j)}[k-1]$, and

$$z_{ji}[k+1] = z_{ji}[k] - \alpha[k]g_{ji}^{(j)}[k], \tag{32}$$

otherwise. The updates for z_{ij} are given by:

$$\begin{aligned} z_{ij}[k+1] = &(1-a_{ij})\mu_{ij}^{(i)}[k] + a_{ij}\mu_{ji}^{(j)}[k] \\ &+ (z_{ij}[k] - \mu_{ij}^{(i)}[k]) + \alpha[k]g_{ij}^{(i)}[k], \end{aligned} \tag{33}$$

if $(j,i) \in \mathcal{E}^{(I)}[k]$ and $\nu_{ji}^{(i)}[k] \neq \nu_{ji}^{(i)}[k-1]$, and

$$z_{ij}[k+1] = z_{ij}[k] - \alpha[k]g_{ij}^{(i)}[k], \tag{34}$$

otherwise.

V. Conclusion

In this work, we proposed frequency controllers for undirected and directed time-varying communication graphs. The controllers guarantee satisfaction of the frequency regulation and load sharing objectives provided some standard assumptions on the communication graphs hold. For undirected communication graphs, we proposed the distributed subgradient method with alternating averaging which ensures convergence when information exchange between neighboring agents is not necessarily simultaneous.

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