A Quasi-Newton Algorithm for Solving the Power Flow Problem in Inverter-Based Power Systems

T. Amuda, O. Ajala, and A. D. Domínguez-García
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Email: {tamuda2, ooajala2, aledan}@illinois.edu

***Abstract*—This paper addresses the power flow problem for electric power networks that are based on inverter-interfaced resources. We consider inverter-based resources operating in grid-forming and grid-feeding modes, and formulate a power flow model that accounts for the terminal relations of these resources. Then, we present Newton's solution method for computing the unknowns in such model, and leverage the structure of the associated iterations to formulate a Quasi-Newton algorithm. We show that such algorithm is closely related to the conventional power flow solution method that is widely adopted in the power systems literature. Numerical results are presented to compare the performance of our proposed Quasi-Newton method to that of Newton's Method.**

***Index Terms*—Grid-Forming Inverters, Power Flow Problem, Quasi-Newton Methods.**

I. Introduction

The conventional power flow problem that appears in most well-known textbooks is formulated assuming that synchronous generators are the only sources of power in the system (see, e.g., [1], p. 327-329). In this formulation, all generators but one, referred to as the slack generator, are assumed to behave as "P-V" sources, i.e., the output voltage of each generator is fixed at some value and so is the power it injects into the system. The slack generator compensates for the difference between generation and load demand and the losses in the system, and is modeled as a voltage source. Loads are described as "P-Q", i.e., the active and reactive power they consume is fixed. Then, given the active power injected by all generators but the slack bus, and the active and reactive power consumed by the loads, the problem is to calculate all remaining variables in the system, i.e., the angles at all buses in the network, voltage magnitudes at load buses, and reactive power injected by all generators (see [1], p. 327).

With the modernization of the power grid, synchronous generators are being replaced by renewable resources such as solar photovoltaic installations and wind turbine generators. These renewable-based resources are interfaced with the grid via inverters. In steady-state, the terminal behavior of such inverters is very different to that of synchronous generators, and depends on how the inverter is operated. Currently, there are two modes in which such inverter interfaces are typically operated; these are referred to as grid-following (GFL) and grid-forming (GFM) modes. When operating in GFL mode, the inverter synchronizes to the grid typically via a phase locked loop [2], and its lower-level controls cause the inverter terminals to operate as a signal controlled current source [3]. When operating in GFM mode, the lower-level controls cause the inverter terminals to operate as a signal controlled voltage source whose voltage magnitude and phase angle are regulated via a primary control scheme [3]. In this paper, we consider the three main such schemes, namely, droop [4]–[6], virtual synchronous machines [7]–[9], and dispatchable virtual oscillator control (dVOC) [10]–[12]. We focus on the power flow problem for power systems with inverter-interfaced resources, and provide numerical algorithms to solve it.

Our main contributions are a formulation of the power flow problem for electric power systems based on inverter-interfaced resources, as well as the development of a Quasi-Newton algorithm to solve such problem. One of the features of the proposed algorithm is that it enables computing the solution by solving two separate problems iteratively. The first of these problems involves solving a conventional power flow problem, whereas the second one comprises a system of linear equations. The main advantage of our proposed solution method over Newton's method is that it facilitates a seamless repurposing of conventional power flow solvers for power-flow problems involving inverter-based resources (IBRs).

To the best of our knowledge, there is no power flow problem formulation in the literature that takes into account the inverter terminal relations for GFM inverters. Nonetheless, several efforts have been made to formulate the power flow problem for systems based on IBRs. Authors in [13] performed unbalanced three-phase load-flow analysis on a low voltage (LV) distribution network with high penetration of IBRs. In [14], a three phase power flow modeling approach for LV networks with high rooftop solar PV penetration is proposed. A three phase power flow algorithm is proposed in [15] for islanded microgrids with droop-based inverters.

This paper is organized as follows. In Section II, we describe the steady-state terminal relations for IBRs, and derive expressions for their active and reactive power injections. In Section III, we utilize the expressions derived in Section II to formulate the power flow problem for electric IBR-based power systems. In Section IV, we present two iterative methods for solving the system of non-linear equations that

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constitute the power flow problem. In Section V, numerical results that compare the performance of both solution methods are presented. Concluding remarks are provided in Section VI.

II. Steady-State Models for GFM Inverters

Here, we derive models describing the steady-state terminal behavior of GFM inverters under droop, virtual synchronous machine, and dispatchable virtual oscillator control. Our starting point to derive such models is the generic primary-control model for GFM inverters proposed in [16]. Due to their behavior, GFL inverters can be modeled as negative loads in the power flow formulation; therefore we do not discuss them here.

Let ω and ω_0 respectively denote the angular frequency of the GFM inverter and the nominal frequency of the system the inverter is connected to, both measured in rad/s. Let V denote the magnitude of the inverter's terminal voltage, and E denote the magnitude of the nominal voltage, both measured in per unit. Let P and Q denote respectively the injected active and reactive power by the inverter into the external system it is connected to, both in per unit. Also, let P^g and Q^g respectively denote reference values for said active and reactive power injections, both in per unit. With the assumption that the internal inverter reactance is negligible, the steady-state behavior of the generic GFM inverter model proposed in [16] is given by

$$\begin{aligned} 0 &= (\omega_0 - \omega) + \kappa_f \mathbf{e}_1^\top T\left(\psi - \frac{\pi}{2}\right)\begin{bmatrix} P^g - P \\ Q^g - Q \end{bmatrix}, \\ 0 &= f(V) + \kappa_v \mathbf{e}_2^\top T\left(\psi - \frac{\pi}{2}\right)\begin{bmatrix} P^g - P \\ Q^g - Q \end{bmatrix}, \end{aligned} \tag{1}$$

where $\mathbf{e}_1 = [1,0]^\top$, $\mathbf{e}_2 = [0,1]^\top$, $\psi \in [0, 2\pi)$ is a rotation angle (see [10], for more details), $T(\phi)$ is a rotational matrix defined by

$$T(\phi) = \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix},$$

and κ_f [rads^{-1}], κ_v [p.u.], $f(V)$ [p.u.] represent the (inverse) frequency and voltage droop coefficients of the primary control, and voltage deviation from nominal voltage, respectively. The values of κ_f, κ_v, and $f(V)$ for droop, VSM and dVOC are given in Table I, where β and γ respectively denote the synchronization gain of the controller and voltage-amplitude control gain, $\alpha = \omega_0\beta$, and φ and ϑ are respectively frequency droop coefficient and voltage droop coefficient. Note that in steady state, droop control and VSM control have the same behavior.

TABLE I: Parameters for the Generic Primary Control Model.

Control	κ_f	κ_v	$f(V)$
droop/VSM	$\frac{1}{\varphi}$	$\frac{1}{\vartheta}$	$-V+E$
dVOC	$\frac{\alpha}{V^2}$	$\frac{\beta}{V\gamma}$	$-V^3+E^2V$

Now, by plugging the values in Table I in the expressions in (1), we obtain steady-state terminal relations as follows. The active and reactive power injected into the external system by a dVOC-based GFM inverter are respectively given by

$$\begin{aligned} P &= P^g + \frac{V^2}{\alpha}\sin(\psi)(\omega_0 - \omega) + \frac{\gamma}{\beta}\cos(\psi)(E^2 - V^2)V^2, \\ Q &= Q^g - \frac{V^2}{\alpha}\cos(\psi)(\omega_0 - \omega) + \frac{\gamma}{\beta}\sin(\psi)(E^2 - V^2)V^2. \end{aligned} \tag{2}$$

The active and reactive power injected into the external system by a droop/VSM-based GFM inverter are respectively given by

$$\begin{aligned} P &= P^g + \varphi\sin(\psi)(\omega_0 - \omega) + \vartheta\cos(\psi)(E - V), \\ Q &= Q^g - \varphi\cos(\psi)(\omega_0 - \omega) + \vartheta\sin(\psi)(E - V). \end{aligned} \tag{3}$$

With the the steady-state terminal relations in (2) – (3) imposed by GFM inverters on the buses they are connected to, we can now formulate the power flow model for inverter-based power systems.

III. Inverter-Based Power Flow Formulation

Consider, a power system with n buses, indexed by the elements in $i \in \mathcal{V} = \{1,..,n\}$, and assume that the system has l dVOC-based inverters, $(m-l)$ droop/VSM-based inverters, and $(n-m)$ constant power loads. Without loss of generality, assume that the l dVOC-based inverter are connected to buses $i = 1, ..., l$, the $m-l$ droop/VSM-based inverters are connected to buses $i = l+1, ..., m$ and the $n-m$ constant power loads are connected to buses $i = m+1, ..., n$. Let V_i [p.u.] and θ_i [rad] respectively denote the magnitude and angle of the phasor associated with the voltage at bus i, $i = 1,..,n$. Without loss of generality, assign bus 1 to be the reference bus and set $\theta_1 = 0$. Let

$$V = \left[(V^d)^\top, (V^g)^\top\right]^\top, \qquad \theta = \left[\theta_2, \ldots, \theta_n\right]^\top,$$

where $V^g = [V_1, \ldots, V_m]^\top$ and $V^d = [V_{m+1}, \ldots, V_n]^\top$, and define

$$\begin{aligned} p_i(\theta, V) &:= \sum_{j=1}^{n} V_i V_j \Big(g_{i,j}\cos(\theta_i - \theta_j) + b_{i,j}\sin(\theta_i - \theta_j)\Big), \\ q_i(\theta, V) &:= \sum_{j=1}^{n} V_i V_j \Big(g_{i,j}\sin(\theta_i - \theta_j) - b_{i,j}\cos(\theta_i - \theta_j)\Big), \end{aligned} \tag{4}$$

with g_{ij} and b_{ij} respectively denoting the real and imaginary parts of the (i,j)-entry of the network admittance matrix.

Let ω^* denote the system steady-state frequency. Let P_i^d and Q_i^d denote the active and reactive power consumed by the load connected to bus i, $i = m+1, \ldots, n$; then, by using the expressions in (2) – (3) and noting that the frequencies of all inverters should be identical in steady-state, we have that

$$\begin{aligned} P_i(\mu, V_i) &= p_i(\theta, V), \\ Q_i(\mu, V_i) &= q_i(\theta, V), \end{aligned} \tag{5}$$

where μ is the mismatch between the system nominal frequency and the steady-state frequency defined as $\mu := \omega_0 - \omega^\star$,

$$J(x_k) = \begin{bmatrix} \frac{\partial \widetilde{p}_1(x)}{\partial \mu}\Big|_{x_k} & \frac{\partial p_1(\theta,V)}{\partial \theta}\Big|_{\theta_k,V_k} & \frac{\partial p_1(\theta,V)}{\partial V^d}\Big|_{\theta_k,V_k} & \frac{\partial \widetilde{p}_1(x)}{\partial V^g}\Big|_{x_k} \\ \frac{\partial \widetilde{p}^g(x)}{\partial \mu}\Big|_{x_k} & \frac{\partial p^g(\theta,V)}{\partial \theta}\Big|_{\theta_k,V_k} & \frac{\partial p^g(\theta,V)}{\partial V^d}\Big|_{\theta_k,V_k} & \frac{\partial \widetilde{p}^g(x)}{\partial V^g}\Big|_{x_k} \\ \frac{\partial \widetilde{p}^d(x)}{\partial \mu}\Big|_{x_k} & \frac{\partial p^d(\theta,V)}{\partial \theta}\Big|_{\theta_k,V_k} & \frac{\partial p^d(\theta,V)}{\partial V^d}\Big|_{\theta_k,V_k} & \frac{\partial \widetilde{p}^d(x)}{\partial V^g}\Big|_{x_k} \\ \frac{\partial \widetilde{q}^g(x)}{\partial \mu}\Big|_{x_k} & \frac{\partial q^g(\theta,V)}{\partial \theta}\Big|_{\theta_k,V_k} & \frac{\partial q^g(\theta,V)}{\partial V^d}\Big|_{\theta_k,V_k} & \frac{\partial \widetilde{q}^g(x)}{\partial V^g}\Big|_{x_k} \\ \frac{\partial \widetilde{q}^d(x)}{\partial \mu}\Big|_{x_k} & \frac{\partial q^d(\theta,V)}{\partial \theta}\Big|_{\theta_k,V_k} & \frac{\partial q^d(\theta,V)}{\partial V^d}\Big|_{\theta_k,V_k} & \frac{\partial \widetilde{q}^d(x)}{\partial V^g}\Big|_{x_k} \end{bmatrix}.$$

Fig. 1: Structure of the Jacobian matrix of $f(x)$ at $x = x_k$.

$$P_i(\mu, V_i) = \begin{cases} P_i^g + \frac{V_i^2}{\alpha_i}\sin(\psi)\mu + \frac{\gamma_i}{\beta_i}\cos(\psi)(E_i^2 - V_i^2)V_i^2, \\ \qquad \text{for} \quad i \in \{1, 2, \dots, l\}, \\ P_i^g + \varphi_i \sin(\psi)\mu + \vartheta_i \cos(\psi)(E_i - V_i), \\ \qquad \text{for} \quad i \in \{l+1, \dots, m\}, \\ -P_i^d, \qquad \text{for} \quad i \in \{m+1, \dots, n\}, \end{cases} \tag{6}$$

and

$$Q_i(\mu, V_i) = \begin{cases} Q_i^g - \frac{V_i^2}{\alpha_i}\cos(\psi)\mu + \frac{\gamma_i}{\beta_i}\sin(\psi)(E_i^2 - V_i^2)V_i^2, \\ \qquad \text{for} \quad i \in \{1, 2, \dots, l\}, \\ Q_i^g - \varphi_i \cos(\psi)\mu + \vartheta_i \sin(\psi)(E_i - V_i), \\ \qquad \text{for} \quad i \in \{l+1, \dots, m\}, \\ -Q_i^d, \qquad \text{for} \quad i \in \{m+1, \dots, n\}. \end{cases} \tag{7}$$

Note that in (5) – (7), there are $2n$ equations and $2n$ unknowns, namely μ, $\theta_2, \dots, \theta_n$ and $V_1, V_2, \dots, V_n$.

Note that the reactance of the transmission lines depend on the system steady-state frequency, ω^*; hence the $b_{i,j}$'s in (4) should be a function of $\mu = \omega_0 - \omega^*$. However, we assume that μ is small enough so that changes in line reactance values are negligible and therefore we assume the $b_{i,j}$'s are constant (their values are computed using ω_0).

IV. Numerical Solution Methods

First, we formulate Newton method to solve for the value of $x := [\mu, \theta^\top, V^\top]^\top$ that satisfies (5), and use it afterwards as the basis for formulating a Quasi-Netwon algorithm.

A. Newton Method

First, let

$$\begin{aligned} p^g(\theta, V) &= [p_2(\theta, V), \dots, p_m(\theta, V)]^\top, \\ p^d(\theta, V) &= [p_{m+1}(\theta, V), \dots, p_n(\theta, V)]^\top, \\ q^g(\theta, V) &= [q_1(\theta, V), \dots, q_m(\theta, V)]^\top, \\ q^d(\theta, V) &= [q_{m+1}(\theta, V), \dots, q_n(\theta, V)]^\top, \\ P^g(\mu, V^g) &= [P_2(\mu, V_2), \dots, P_m(\mu, V_m)]^\top, \\ Q^g(\mu, V^g) &= [Q_1(\mu, V_1), \dots, Q_m(\mu, V_m)]^\top, \\ P^d &= [P_{m+1}^d, \dots, P_n^d]^\top, \\ Q^d &= [Q_{m+1}^d, \dots, Q_n^d]^\top, \end{aligned}$$

and define

$$f(x) := \begin{bmatrix} \widetilde{p}_1(x) \\ \widetilde{p}^g(x) \\ \widetilde{p}^d(x) \\ \widetilde{q}^g(x) \\ \widetilde{q}^d(x) \end{bmatrix}, \tag{8}$$

where

$$\widetilde{p}_1(x) := p_1(\theta, V) - P_1(\mu, V_1), \tag{9a}$$
$$\widetilde{p}^g(x) := p^g(\theta, V) - P^g(\mu, V^g), \tag{9b}$$
$$\widetilde{p}^d(x) := p^d(\theta, V) + P^d, \tag{9c}$$
$$\widetilde{q}^g(x) := q^g(\theta, V) - Q^g(\mu, V^g), \tag{9d}$$
$$\widetilde{q}^d(x) := q^d(\theta, V) + Q^d; \tag{9e}$$

we refer to $f(\cdot)$ as the *mismatch function* for the power flow model in (5) – (7).

Newton's method for computing the solution to the power flow model in (5) – (7) can be formulated as follows:

$$J(x_k)\Delta x_k = -f(x_k), \tag{10}$$
$$x_{k+1} = x_k + \Delta x_k, \quad k = 0, 1, 2, \dots, \tag{11}$$

with $\mu_0 = 0$, $\theta_0 = \mathbf{0}_{n-1}$, and $V_0 = \mathbf{1}_n$, where $\mathbf{0}_{n-1}$ denotes the $(n-1)$-dimensional all-zeros vector and $\mathbf{1}_n$ denotes the n-dimensional all-ones vector, and $J(x_k)$ denoting the Jacobian matrix of $f(x)$ evaluated at $x_k := [\mu_k, \theta_k^\top, V_k^\top]^\top$. The structure of $J(x_k)$ is given in Fig. 1.

B. Quasi-Newton Method

Next, we develop a Quasi-Newton algorithm, which essentially decouple the problem of computing the solution into two separate problems. An advantage of this alternative algorithm over Newton method is that it enables repurposing standard power flow solvers implemented in commercial packages to solve the power flow problem associated with inverter-based power systems.

$$L(x_k) = \begin{bmatrix} \frac{\partial \widetilde{p}_1(x)}{\partial \mu}\big|_{x_k} & \frac{\partial p_1(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k} & \frac{\partial p_1(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k} & \mathbf{0}_m^\top \\ \mathbf{0}_{m-1} & \frac{\partial p^g(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k} & \frac{\partial p^g(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k} & \mathbf{0}_{(m-1)\times m} \\ \mathbf{0}_{n-m} & \frac{\partial p^d(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k} & \frac{\partial p^d(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k} & \mathbf{0}_{(n-m)\times m} \\ \mathbf{0}_{n-m} & \frac{\partial q^d(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k} & \frac{\partial q^d(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k} & \mathbf{0}_{(n-m)\times m} \\ \mathbf{0}_m & \mathbf{0}_{m\times(n-1)} & \mathbf{0}_{m\times(n-m)} & \frac{\partial \widetilde{q}^g(x)}{\partial V^g}\big|_{x_k} \end{bmatrix}.$$

Fig. 2: Matrix associated with the proposed Quasi-Newton algorithm.

The first step in formulating our proposed Quasi-Newton algorithm is to reorder the columns of $f(x_k)$ and define a new mismatch function, $g(\cdot)$, as follows:

$$g(x_k) := \begin{bmatrix} \widetilde{p}_1(x_k) \\ \widetilde{p}^g(x_k) \\ \widetilde{p}^d(x_k) \\ \widetilde{q}^d(x_k) \\ \widetilde{q}^g(x_k) \end{bmatrix}.$$

Next, we reorder the blocks in $J(x_k)$ to be consistent with $g(x_k)$ and set some of the entries of the resulting matrix to zero. In particular, we set to zero the blocks $\frac{\partial \widetilde{p}^d(x)}{\partial \mu}\big|_{x_k}$ and $\frac{\partial \widetilde{q}^d(x)}{\partial \mu}\big|_{x_k}$ corresponding to buses with constant power loads. Also, the entries of $\frac{\partial \widetilde{q}^g(x)}{\partial \mu}\big|_{x_k}$ are small for inductive power system (see [10]); therefore we set this block to zero. Then, using the same argument as that used when developing the standard decoupled power flow algorithm, we set to zero the following blocks: $\frac{\partial \widetilde{p}_1(x)}{\partial V^g}\big|_{x_k}$, $\frac{\partial \widetilde{p}^g(x)}{\partial V^g}\big|_{x_k}$, and, $\frac{\partial q^g(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k}$, as the entries of these matrices are very small compared to those of the other entries of the Jacobian matrix (see e.g., [17]). Similarly, the entries of $\frac{\partial \widetilde{p}^g(x)}{\partial \mu}\big|_{x_k}$, $\frac{\partial \widetilde{q}^d(x)}{\partial V^g}\big|_{x_k}$ and $\frac{\partial q^g(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k}$ are small; therefore, we set these blocks to zero as well. The matrix that results from this process, which we denote by $L(x_k)$, is given in Fig. 2.

We can now use $L(x_k)$ and $g(x_k)$ to formulate the following Quasi-Newton algorithm:

$$L(x_k)\Delta x_k = -g(x_k), \tag{12a}$$
$$x_{k+1} = x_k + s\Delta x_k, \tag{12b}$$

with $\mu_0 = 0$, $\theta_0 = \mathbf{0}_{n-1}$, and $V_0 = \mathbf{1}_n$, and $s \in (0,1]$. Now, because of the sparsity structure of $L(x_k)$, we can rewrite (12a) as follows

$$\Delta\mu_k = -\frac{\widetilde{p}_1(x_k) + \frac{\partial p_1(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k}\Delta\theta_k + \frac{\partial p_1(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k}\Delta V_k^d}{\frac{\partial \widetilde{p}_1(x)}{\partial \mu}\big|_{x_k}}, \tag{13}$$

$$\underbrace{\begin{bmatrix} \frac{\partial p^g(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k} & \frac{\partial p^g(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k} \\ \frac{\partial p^d(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k} & \frac{\partial p^d(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k} \\ \frac{\partial q^d(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k} & \frac{\partial q^d(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k} \end{bmatrix}}_{=:M(x_k)} \begin{bmatrix} \Delta\theta_k \\ \Delta V_k^d \end{bmatrix} = -\begin{bmatrix} \widetilde{p}^g(x_k) \\ \widetilde{p}^d(x_k) \\ \widetilde{q}^d(x_k) \end{bmatrix}, \tag{14}$$

$$\frac{\partial \widetilde{q}^g(x)}{\partial V^g}\big|_{x_k}\Delta V_k^g = -\widetilde{q}^g(x_k). \tag{15}$$

In (13), note that if the Quasi-Newton algorithm converges, $\Delta\mu_k \to 0$ as $k \to \infty$.

Note that the matrix

$$M(x_k) = \begin{bmatrix} \frac{\partial p^g(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k} & \frac{\partial p^g(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k} \\ \frac{\partial p^d(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k} & \frac{\partial p^d(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k} \\ \frac{\partial q^d(\theta,V)}{\partial \theta}\big|_{\theta_k,V_k} & \frac{\partial q^d(\theta,V)}{\partial V^d}\big|_{\theta_k,V_k} \end{bmatrix}$$

is the Jacobian matrix for the standard power flow problem (see, e.g., [1]); thus, the expression in (14) coincides with that of Newton algorithm for the conventional power flow problem. Therefore, this portion of the proposed Quasi-Newton algorithm can be implemented using a commercial power flow solver.

The proposed Quasi-Newton algorithm can be implemented as follows. First, at each iteration k, compute $\Delta\theta_k$ and ΔV_k^d from (14). Then, compute $\Delta\mu_k$ and ΔV_k^g concurrently in (13) and (15) respectively; see Algorithm 1 for the pseudocode of such implementation.

V. Numerical Results

In this section, we present simulation results for both the standard Newton method and the proposed Quasi-Newton method. The two algorithms are tested on modified versions of the IEEE-14 bus test system [18], IEEE-57 bus test system [19], and IEEE-300 bus test system [20]. The five synchronous generators in the IEEE-14 bus test system are replaced by three GFM dVOC inverters and two GFM droop/VSM inverters. The

Algorithm 1: Implementation of the Quasi-Newton algorithm

initialize $\mu_0 = 0$, $\theta_0 = \mathbf{0}_{n-1}$, $V_0 = \mathbf{1}_n$, $\varepsilon = \infty$, and $k = 0$;
set s and tolerance tol;
while $\varepsilon > tol$ **do**
  compute $\Delta\theta_k$ and ΔV_k^d using (14);
  compute $\Delta\mu_k$ in (13) and ΔV_k^g using (15) ;
  compute x_{k+1} using (12b);
  $\varepsilon = \|g(x_k)\|_\infty$;
  $k = k + 1$.
end

TABLE II: Simulation parameters.

Symbol	Value
φ	0.8038 s/rad
ϑ	25 p.u.
β	0.0033 p.u.
γ	0.0796 p.u.
ψ	$\pi/2$ rad

seven synchronous generators in the IEEE-57 bus test system are replaced by four GFM dVOC inverters and three GFM droop/VSM inverters. The sixty-nine synchronous generators in the IEEE-300 bus test system are replaced by thirty-five GFM dVOC inverters and thirty-four GFM droop/VSM inverters. Parameters in Table II from [21] are used in all simulations.

TABLE III: Comparison of iteration number and execution time for Newton method and Quasi-Newton method for 10^{-4} error tolerance.

# of buses	Newton		Quasi-Newton	
	Iteration #	time [s]	Iteration #	time [s]
14	6	0.0165	25	0.0179
57	5	0.2164	32	0.4641
300	6	8.574	29	16.231

Total convergence time as shown in Table III for the Quasi-Newton method is approximately twice the Newton method; however, the time per iteration for the Quasi-Newton method is much less. Commercial power flow solvers can be exploited to solve equation (14) of the Quasi-Newton algorithm. A traditional power network has more loads connected than generators, hence the greater part of the floating-point operations of the Quasi-Newton algorithm is in equation (14), which can be solved using commercial power flow solvers. Since commercial power flow solvers are optimized to perform faster than regular solvers implemented on ordinary computer systems, this will reduce the time to convergence of the algorithm.

VI. Concluding Remarks

This paper presents a power flow problem formulation for inverter-based power systems. The steady-state behavior of GFM inverters control strategies are used to set the terminal constraints on the bus variables. Solution methods to the problem were discussed and numerical results illustrating the solution methods were presented.

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